MILNE'S
NEW YORK STATE
ARITHMETIC

SECOND BOOK

WITHOUT ANSWERS

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NEW YORK STATE
ARITHMETIC

SECOND BOOK

BY

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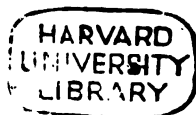
PREFACE

THIS series of arithmetics has been written to supply the demand for a sound and modern interpretation of present-day educational ideas concerning the teaching of arithmetic. It is intended especially to meet the need for a definite and complete interpretation of the Course of Study in Arithmetic prescribed by the Education Department of the State of New York.

The series consists of two books. The "First Book" for years three, four, and five, is divided into half-year units, but within each half year the treatment is topical. The "Second Book" is a complete topical arithmetic, covering, in order, the Syllabus requirements for years six, seven, and eight.

The first two chapters of the "Second Book" are designed to gather up in a compact and comprehensive review so much of the work of the "First Book" as the Syllabus does not mention again in the last three years of the course. The needs of any class about to enter upon the sixth year work will determine whether or not any or all of this part of the book should be used. The next two chapters are review chapters included by the Syllabus in the sixth year.

Certain topics not mentioned in the Syllabus are treated where they should logically appear in the body of the book for the benefit of those schools desiring to teach them. They are so arranged, however, that their omission will not impair the sequence of the Syllabus course. Other topics are given in the Appendix.

The following features of the book are worthy of attention :
1. The logical, common-sense method of presentation.

2. The thorough, clear-cut explanations. These give help where it is most needed by illustrating the more difficult rather than the easier instances in each case.

3. The easy steps in gradation, which carry the work at every stage to some complete result.

4. The definiteness of the instruction, which leaves nothing to embarrass teachers and pupils.

5. The exceptionally large number of oral and written exercises for practice.

6. The nature of the problems. They are practical, clearly stated, and adapted to the child's experience. Industrial problems relating to commerce, manufacture, agriculture, and household and manual arts are abundant.

7. The importance placed on correct interpretation of problems and on choice of the most economical method of solution.

8. The up-to-date character of the subject matter, which includes problems suggested by recent legislation in the state and nation, such as the Workmen's Compensation Act, the Federal Income Tax, Federal Reserve Banks, Postal Savings Banks, the Parcel Post, and the latest and most practical business usages, business forms, and commercial paper.

9. The systematic reviews, both topical and miscellaneous.

10. The training for efficiency provided by short methods, exercises in mental estimating, and numerous tests and time drills.

<div align="right">WILLIAM J. MILNE.</div>

CONTENTS

Chapters and topics that are starred are not required by the Syllabus.

INTRODUCTORY CHAPTERS

(Pages 9–78)

SIXTH YEAR

(Pages 79–240)

5

6 CONTENTS

CONTENTS 7

8   CONTENTS

REVIEW OF
INTEGERS AND DECIMALS

NOTATION AND NUMERATION

THE present system of arithmetic is the result of growth through many centuries. Primitive men counted on their fingers, calling *five* "one hand" and *ten* "two hands" or the "double hand." When it became necessary to count more than ten, they used their toes also, and called *twenty* "one man." In earliest times men kept track of their count by marks in the sand or by sticks and pebbles ; but as civilization advanced, a method providing for a more permanent record of numbers became necessary. Because of the primitive habit of counting on the fingers, most of the ancient nations used a system of counting by groups of ten, but the symbols representing the numbers and the methods of using them varied greatly.

It was hundreds of years before the figures now commonly used were introduced. At first there was no symbol like our zero to stand for an unoccupied place ; for there was no notion of place value, which is the real basis of our system. It was not until the zero was used to fill vacant places that the system became useful for difficult computations.

The system in present use had its origin with the Hindus ; but it took its name from the Arabs who, having become acquainted with it, were instrumental in introducing it into Europe. For a long time the Arabs were thought to be its originators.

9

Any one thing is a **unit**.
One, a book, a dollar, a gallon are units.

A unit or a collection of units is a **number**.

Numbers are represented by characters, as *figures* or *letters*, and are expressed by *words*.

Any method of representing numbers by characters is **notation**.

The notation by *figures*, in common use, is the **Arabic notation**, sometimes called the *Indian*, or *Hindu notation*.

For certain purposes we use a notation by *letters*, the **Roman notation**, named from the ancient Romans who used it.

Any method of naming, or reading, numbers represented by characters is **numeration**.

The method commonly used in America is the *French numeration*.

ARABIC SYSTEM

In counting a large number of objects it is natural to arrange them in equal groups. The system of grouping that is most commonly used is that of grouping by *tens*, the **decimal** system, from the Latin word *decem* meaning *ten*.

In such a system 10 units make a group, 10 of these groups make a larger group, 10 of these larger groups make the next larger group, and so on.

The Arabic system of notation is a decimal system. It represents numbers by means of these ten figures:

one	two	three	four	five	six	seven	eight	nine	naught
1	2	3	4	5	6	7	8	9	0

Naught, called also **zero** and **cipher**, represents no value and is used to fill vacant places. The others are *significant* figures, because each represents a value.

In this system a group of ten is **one ten**, written 10; a group of 10 tens is **one hundred**, written 100; a group of 10 hundreds is **one thousand**, written 1000; etc.

In the number 1111, the first 1 beginning at the right stands for 1 unit ; the others in order toward the left represent 1 ten, 1 hundred, and 1 thousand.

The decimal system may be extended indefinitely to the left or to the right. Before extending it to the right, a dot, or **decimal point,** is placed after the figure that stands for units.

In 1111.111, each 1 stands for 10 times as much as the next 1 toward the right, the first 1 following the decimal point stands for 1 of the 10 equal parts of a unit, one tenth ; the second for 1 of the 10 equal parts of one tenth, or 1 of the 100 equal parts of a unit, one hundredth ; the third for one thousandth ; etc.

What does each 3 in 33.333 represent? each 5 in 555.55 ?

All figures at the left of the decimal point represent whole units, or **integers.**

A decimal point is assumed to follow every integer.

All figures at the right of the decimal point represent parts of units, or **decimals.**

A decimal point is placed before every decimal.

A number expressed by an integer and a decimal is a **mixed decimal.**

Figures in *units'* place express *units of the first order ;* in *tens'* place, *units of the second order ;* in *hundreds'* place, *units of the third order ;* etc.

The decimal orders are numbered *first, second, third,* etc., from the decimal point toward the right.

For convenience in writing and reading large numbers, the figures are separated by commas into groups of three figures each, called **periods.**

The highest or left-hand period may contain less than three figures.

The first period of integers, counting from the right, is used to denote any number from 1 *unit* to 999 *units ;* the second period, any number from 1 *thousand* to 999 *thousand ;* the third period, any number from 1 *million* to 999 *million ;* etc.

The general arrangement of the *orders* of *units* and their separation into *periods* is here illustrated:

Orders: (Ten-trillions, Trillions, Hundred-billions, Ten-billions, Billions, Hundred-millions, Ten-millions, Millions, Hundred-thousands, Ten-thousands, Thousands, Hundreds, Tens, Units)

3 7 , 2 4 0 , 0 8 0 , 0 0 0 , 4 3 8

Periods: TRILLIONS BILLIONS MILLIONS THOUSANDS UNITS

This number is read, " 37 trillion, 240 billion, 80 million, 438."

Each period beginning at the left is read with its name as if it stood alone, except that a period of 0's is passed over and the name of the units' period is omitted.

The word " and " is not used between periods.

Oral Exercises

Read the following:

1. 471	7. 2492	13. 66,813	19. 97,020,001
2. 806	8. 9008	14. 16,010	20. 40,007,082
3. 6345	9. 20,721	15. 780,062	21. 299,009,009
4. 2061	10. 92,007	16. 400,600	22. 643,250,699
5. 43,834	11. 865,990	17. 6,445,386	23. 3,754,168,260
6. 45,045	12. 530,005	18. 1,496,300	24. 8,766,776,677

25. The area of New York state is 49,170 square miles ; recently its population was 9,113,279.

In a certain year the Panama Canal Commissary imported :

26. Milk, 57,016 gallons   30. Eggs, 692,060 dozen

27. Cream, 29,950 gallons   31. Poultry, 554,028 pounds

28. Butter, 429,267 pounds   32. Meats, 9,241,858 pounds

29. Cheese, 110,415 pounds   33. Vegetables, 8,787,140 pounds

Copy, point off into periods, and write in words :

1. 14039	**5.** 873002	**9.** 9625148	**13.** 504008010
2. 90074	**6.** 290564	**10.** 8000001	**14.** 647101566
3. 34008	**7.** 601006	**11.** 25051206	**15.** 1348613259
4. 99965	**8.** 430830	**12.** 34584383	**16.** 5007829345

Express by figures :

17. Twelve thousand, two hundred sixty-five.

18. One hundred eleven thousand, one hundred six.

19. Four hundred ten thousand, one hundred forty.

20. Six hundred thousand, seven hundred twenty-one.

21. Five hundred eight ; nine thousand twenty-two.

22. One thousand two hundred two ; six thousand six.

23. Two thousand forty-two ; ten thousand sixty-five.

24. Eight hundred nine ; two thousand one hundred six.

25. Twelve thousand, six ; six million, one hundred one.

26. Five thousand ten , twenty-five thousand, eighty-two.

27. Forty-two million , sixty-one million, five thousand.

28. Ninety-six billion ; ten million, forty thousand, two.

29. Seventy-three billion, eighty-three ; eight trillion.

30. Five million, twenty thousand, one hundred eight; forty-eight million, three thousand, five hundred.

31. One billion, one hundred twenty million, two hundred fifty thousand, five; two hundred million, two hundred three.

32. Twelve billion, twelve million, twelve thousand, twelve; five hundred two million, eight hundred seventy-five.

33. Three trillion, three billion, three million, three thousand, three ; nine million, nine.

This table shows *decimal orders* and their relation to *integral orders:*

Hundreds	Tens	Units	(Decimal Point)	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
3	6	5	.	7	4	3	2	9	8

This number is read " 365 *and* 743,298 *millionths.*"

The orders below millionths are ten-millionths, hundred-millionths, billionths, ten-billionths, etc.   They are seldom used.

A decimal should be read as an integer, with the denomination of the right-hand figure added.

The word "and" is used between the integral and decimal parts of a mixed decimal and not elsewhere.

The fundamental principle of the decimal system is :
Ten units of any order make one unit of the next higher order.

Oral Exercises

Read:

1. 2.3	10. 17.4	19. 84.26	28. 521.09	37. 406.175
2. .45	11. 8.25	20. 174.5	29. 1.6734	38. 5172.86
3. 8.6	12. .267	21. .6397	30. 3452.8	39. 82.3742
4. .09	13. 5.36	22. 8.484	31. .27673	40. .092406
5. .67	14. .058	23. .0351	32. 49.036	41. 36187.4
6. 4.1	15. 68.8	24. .0079	33. 6.4902	42. 7.04329
7. .84	16. .009	25. 56.28	34. .05647	43. .283674
8. .07	17. 4.12	26. 4.936	35. 74.325	44. .000003
9. 6.8	18. .831	27. 851.2	36. .00098	45. 4.82589

1. Express decimally 54 thousandths.

Since 54 thousandths is equal to 5 hundredths and 4 thousandths, 4 is written in thousandths' place, 5 in hundredths' place, and since there are no tenths, 0 is written in tenths' place. A decimal point is then placed before the tenths. Hence, 54 thousandths = .054.

Express as a decimal or as a mixed decimal:

2. Two tenths; seven tenths; twelve hundredths.

3. 5 hundredths; 3 and 5 tenths; 16 and 9 tenths.

4. Eleven thousandths; eight and four hundredths.

5. Sixty-five hundredths; two and twelve thousandths.

6. Six and two tenths; one and twenty-five hundredths.

7. 25 and 3112 ten-thousandths; 18 and 18 thousandths.

8. 8 hundredths; 125 thousandths; 246 ten-thousandths.

9. Fifty-one and fifteen thousandths; eight thousandths.

10. 15 and 14 ten-thousandths; 1675 hundred-thousandths.

11. 400 and 49 thousandths; 7 and 5 hundred-thousandths.

12. 34 and 81 hundred-thousandths; 2 and 148,202 millionths.

13. Ten and five thousand one hundred seventeen ten-thousandths; seven hundred nine millionths.

14. Sixty-five ten-thousandths; one hundred fifteen and two hundred thirty-five hundred-thousandths.

15. Nine and one hundred fifty-five millionths; ninety-nine thousand nine hundred ninety-nine hundred-thousandths.

16. Eighty-nine millionths; five thousand one hundred sixty-nine and two thousand one hundred six millionths.

Write in words:

17. 92.27   **19.** 6.0725   **21.** .00008   **23.** 407.349

18. 4.002   **20.** .02164   **22.** 295.54   **24.** .640003

In the money of the United States:

10 mills = 1 cent	10 dimes = 1 dollar
10 cents = 1 dime	100 cents = 1 dollar

The **dollar sign** is $; the sign for cent or cents is ¢.

Since 10 units of one denomination make 1 unit of the next higher denomination, United States money is conveniently represented in figures by the decimal system, dimes and cents being written as hundredths, and mills as thousandths of a *dollar*, which is the unit of the system.

Thus, 46 dollars and 78 cents is written $46.78.

Also, $.01 is read. "1 cent"; $.001, "1 mill"; $.256, "25 cents 6 mills"; 25¢, "25 cents"; and 56.7¢, "56 and 7 tenths cents."

Oral and Written Exercises

Read as dollars and cents or as dollars, cents, and mills:

1. $2.50	6. $10.05	11. $25.25	16. $4.125	21. $36.214
2. $1.76	7. $12.38	12. $32.07	17. $8.253	22. $53.052
3. $3.84	8. $14.06	13. $40.65	18. $7.045	23. $75.258
4. $7.42	9. $18.67	14. $50.01	19. $5.102	24. $28.043
5. $8.03	10. $20.02	15. $26.93	20. $9.008	25. $96.077

Write, using the sign $:

26. 25¢	31. 2 dollars and 1 cent	36. 10 cents 1 mill
27. 50¢	32. 5 dollars and 4 cents	37. 24 cents 5 mills
28. 12¢	33. 3 dollars and 9 cents	38. 35 cents 8 mills
29. 28¢	34. 8 dollars and 2 cents	39. 67 cents 3 mills
30. 37¢	35. 6 dollars and 5 cents	40. 19 cents 4 mills

The Roman notation uses seven letters, namely:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

The letters are combined according to these principles:

1. *When a letter is followed by the same letter or a letter of less value, the values of the two letters are united.*

Thus, II represents 2; XXX, 30; VI, 6; DC, 600.
Also XII stands for 10 + 2, or 12.

2. *When a letter is followed by a letter of greater value, the value of the less is taken from that of the greater.*

Thus, IV represents 4; IX, 9; XL, 40; CD, 400.
Just as XII stands for 10 + 2, or 12, so XIX stands for 10 + 9, or 19.

3. *A letter with a bar placed over it represents a thousand times as much as it does without the bar.*

Thus, $\overline{V}$ represents 5000; $\overline{X}$, 10,000; $\overline{IV}$, 4000.

This table further illustrates the method of combination:

III, 3	XIV, 14	XLI, 41	XCIX, 99	MC, 1100
VII, 7	LIX, 59	LXX, 70	DCCC, 800	$\overline{XX}$, 20,000

Oral and Written Exercises

Read:

1. $\overline{M}$	4. LXXIX	7. CCV	10. MDXC
2. XCIX	5. CXVIII	8. DXIX	11. MMDC
3. DLXXV	6. CDLXIII	9. DCXL	12. $\overline{IV}$CCXL

Express in the Roman notation:

13. 55	16. 61	19. 101	22. 496	25. 1607
14. 66	17. 58	20. 114	23. 509	26. 6000
15. 79	18. 97	21. 325	24. 1900	27. 1914

ADDITION

You have learned how to *add*, *subtract*, *multiply*, and *divide*, but you should practice on these operations until you can perform them **rapidly** and with **absolute accuracy.**

The process of uniting two or more numbers into one number is **addition.**

The result obtained by adding is the **sum.**

The numbers added are **addends.**

The sign **+** indicates *addition* and is read **plus;** the sign **=** indicates *equality* and is read **equals,** or **is equal to.**

Numbers that are made up of the same kind of units are **like numbers.**

$4 and $6 are like numbers; also 8 feet and 5 feet.

Numbers that are made up of different kinds of units are **unlike numbers.**

7 pounds and 9 gallons are unlike numbers.

1. Can you add $4 and $6?   7 pounds and 9 gallons?

Only like numbers can be added.

2. What is the sum of 6 and 2? of 2 and 6? of 3, 4, and 2? of 2, 4, and 3? of 4, 3, and 2?

The sum of two or more numbers is the same in whatever order they are added.

Addition by groups.

(1) (2)   You have learned to add as in (1), naming re-
3   3 ⎫ sults, thus, "9, 15, 20, 23, 27, 30."
4   4 ⎪   You can add much more rapidly, if you learn to
8   3 ⎬ catch the sums of *groups* of numbers, reading up the
5   5 ⎪ column as in (2), thus, "9, 20, 30."
6   6 ⎭   NOTE. — Where possible, group numbers whose sum is 10
9   9   or 11, as these sums are more easily added, but by practice
——   ——   you will be able to use groups whose sums are larger.
30   30

Written Exercises

Add upward and *test* each answer by adding downward:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
5	7	6	2	7	9	8	2	5	4	3	7
6	4	3	8	1	6	3	4	8	5	9	8
7	9	9	2	3	2	5	7	1	6	5	9
3	8	5	7	2	4	7	9	6	7	8	6
5	2	4	1	5	6	3	1	3	2	3	4

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
3	5	4	6	8	6	5	6	4	6	5	7
7	6	5	2	2	6	8	8	3	4	8	9
9	2	3	7	9	3	4	3	3	8	7	3
4	7	8	5	5	8	7	9	8	2	6	1
6	9	1	3	7	6	2	4	7	5	4	5

25.	26.	27.	28.	29.	30.	31.	32.	33.	34.	35.	36.
5	3	8	5	8	5	8	8	3	5	4	8
6	8	9	4	3	7	3	8	9	4	3	8
8	1	7	6	9	6	2	7	2	3	8	9
7	9	6	8	2	9	9	7	1	7	5	9
9	3	8	7	1	8	5	6	8	6	7	4
2	5	4	3	9	4	1	6	2	8	9	4

37.	38.	39.	40.	41.	42.	43.	44.	45.	46.	47.	48.
7	4	3	4	9	1	5	9	8	5	4	4
4	5	2	3	2	2	5	8	1	6	4	9
9	6	3	4	3	3	5	4	4	8	5	1
3	4	5	3	7	4	6	5	6	4	3	8
8	6	4	7	1	5	6	3	5	3	8	2
6	8	7	6	5	6	6	1	7	9	7	9
5	3	6	1	4	7	7	2	3	6	6	3

Written Exercises

Add and test:

	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
$a.$	7	8	3	6	4	5	1	2	9	8	4	6
$b.$	6	5	4	6	3	8	7	5	4	7	3	5
$c.$	3	9	6	5	4	1	9	3	2	3	5	4
$d.$	8	1	6	3	4	3	6	7	8	2	8	8
$e.$	5	4	9	3	8	9	6	1	7	1	7	4
$f.$	8	3	1	9	4	2	·2	6	4	4	1	3
$g.$	4	1	3	2	6	8	8	3	5	6	6	7
$h.$	2	7	9	1	8	5	7	3	6	3	4	9
$i.$	9	1	6	8	4	9	3	2	1	8	3	6
$j.$	8	3	4	2	6	8	5	6	9	7	7	4

	13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
$k.$	3	5	4	3	3	8	4	6	5	7	3	·5
$l.$	4	2	2	9	6	5	2	9	7	8	5	8
$m.$	2	9	5	5	4	3	8	4	2	9	6	3
$n.$	8	8	8	7	5	7	2	6	3	9	1	7
$o.$	7	3	2	5	1	8	9	2	4	5	7	6
$p.$	6	4	6	8	2	8	3	6	5	7	6	9
$q.$	9	5	3	1	5	2	4	5	8	2	7	5
$r.$	5	3	9	7	7	8	3	4	2	6	5	8
$s.$	2	8	4	4	1	8	5	4	3	1	4	4
$t.$	3	1	8	2	7	9	2	6	5	4	2	2
$u.$	1	7	2	3	5	4	3	9	8	7	8	3

Copy in a column and add the numbers in row:

25. a	28. d	31. g	34. j	37. m	40. p	43. s
26. b	29. e	32. h	35. k	38. n	41. q	44. t
27. c	30. f	33. i	36. l	39. o	42. r	45. u

1. Add 23, 12, 34, and 57.

23
12
34
57
———
126

The numbers are written so that units of the same order are in a column, and the columns are added separately.

Adding the units, we find that the sum is 16; 16 means 1 ten and 6 units; therefore, we write 6 under the units and add 1 ten to the tens.

Adding the tens, we find there are 12 tens. We write 12 under the tens. The sum of 23, 12, 34, and 57 is 126.

Add quickly:

	2.	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**	**11.**
a.	72	73	84	57	27	21	96	20	14	48
b.	85	97	17	46	72	53	81	38	32	71
c.	90	16	52	93	18	38	45	47	27	65
d.	36	68	78	16	19	73	78	93	55	89

	12.	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**	**21.**
e.	39	54	89	28	59	25	45	68	56	20
f.	29	90	24	77	83	27	17	39	28	74
g.	72	71	74	50	17	48	18	74	47	85
h.	45	29	93	33	26	16	13	56	94	68
i.	13	36	35	86	28	36	14	23	89	49

	22.	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**	**31.**
j.	45	62	42	65	43	68	56	42	38	46
k.	58	87	24	89	69	42	49	46	67	80
l.	34	49	85	38	22	25	17	34	79	58
m.	22	36	48	76	85	34	21	58	54	95
n.	67	55	34	45	74	28	36	91	83	78

Copy and find the sum of the numbers in row :

32. *a*   **34.** *c*   **36.** *e*   **38.** *g*   **40.** *i*   **42.** *k*   **44.** *m*

33. *b*   **35.** *d*   **37.** *f*   **39.** *h*   **41.** *j*   **43.** *l*   **45.** *n*

Written Exercises

Add and test:

1.	267	2.	$4.09	3.	29.63	4.	6.423
	538		7.23		78.04		9.275
	496		5.86		89.12		4.368
	1301		$17.18		196.79		20.066

Decimal expressions may easily be written with units of the same order in a column by writing them with the decimal points in a column. The decimal point is then written in the sum directly under the other decimal points.

5.	324	6.	$7.35	7.	2071	8.	42.09	9.	16.423
	163		2.09		4683		35.85		53.694
	747		6.41		8939		94.21		7.538
	508		4.83		1746		58.34		85.045

10.	287	11.	$5.15	12.	3768	13.	18.26	14.	20.67
	412		1.87		4691		63.97		99.807
	865		6.58		9057		57.84		37.258
	374		.93		2846		86.56		15.5
	191		8.60		5924		24.63		64.722

15.	568	16.	$6.49	17.	4036	18.	5.473	19.	36.418
	796		4.87		7985		1.849		5.98
	175		.72		873		9.76		88.765
	347		3.08		8437		4.524		27.078
	634		7.56		2764		8.475		56.823

20.	247	21.	$4.25	22.	3247	23.	.6524	24.	70.362
	682		1.76		8575		.5896		8.947
	537		8.58		436		.3637		59.258
	396		5.82		4898		.9064		3.164
	463		6.09		7623		.2452		45.723
	856		9.61		1054		.4349		96.671

Add and test:

25.	26.	27.	28.	29.	30.
457	6.09	5872	7.436	53.739	9.1758
386	8.97	7709	9.807	78.964	6.8047
773	4.87	4568	3.695	34.678	7.9694
238	5.48	8316	8.563	67.841	3.0823
842	6.76	6647	5.748	12.085	8.7162
564	7.65	2354	8.354	86.547	4.6719

Add by columns; then by rows:

	31.	32.	33.	34.	35.	36.	37.	38.
39.	5.24	+ 27.6	+ .031	+ 42.78	+ 169.3	+ 3.705	+ .0243	+ 85.637
40.	4.09	+ 56.7	+ .468	+ 67.49	+ 820.9	+ 7.849	+ .9706	+ 59.386
41.	7.95	+ 84.4	+ .875	+ 18.56	+ 637.8	+ 5.437	+ .6897	+ 74.568
42.	9.86	+ 78.9	+ .509	+ 83.47	+ 748.7	+ 3.218	+ .4154	+ 27.875
43.	8.57	+ 63.3	+ .786	+ 29.85	+ 695.4	+ 9.536	+ .8963	+ 96.794

* Add and test, timing yourself on each exercise:

44.	45.	46.	47.	48.	49.
3206	54.93	63,745	45.109	279.435	70.6398
7483	81.69	70,924	82.782	724.568	68.3456
4875	65.48	38,268	76.834	437.049	16.4789
6138	77.77	85,607	57.963	893.775	87.7647
8767	48.35	47,382	98.645	935.856	94.8574
9692	32.56	72,479	63.272	548.583	55.9043
5564	93.84	54,833	59.597	687.937	23.5874
9345	10.23	91,862	37.346	306.376	79.5965

* In all time exercises throughout the book, it is intended that teachers shall keep the records of all the pupils, and a few days later give the same exercises again with the idea of having the pupils beat their previous records.

Written Exercises

1. A poultry dealer sold 4800 eggs in January, 2160 eggs in February, and 6708 eggs in March. How many eggs did he sell in these three months?

2. A lady had a hat made, costing as follows: frame, $.25; plume, $5; ribbon, $.75; velvet, $2.25; buckle, $1; and making, $1. How much did she spend for the hat?

3. Westport is 127.78 miles north of Albany and 63.13 miles south of Rouses Point, on a railroad joining these two places. How far is it from Albany to Rouses Point?

4. A steamship carried, on one trip, 467 first cabin passengers, 300 second cabin passengers, 300 steerage passengers, and a crew of 550 men. Find the number of people on board.

5. How many acres are there in a farm, if there are 48.25 acres of grain, 40.3 acres of meadow, and 48.7 acres of pasture?

6. A Boy Scout party on a tramp walked 9.5 miles the first day, 10.25 miles the second, 9.75 miles the third, 10.375 miles the fourth, and 12.125 miles the fifth. How far did they walk?

7. A traction company employs 951 conductors and motormen, 42 switchmen, 76 track cleaners, 543 common laborers, and 56 officials. How many employees has it?

8. A cow's butter fat record in pounds, for five years, was: 240.5, 339, 209.9, 297.6, and 275.1, respectively. Find the total yield of butter fat during this time.

9. A man bought the following articles to furnish his office: desk, $28.50; desk chair, $5.75; typewriter, $85; table, $12; and chairs, $25.50. Find the cost of all.

10. How many rods of fence are needed to inclose a field whose sides are 27.5 rods, 84.125 rods, 46.08 rods, 62.504 rods, and 18.32 rods long, respectively?

Drill Exercises

Add and test :

	1.	2.	3.	4.	5.	6.
a.	$5.14	$6.50	$14.85	$20.48	$50.25	$125.40
b.	3.78	2.89	8.13	.98	17.50	56.36
c.	7.06	.75	12.64	16.10	9.48	150.89
d.	9.12	8.04	28.12	9.42	61.72	225.75
e.	4.25	5.47	80.05	32.50	45.84	85.25

Copy and find the sum of the numbers in row :

7. *a*   **8.** *b*   **9.** *c*   **10.** *d*   **11.** *e*

Add and test, timing yourself on each exercise :

12.	13.	14.	15.	16.
42,653	27,435	16,736	75,886	83,109
81,515	41,203	40,275	40,187	45,731
27,294	84,546	31,014	38,265	23,015
16,248	28,711	27,901	25,726	87,893
11,612	84,552	63,283	53,105	45,637
73,253	73,216	15,436	44,682	58,448
46,210	30,481	51,602	37,426	29,705
43,824	17,104	36,584	21,109	83,892
25,276	81,023	44,351	63,584	35,219
62,504	53,665	12,348	92,435	42,753
31,243	31,704	60,095	18,046	28,465
15,435	28,567	59,643	45,912	53,234
30,903	41,523	43,526	37,234	64,582
41,006	62,048	35,400	61,392	43,819
57,384	57,632	27,389	50,018	70,081
34,825	23,456	67,556	37,603	59,524
72,403	81,007	43,213	21,547	32,635
14,912	19,285	51,781	73,214	47,387
54,045	78,642	80,425	48,902	64,924
46,266	35,480	78,309	54,974	75,293

SUBTRACTION

The process of finding how much greater one number is than another, or the process of finding what number added to one of two numbers will give the other, is **subtraction**.

The number from which another is subtracted is the **minuend**.

The number subtracted is the **subtrahend**.

The result obtained by subtracting is the **difference**, or **remainder**.

The sign — indicates *subtraction.* It is read **less** or **minus**.

1. Can you subtract $7 from $10 ?   8 feet from 12 gallons ?

The minuend and the subtrahend must be like numbers.

2. Find the difference between 7 and 5; $7 + 3$ and $5 + 3$; $7 - 2$ and $5 - 2$.   How do these differences compare ?

Increasing or decreasing the minuend and the subtrahend by the same number does not change the difference or remainder.

Oral Exercises

What number added to the lower one gives the upper one ?

1. 6	**2.** 5	**3.** 9	**4.** 14	**5.** 13	**6.** 14	**7.** 16
3	2	4	7	8	5	8

How much greater is the upper number than the lower one ?

8. 9	**9.** 7	**10.** 8	**11.** 12	**12.** 11	**13.** 16	**14.** 12
3	2	6	5	4	9	4

Subtract the lower number from the upper one :

15. 7	**16.** 9	**17.** 8	**18.** 15	**19.** 13	**20.** 17	**21.** 14
3	2	1	7	6	9	8

22. Point out the minuend in exercise **7**; the subtrahend in exercise **8**.

1. From 498 subtract 165.

AUSTRIAN, OR "ADDITION," METHOD

By this method the object is to find what number *added* to the subtrahend gives the minuend.

As no arithmetical number added to 5 gives 3, enough is added to 5 to give 10 + 3, or 13. The minuend is thereby increased by 10, but in order that the remainder shall not be changed (p. 26, 2), the subtrahend must be increased by the same number, which is done by "carrying" 1 ten to the 6 tens, making it 7 tens. Thus:

498	For the units, 5 and 8 are 13; write 8.
165	For the tens, 7 and 2 are 9; write 2.
328	For the hundreds, 1 and 3 are 4; write 3.

The remainder is 328.

COMMON, OR "TAKING-AWAY," METHOD

As 5 units cannot be taken from 3 units, take 1 ten (= 10 units) from the 9 tens to unite with the 3 units, making 13 units. This changes the 9 tens in the minuend to 8 tens. Thus:

498	For the units, 5 from 13, 8; write 8.
165	For the tens, 6 from 8, 2; write 2.
328	For the hundreds, 1 from 4, 3; write 3.

The remainder is 328.

NOTE. — The teacher is advised not to confuse pupils by teaching both of these methods to the same class. The method that is preferred should be adopted and followed.

Subtract, and *test* the result by adding it to the subtrahend; the sum thus obtained should be equal to the minuend:

2. 374	3. 578	4. 680	5. 486	6. 891	7. 906
236	182	432	237	384	584

8. 529	9. 634	10. 758	11. 878	12. 980	13. 408
372	241	129	489	348	276

14. 765	15. 920	16. 853	17. 642	18. 754	19. 987
189	546	456	268	467	389

Subtract and test:

1. 4328	2. $24.37	3. 5.175	4. 8000	5. $60.00
1459	13.94	2.437	2376	28.54
2869	$10.43	2.738	5624	$31.46

In subtracting decimal expressions, write the subtrahend under the minuend so that the decimal points are in a column. Write the decimal point in the remainder directly under the other decimal points.

In subtracting a decimal from an integer, 0's may be assumed to occupy decimal places after the integer.

6. 647	11. $6.15	16. 4700	21. 50.39	26. 7586.1
466	2.37	2590	35.42	1438.5

7. 721	12. $3.81	17. 2035	22. 81.09	27. 627.91
345	.49	527	45.56	47.63

8. 502	13. $8.00	18. 3106	23. 24.25	28. 80.047
467	3.45	1247	5.45	32.564

9. 940	14. $7.12	19. 6000	24. 50	29. 4.0235
759	5.14	2428	42.24	2.2457

10. 734	15. $.543	20. 8435	25. 42.6	30. 17
78	.447	968	15.75	5.125

31–35. From 89.24 subtract: 48; 5.36; .237; 29.6; 32.68.

36–40. From 275 subtract: 5.18; .09; 29.4; .725; 82.74.

Find answers and test, timing yourself:

41. $3.48 - 2.39$	47. $24.51 - 16.6$	53. $346.8 - 27.45$
42. $5.09 - 3.25$	48. $5.342 - .275$	54. $5.203 - 1.426$
43. $75.1 - 43.4$	49. $498.7 - 8.87$	55. $82.15 - 5.368$
44. $42.9 - 9.38$	50. $65.02 - .329$	56. $6.014 - .3789$
45. $63.7 - 2.45$	51. $.8414 - .518$	57. $700.8 - 4.223$
46. $8.36 - .687$	52. $7.035 - 4.27$	58. $92.16 - .0914$

Written Exercises

1. Seneca Lake is 618 feet deep. If it is 183 feet deeper than Cayuga Lake, how deep is Cayuga Lake?

2. Find the difference between the cost per ton of mining coal by hand labor at 78.8¢, and by machine labor at 64¢.

3. George Washington was born in the year 1732 and died in 1799. How old was he when he died?

4. A man who had a farm of 120.75 acres sold 44.37 acres of it. How many acres had he left?

5. America was discovered by Columbus in the year 1492. How many years have passed since that event?

6. Sterling silver is made of pure silver and copper. If .025 of it is copper, what part is pure silver?

7. Hook-and-ladder truck No. 1 weighs 9655 pounds. Engine No. 1 weighs 7985 pounds. Find the difference in weight between them.

8. In order to catch a train a man had to drive 14.75 miles in two hours. If he drove 7.125 miles the first hour, how many miles did he drive the second hour?

9. By efficient management the cost of excavation on the Panama Canal was reduced from 11.5¢ to 8.88¢ per cubic yard. How much was saved per cubic yard?

10. One year 9620 books were published in this country, of which 695 were reprints. How many new books appeared?

11. The world's record cow recently increased her butter record for a hundred days from 452 pounds to 467.23 pounds. Find the gain.

12. The heaviest bell in the world weighs 432,000 pounds. How much heavier is it than the bell in the City Hall, New York, which weighs 22,500 pounds?

13. From the sum of 58.9 and 2.65, subtract their difference.

14. A large water tank when filled weighs 72,000 tons. If the water alone weighs 48,000 tons, what is the weight of the tank?

15. After a man had traveled 429 miles on a 1000-mile mileage book, how many mileage coupons had he left?

16. *Dairymaid* and *Jedetta* are Guernsey cows. One year Dairymaid gave 14,562.4 pounds of milk and Jedetta, 15,109.1 pounds. How much more milk did Jedetta give than Dairymaid?

17. A locomotive and tender weigh 596,000 pounds. If the tender weighs 170,000 pounds, what is the weight of the locomotive?

18. The weight of an English oak beam when wet was 972.25 pounds and when seasoned 630.5 pounds. How many pounds was the weight reduced by seasoning?

19. If 13,362 school children are enrolled in a certain city and 6693 of them are boys, how many girls are enrolled?

20. From thirty-four and two hundred forty-five thousandths subtract eighteen and nine hundredths.

21. The sum of two numbers is 3,042,801 and one of them is 2,400,037. What is the other number?

22. A boy could jump 3.25 feet high. After practicing a month he found that he could clear the bar when it was 4.125 feet above the ground. How much did he gain by practice?

23. The Panama Canal reduces the water route between New York and San Francisco from 13,400 miles to 5300 miles. How many miles is the route reduced?

24. The German mark is worth $.2385, the French franc $.193, and the English shilling $.2434. How much less than a quarter of a dollar is each of these foreign coins worth?

Subtract and test, timing yourself on the second column :

1. 412 − 81	15. 7596 − 189	29. 39,456 − 3167
2. 541 − 38	16. 8493 − 576	30. 54,831 − 2759
3. 767 − 99	17. 5684 − 839	31. 78,759 − 3586
4. 584 − 85	18. 6897 − 928	32. 92,813 − 6468
5. 295 − 76	19. 3152 − 425	33. 53,773 − 5864
6. 734 − 49	20. 4253 − 864	34. 92,884 − 9271
7. 420 − 57	21. 6845 − 986	35. 14,638 − 8397
8. 913 − 426	22. 7324 − 5659	36. 54,579 − 28,312
9. 835 − 341	23. 4427 − 3588	37. 65,721 − 56,325
10. 786 − 453	24. 6678 − 3199	38. 94,219 − 49,836
11. 913 − 746	25. 8634 − 7824	39. 81,848 − 43,914
12. 583 − 279	26. 5885 − 3527	40. 73,857 − 18,476
13. 468 − 349	27. 9346 − 7752	41. 93,762 − 59,318
14. 592 − 176	28. 7148 − 2956	42. 81,935 − 46,853

Subtract from 90.12 :

43. 5.4	47. 44.2	51. .341	55. 60.04	59. 2.753
44. .38	48. 37.8	52. 84.4	56. 5.391	60. 42.76
45. .69	49. .815	53. 41.9	57. 36.51	61. .5347
46. 9.8	50. 7.63	54. 9.06	58. 1.859	62. 67.38

63–82. Subtract the numbers in exercises 43–62 from 200.

Subtract from $50.25 :

83. $.79	87. $4.87	91. $5.93	95. $11.18	99. $3.965
84. $.83	88. $2.16	92. $3.78	96. $18.46	100. $5.456
85. $.58	89. $7.26	93. $9.57	97. $23.14	101. $8.018
86. $.89	90. $3.09	94. $8.34	98. $31.98	102. $9.235

103–122. Subtract the numbers in exercises 83–102 from $100.

MULTIPLICATION

The process of taking one number as many times as there are units in another is **multiplication.**

The number multiplied is the **multiplicand.**

The number by which we multiply is the **multiplier.**

The result found by multiplying is the **product.** The multiplicand and multiplier are **factors** of the product.

The sign × indicates *multiplication.* It is read **times** when it precedes the multiplicand, and **multiplied by** when it follows the multiplicand.

Thus, 3 × 5 is read "3 times 5," if 5 is the multiplicand, but "3 multiplied by 5," if 3 is the multiplicand.

The pupil should know the multiplication tables perfectly.

NOTE. — The complete tables are given for reference on page 500.

Oral Exercises

Tell products instantly by rows and by columns:

	6.	**7.**	**8.**	**9.**	**10.**
1.	3 × 7	3 × 3	4 × 7	5 × 6	7 × 9
2.	4 × 6	6 × 2	3 × 9	7 × 5	6 × 3
3.	5 × 3	4 × 3	7 × 6	6 × 6	8 × 6
4.	4 × 4	7 × 2	8 × 3	5 × 9	9 × 4
5.	8 × 2	8 × 5	9 × 2	8 × 4	8 × 8
	16.	**17.**	**18.**	**19.**	**20.**
11.	9 × 6	8 × 10	10 × 5	11 × 9	10 × 11
12.	7 × 8	5 × 11	11 × 3	10 × 7	11 × 12
13.	7 × 7	4 × 12	12 × 6	12 × 8	11 × 11
14.	8 × 9	8 × 11	10 × 9	11 × 6	10 × 12
15.	9 × 9	8 × 12	12 × 7	12 × 9	12 × 12

A number that is used with reference to some particular kind of object or unit is a **concrete number**.

6 pears, $12, and 40 miles are concrete numbers.

A number that is used without reference to any particular kind of object or unit is an **abstract number**.

6, 12, and 40 are abstract numbers.

1. How many dollars are 4 times $5? $5 multiplied by 4?

2. Can you find $5 times 4? Can you multiply 4 by $5?

The multiplier must be an abstract number or be regarded as abstract.

Complete and read:

3. 8 × 3 pounds = ?   **5.** 3 pounds × 8 = ?

4. 7 × 6 bushels = ?   **6.** 6 bushels × 7 = ?

7. How does the name of the unit in the product compare with the name of the unit in the multiplicand?

The product and the multiplicand are like numbers.

8. How does 5 times 11 compare with 11 times 5? 2 times 17 with 17 times 2? 3 times 21 with 21 times 3?

The product is numerically the same whichever number is regarded as the multiplier.

In practice it is generally more convenient to use the smaller number as the multiplier.

Oral Exercises

Read and complete the following. Name the multiplicand. the multiplier, and the product. Describe each as concrete or abstract.

1. 3 × 7 days =   **4.** 5 pecks × 11 =   **7.** 2 × 16 ounces =

2. 4 feet × 5 =   **5.** 10 × 8 quarts =   **8.** 4 quarts × 13 =

3. 6 × 2 pints =   **6.** 12 inches × 9 =   **9.** 20 × 12 hours =

MULTIPLICATION BY INTEGERS

Written Exercises

1. Multiply 954 by 8.

Write the multiplier under the multiplicand and multiply the units, tens, and so on in order, carrying as in addition. Thus:

954   For the units, $8 \times 4 = 32$; write **2**.
8   For the tens, $8 \times 5 = 40$; $40 + 3 = 43$; write **3**.
———   For the hundreds, $8 \times 9 = 72$; $72 + 4 = 76$; write **76**.
7632   The product is 7632.

Find the product:

2. 5×46	**7.** 4×98	**12.** 415×7	**17.** 3245×4
3. 6×37	**8.** 3×69	**13.** 372×8	**18.** 2861×5
4. 7×28	**9.** 8×58	**14.** 846×5	**19.** 3942×3
5. 6×93	**10.** 9×43	**15.** 364×4	**20.** 5687×6
6. 7×86	**11.** 6×85	**16.** 918×3	**21.** 3834×8

Multiply:

22. 378	**23.** $5.62	**24.** 3.75	**25.** 21.25	**26.** 7.345
11	4	5	8	4
4158	$22.48	18.75	170.00	29.380

Multiply by 11 and 12 just as by single-figure multipliers.

Decimal expressions are multiplied as shown above, the decimal point being written in the product directly under that in the multiplicand.

Zeros at the right of a decimal may be *omitted*.

Multiply by 4:

27. 287	**30.** $3.46	**33.** 2648	**36.** $15.34	**39.** 68,423
28. 493	**31.** $5.29	**34.** 5732	**37.** $34.02	**40.** 43,768
29. 789	**32.** $8.97	**35.** 8917	**38.** $76.38	**41.** 94,089

42–176. Multiply each of the numbers in exercises **27–41** by 3; by 2; by 5; by 7; by 9; by 6; by 8; by 11; by 12.

Find the cost of :

1. 8 watermelons at $.65 each.

2. 6 pounds of ham at $.31 a pound.

3. 12 pounds of butter at $.42 a pound.

4. 16 boxes of Kiefer pears at $2 per box.

5. 5 barrels of flour at $6.75 per barrel.

6. 9 barrels of apples at $3.85 per barrel.

7. 34 pairs of canvasback ducks at $2 per pair.

8. A mining company owns 25 ore cars with a capacity of 6 tons each.   Find the total capacity of the cars.

9. A square garden 95 feet on each side is inclosed by a fence.   How long is the fence?

10. If a fall of snow averaged .597 of an inch per hour, how many inches of snow fell in 8 hours?

11. Find the total yield from an 11-acre field of oats, averaging 36 bushels per acre.

12. If a laborer breaks 1.75 cubic yards of stone per day, how many cubic yards of stone does he break in 12 days?

13. A certain railroad annually uses 1725 tons of coal to dry sand.   Find the cost of this coal at $2 per ton.

14. Mr. Allen's salary is $87.50 per month.   Find his salary per year.

15. With a reaper and binder a man harvested 6.375 acres of grain per day.   At that rate how many acres would he harvest in 8 days?

16. There are 5280 feet in a mile.   How many feet are there in 6 miles?

17. A contractor built 8 houses at a cost of $7325 each. How much did they all cost ?

1. Every cipher that is annexed to an integer moves each figure one place toward the left, and therefore (page 11) changes the units to tens, the tens to hundreds, and so on. Hence,

· *To multiply an integer by* 10, *annex one* 0; *by* 100, *two* 0's; *by* 1000, *three* 0's; *etc.*

2. In a decimal expression, every removal of the decimal point one place toward the right corresponds to moving each figure one place toward the left. Hence,

To multiply a decimal by 10, *move the decimal point toward the right one place; by* 100, *two places; by* 1000, *three places; etc.*

Thus, $10 \times 5.25 = 52.5$; $100 \times 5.25 = 525$. If necessary, annex 0's, as in $1000 \times 5.25 = 5250$.

Written Exercises

1–6. Multiply by 10: 64; 352; $9.50; .33; .024; 29.8.

7–12. Multiply by 100 : 81; $.16; .302; 62.7; 5.41; 306.

13–18. Multiply by 1000: 27; 934; 3.706; $2.60; $.009; 3.8.

19–23. Multiply by 10,000 : 45; $.193; 4.4; .304; 6.49.

Multiply :

24.	46	**25.**	9.875
	80		300
	3680		2962.5

Since 80 equals 8×10, we multiply by 8 and then by 10; that is, we multiply by 8 and annex *one* 0.

We multiply by 3, then by 100; that is, we multiply by 3 and move the decimal point *two* places to the right.

26.	34 by 20	**32.**	$.08 by 30	**38.**	397.8 by 600
27.	68 by 50	**33.**	$.38 by 90	**39.**	17.75 by 400
28.	49 by 70	**34.**	$2.20 by 50	**40.**	5.328 by 300
29.	77 by 600	**35.**	$4.35 by 40	**41.**	.7061 by 800
30.	89 by 800	**36.**	$.348 by 20	**42.**	21.54 by 7000
31.	93 by 700	**37.**	$.076 by 60	**43.**	9.009 by 9000

1. Find the cost of 10 horses at $275 each.

2. How many dozen eggs are there in 25 cases of 30 dozen each?

3. The wear on the granite pavement of a certain bridge for a year was .22 of an inch. Find the wear for 20 years.

4. A case of cotton lace consisted of 1000 cards each containing 6 yards. How many yards of lace were there in the case?

5. A farmer sold 30 tons of hay at $18.25 per ton. How much did he receive for it?

6. A bushel of shelled corn weighs 56 pounds. Find the weight of 800 bushels.

7. A barrel of flour weighs 196 pounds. How many pounds of flour are required to fill 40 barrels?

8. If anthracite coal is .94 carbon, how much carbon do 2000 pounds of anthracite coal contain?

9. A machine for sharpening drills strikes 1600 blows per minute. How many blows does it strike per hour?

10. From a 20-acre field of barley, the yield was 48.25 bushels per acre. Find the total yield.

11. A field yielded 50 bales of cotton averaging 496 pounds each. How many pounds of cotton did it yield?

12. A paper company uses 30 tons of coal per day. Find the cost of the coal used per week of 6 days at $2.50 per ton.

13. A box contained 100 cases of needles each holding 10 bundles of 25 needles. How many needles were there in the box?

14. If it takes 10.5 cubic feet of natural gas to produce as much heat as 1 pound of coal, how many cubic feet of gas are required to give as much heat as 1 ton (2000 pounds) of coal?

Written Exercises

1. Multiply 836 by 428 and test the result.

	(1)	(2)	TEST
	836	836	428
	428	428	836
8 times 836 =	6688	6688	2568
20 times 836 =	16720	1672	1284
400 times 836 =	334400	3344	3424
428 times 836 =	357808	357808	357808

The full partial products are 6688, 16720, and 334400 as shown in (1), but in practice they are written with the ciphers omitted as in (2).

Observe from (2), that the *right-hand figure of each partial product is written directly under the figure of the multiplier used to obtain it.*

Test. — Multiplying the multiplier by the multiplicand, you find the product to be the same as in (2). The result, therefore, is probably correct.

Find the product and test :

2. 25 × 16   **8.** 19 × 358   **14.** 124 × 433   **20.** 2146 × 358

3. 37 × 54   **9.** 35 × 261   **15.** 463 × 725   **21.** 3921 × 426

4. 45 × 62   **10.** 52 × 735   **16.** 693 × 852   **22.** 5438 × 237

5. 76 × 85   **11.** 71 × 943   **17.** 836 × 574   **23.** 7146 × 785

6. 69 × 73   **12.** 86 × 852   **18.** 789 × 694   **24.** 9732 × 846

7. 84 × 97   **13.** 94 × 687   **19.** 919 × 425   **25.** 8817 × 967

26.	$27.64	**27.**	68.95	**28.**	24.68
	56		407		4800
	165 84		482 65		19744
	1382 0		27580		9872
	$1547.84		28062.65		118464

Dollar signs and decimal points are omitted from partial products.

In exercise **27**, the partial product by 0 (tens), which is 0, is not written.

In exercise **28**, first multiply by 48 and then by 100 by moving the decimal point two places toward the right.

Multiply, timing yourself on the second column:

29. $4.25 by 18	**39.** 37.2 by 28	**49.** 579.6 by 348
30. $3.76 by 24	**40.** .209 by 33	**50.** 4.837 by 805
31. $5.08 by 43	**41.** 4.82 by 65	**51.** .8406 by 786
32. $8.16 by 37	**42.** .896 by 720	**52.** 95.47 by 972
33. $7.39 by 58	**43.** 9.32 by 490	**53.** 2.536 by 609
34. $9.56 by 89	**44.** 2.893 by 18	**54.** 43.82 by 4100
35. $3.89 by 680	**45.** 308.5 by 46	**55.** 367.5 by 2007
36. $7.62 by 708	**46.** 61.48 by 390	**56.** .7061 by 8600
37. $5.40 by 236	**47.** 78.23 by 506	**57.** 3.875 by 2360
38. $8.94 by 475	**48.** .8745 by 738	**58.** 89.64 by 9023

59. A tract of 15 acres of land yielded an average of 42 tons of sugar cane per acre. Find the total yield.

60. Find the amount of red clover seed required to seed 16 acres of land, .25 of a bushel being used per acre.

61. How much must a harness dealer pay for 36 mohair carriage robes at $22 each?

62. It costs $1.22 a word to cable from New York to Hongkong. How much does it cost to send a message of 27 words?

63. A merchant bought 75 pieces of muslin, each containing 39 yards. How many yards of muslin did he buy?

64. The daily pay of a locomotive engineer was $4.10. What was his salary for a year of 313 working days?

65. When the wind blows with a velocity of eight miles per hour it exerts a pressure of .32 of a pound per square foot. Find the pressure on 64 square feet.

66. If 2.002 acres of land are required to grow the crops for the maintenance of 1 cow, how many acres are required for the maintenance of 25 cows?

Find the cost of:

67. 14 lawn mowers at $3.25 each.

68. 20 tons of coal at $6.75 per ton.

69. 109 acres of land at $124 per acre.

70. 15 sewing machines at $21.50 each.

71. 40 boxes of laundry soap at $3.19 each.

72. 150 gallons of sirup at $1.10 per gallon.

73. 24 224-pound bags of salt at $1.74 each.

74. 250 5-pound cans of coffee at $1.64 each.

75. 16 barrels of oatmeal at $5.52 per barrel.

76. 36 100-pound sacks of sugar at $5.25 each.

77. 206 40-quart cans of milk at $1.75 per can.

78. A barrel of salt weighs 280 pounds. Find the weight of 25 barrels of salt.

79. How many pounds of metal are needed to mold 375 bullets weighing .313 of a pound each?

80. If 1 ton of coal produces 1500 pounds of coke, what amount of coke may be obtained from 275 tons of coal?

81. A slate roof weighs 6.75 pounds per square foot. Find the weight of 864 square feet of this roof.

82. Find the weight of 794 bales of raw silk, weighing 220 pounds each.

83. The ties for an electric railway cost $1584 per mile of track. Find the cost of the ties for 208 miles of track.

84. A farm horse was fed an average daily ration of 17.125 pounds of alfalfa hay and 3.075 pounds of oats. Find the amount of each required for 14 days.

85. A paper mill contained 14 paper-making machines each of which made 1.32 tons of paper per hour. Find the output for a day of 24 hours.

MULTIPLICATION BY DECIMALS

Since **10** units of any order make **1** unit of the next higher order, **1** unit of any order is 1 tenth of a unit of the next higher order, or it is a unit of the next higher order multiplied by .1.

Hence, every removal of a figure one place toward the right, or of the decimal point one place toward the left, changes tens to units, units to tenths, tenths to hundredths, etc.   That is,

To multiply an integer or a decimal by .1 move the decimal point toward the left one place; by .01, two places; by .001, three places; etc.

If necessary, prefix 0's; thus, .001 × 2.5 = .0025.

Written Exercises

When doing exercises 1-20, compare the number of decimal places in each product with the number in the multiplicand *plus* the number in the multiplier.

1–6. Multiply by .1: 7; 45; 3.6; 8.2; .6; .75.

7–11. Multiply by .01: 56; 7492; 87.5; 8.75; .432.

12–16. Multiply by .001: 965; 18,609; 75; 6.25; .4.

17–20. Multiply by .0001: 2387; 6381.6; 8.46; 40.36.

The number of decimal places in the product is equal to the number in both multiplicand and multiplier.

21. Multiply 235 by .3.   **22.** Multiply 74.2 by .07.

$$
\begin{array}{r} 235 \\ .3 \\ \hline 70.5 \end{array}
\qquad
\begin{array}{r} 74.2 \\ .07 \\ \hline 5.194 \end{array}
$$

Since .3 equals 3 × .1, multiply by 3 and then by .1; or, multiply as if the numbers were integers and point off 0 + 1, or 1, decimal place.

Since .07 = 7 × .01, multiply by 7 and then by .01; or, multiply as if the numbers were integers and point off 1 + 2, or 3, decimal places.

Multiply as if the numbers were integers, and from the right of the product point off as many decimal places as there are decimal places in both multiplicand and multiplier.

Multiply:

23. 84 by .6	**27.** $2.35 by .5	**31.** 3.24 by .8
24. 79 by .05	**28.** $3.48 by .7	**32.** 60.8 by .04
25. 56 by .08	**29.** $7.67 by .6	**33.** .596 by .09
26. 48 by .009	**30.** $5.89 by .8	**34.** 74.8 by .007

35. Multiply .4365 by .058.

    .4365   The product has *four* decimal places for the decimal in
     .058   the multiplicand, and *three* more for the decimal in the
    34920   multiplier, or *seven* decimal places, a 0 being prefixed to give
    21825   enough places.   After the seven decimal places are pointed
   .0253170   off, the 0 at the right is omitted.

Multiply and test, timing yourself on the second column:

36. 84 by 7.5	**45.** 342 by 8.2	**54.** 246.5 by 10.2
37. 21 by .38	**46.** 224 by 4.5	**55.** 38.75 by 8.04
38. 96 by .65	**47.** 365 by .32	**56.** 720.6 by .035
39. 64 by 2.7	**48.** 221 by 1.8	**57.** 32.15 by .068
40. 3.7 by 5.6	**49.** 4.17 by .72	**58.** 70.41 by 70.4
41. 7.4 by 3.2	**50.** .228 by 6.5	**59.** 907.4 by .0345
42. .35 by 4.8	**51.** .655 by .32	**60.** 27.68 by .1625
43. .44 by 8.5	**52.** 9.18 by .85	**61.** 66.55 by 3.044
44. .78 by .96	**53.** 4.76 by 4.2	**62.** 71.62 by 50.85

Multiply:

63. $4.75 by 2.4	**69.** $12.50 by .63	**75.** 4376 by .058
64. $6.08 by .45	**70.** $31.34 by 7.5	**76.** 5089 by 26.7
65. $7.53 by 3.7	**71.** $48.70 by .89	**77.** 7628 by 8.245
66. $5.25 by .56	**72.** $27.65 by 4.6	**78.** 3145 by 16.32
67. $8.46 by 7.3	**73.** $54.83 by 6.7	**79.** 6278 by 207.8
68. $9.34 by 8.1	**74.** $65.80 by .94	**80.** 5094 by .4595

Written Exercises

1. Find the cost of 7.5 bushels of wheat at $.88 a bushel.

2. How much do 8.04 tons of straw cost at $15.75 a ton?

3. Find the cost of 60.25 bushels of oats at $.48 a bushel.

4. On a dry track a horse could trot at the rate of .45 of a mile per minute, and on a slippery track he could trot only .8 as fast. Find his speed per minute on the slippery track.

5. When snow melts it forms about .18 of its bulk in water. What depth of water will result when 16 inches of snow melt?

6. The weight of a gallon of cream that is .25 fat is 8.4 pounds. How many pounds of fat does it contain?

7. If there are 18.6 cubic feet of coarse sand in a ton, how many cubic feet of coarse sand are there in 2.25 tons?

8. The castor bean is .25 oil. Find the number of pounds of oil produced from 36 pounds of castor beans.

9. If 1 ton of alfalfa hay removes 50.95 pounds of potash from the soil, how many pounds of potash will be removed from an acre of land that produces 3.8 tons of alfalfa hay?

10. Green oak timber weighs 67.7 pounds per cubic foot. Find the weight of a stick that contains 24.5 cubic feet.

11. How long will it take a train to make the run from Troy to New York, if it travels at the rate of 1 mile in 1.75 minutes, the distance being 148.32 miles?

12. A cubic foot of water weighs 62.5 pounds. What is the weight of a cubic foot of tin, if it is 7.29 times as heavy as water?

13. At a certain paper mill 1.95 tons of water had to be evaporated for every ton of paper made. How many tons were evaporated per day, if the output of paper was 443.52 tons?

Drill Exercises

Find products :

1. 4 × 56	**27.** 65.57 × .2	**53.** $45.60 × 40
2. 6 × 49	**28.** .00045 × 7	**54.** $54.10 × .07
3. 8 × 75	**29.** 18,000 × .6	**55.** 160.45 × 12
4. 5 × 8.4	**30.** $8.38 × 50	**56.** .07236 × 28
5. 9 × .67	**31.** .6795 × 60	**57.** 47.893 × 4.5
6. .7 × 9.8	**32.** $7.63 × 12	**58.** .00436 × 39
7. .04 × 73	**33.** $6.94 × 11	**59.** 68,045 × 5.6
8. 11 × 46	**34.** $5.75 × 2.5	**60.** 8468 × .008
9. 12 × 57	**35.** 2192 × .09	**61.** 7.345 × 200
10. 20 × 3.5	**36.** $5408 × 40	**62.** $6594 × 400
11. 4.6 × 47	**37.** 37.21 × 12	**63.** 423.8 × .067
12. 38 × .75	**38.** 4.638 × 11	**64.** .0404 × 12.5
13. 63 × 8.3	**39.** .6045 × 36	**65.** 8236 × .509
14. .79 × .58	**40.** .0148 × 4.3	**66.** 91.27 × 8.36
15. 6 × $2.50	**41.** 78.16 × .54	**67.** $27.50 × 600
16. 4 × $4.25	**42.** .0084 × 35	**68.** $18.95 × 150
17. 7 × $5.85	**43.** 7.63 × 500	**69.** $37.64 × 320
18. .08 × 245	**44.** 537 × .005	**70.** $59.78 × 435
19. 11 × 3.06	**45.** 68.9 × .046	**71.** 134.06 × .009
20. 12 × 61.3	**46.** 307 × $138	**72.** 45.725 × 608
21. 30 × 537	**47.** 8.24 × 23.7	**73.** .00009 × 325
22. 60 × 2.86	**48.** 209 × 43.8	**74.** 160.49 × .087
23. .19 × .028	**49.** 57.6 × .359	**75.** 5.308 × 17.26
24. 54 × 73.3	**50.** 706 × 4.19	**76.** 47.36 × .2401
25. 4.8 × 6.88	**51.** .625 × 34.8	**77.** $6340 × .0035
26. .67 × 909	**52.** 80.9 × 5.76	**78.** 796.5 × 36.49

Miscellaneous Exercises

1. A large orchard contained 18 rows of 86 trees each. How many trees were there in the orchard?

2. At the bank a boy saw 18 bags, each of which contained $5000 in gold coin. Find the value of the gold in all the bags.

3. On an acre of land a farmer used 48.5 bushels of lime, weighing 90 pounds per bushel. How many pounds of lime did he use?

4. A coffee plantation of 62 acres contained 500 trees to the acre. Find the whole number of coffee trees.

5. The clip from a flock of Angora goats was 850 pounds of mohair. Find its value at $.35 per pound.

6. A machine can set 50 screws per minute. At the same rate, how many screws can it set in a day of 8 hours?

7. At a shoe sale, 18,000 pairs of shoes were sold at $2.20 per pair. Find the total amount of the sales.

8. A rice grower sold 2592 bushels of rough rice at $.85 a bushel. How much did he receive for it?

9. A cargo of lemons consisted of 25,000 boxes each weighing 92 pounds. Find the weight of the cargo.

10. A merchant sold 7 pieces of cloth, each containing 34 yards, at $2.25 per yard. How much did he receive for it?

11. In building a factory, 318 loads of bricks averaging 932 bricks each were required. How many bricks were required?

12. If a bushel of wheat absorbs 1.184 pounds of potassium from the soil, how many pounds do 12.5 bushels absorb?

13. If copper trolley wire weighs 2128 pounds per mile, how much do 4.25 miles of such wire weigh?

Find the cost of :

14. 36 bicycles at $18.25 each.

15. 54 magic lanterns at $3.75 each.

16. 75 Boy Scout hats at $1.15 each.

17. 132 Boy Scout coats at $1.35 each.

18. 32.5 yards of duck canvas at $.18 a yard.

19. 8.75 yards of khaki suiting at $.28 a yard.

20. A letter-stamping machine stamped 1800 letters per minute. At this rate how many letters would it stamp per hour ?

21. It is estimated that each person in New York state consumes .473 of a barrel of flour and meal per year. Find the amount consumed per year in a city of 5.58 million people.

22. A single poppy has been known to contain 32,000 seeds. How many seeds will 335 such poppies contain ?

23. How much less will it cost to send 78,000 hundredweight of wheat by water at 7.85¢ per hundredweight than by rail at 16.5¢ per hundredweight ?

24. How many barrels of each material were used in making a cement sidewalk 7 yards long, if each yard required .397 of a barrel of cement, .775 of a barrel of sand, and .9 of a barrel of gravel ?

25. A silver dollar weighs .859375 of an ounce. Find the weight in ounces of 208 silver dollars.

26. During 288 days a cow produced an average of 36.25 pounds of milk per day. Find the total amount produced.

27. If the multiplicand is 57.639 and the multiplier is 42.08, what is the product ?

28. The following tickets were sold for the Symphony Club's concert : 420 at $2.50 each, 366 at $2 each, 474 at $1.25 each, and 596 at $.75 each. What were the box-office receipts?

. DIVISION

The process of finding how many times one number contains another is **division**.

Division is also the process of separating a number into equal parts. This kind of division is **partition**.

The number divided is the **dividend**.

The number by which we divide is the **divisor**.

The result obtained by dividing is the **quotient**.

When all the dividend is divided and the quotient is an integer, the division is **exact**.

The part of the dividend that is left when the division is not exact is the **remainder**.

The sign ÷ indicates *division*. It is read **divided by**.

Division is also indicated by writing the dividend above the divisor with a straight line between them; or by writing the dividend at the right of the divisor with a curved line between.

Thus, $\frac{24}{6}$, or 6)24, indicates that 24 is to be divided by 6.

Oral Exercises

Tell quotients instantly by rows and by columns:

	8.	9.	10.	11.	12.	13.
1.	14 ÷ 2	$\frac{18}{9}$	6)36	30 ÷ 5	$\frac{48}{4}$	9)108
2.	15 ÷ 5	$\frac{16}{8}$	5)40	48 ÷ 6	$\frac{60}{5}$	10)100
3.	12 ÷ 6	$\frac{32}{4}$	9)45	36 ÷ 4	$\frac{77}{7}$	11)121
4.	16 ÷ 4	$\frac{27}{3}$	8)80	49 ÷ 7	$\frac{56}{8}$	10)110
5.	21 ÷ 3	$\frac{45}{5}$	5)55	64 ÷ 8	$\frac{66}{6}$	12)132
6.	24 ÷ 8	$\frac{42}{6}$	9)90	54 ÷ 6	$\frac{81}{9}$	10)120
7.	28 ÷ 7	$\frac{72}{9}$	7)84	63 ÷ 9	$\frac{72}{6}$	12)144

Complete and read:

1. $20 + $4 =   **2.** 36 ¢ + 9 ¢ =   **3.** 18 rods + 6 rods =

If the dividend and the divisor are like numbers, the quotient is an abstract number.

This kind of division is finding how many times one number is contained in another. It is *measuring.*

Complete and read:

4. $20 + 5 =   **5.** 36 ¢ + 4 =   **6.** 18 rods + 3 =

If the dividend is concrete and the divisor is abstract, the dividend and the quotient are like numbers.

This kind of division is separating into equal parts and finding how many there are in each part. It is *partition.*

DIVISION BY INTEGERS

Written Exercises

1. Divide 648 by 2.

Divide the hundreds, tens, and units, respectively, by 2. Thus:

For the hundreds, 2 in 6, 3; write 3.

2)648   For the tens,   2 in 4, 2; write 2.
 324   For the units,   2 in 8, 4; write 4.

The quotient is 324.

Divide:

2. 3)364
  121, 1 rem.

3. 4)807
  201¾

The remainder may be written in either of these ways. The latter way makes it a part of the quotient, the division of 3 by 4 being indicated.

4. 246 by 2	**8.** 607 by 6	**12.** 558 by 5	**16.** 2486 by 2
5. 488 by 4	**9.** 778 by 7	**13.** 363 by 3	**17.** 3698 by 3
6. 639 by 3	**10.** 848 by 4	**14.** 889 by 8	**18.** 8040 by 4
7. 506 by 5	**11.** 647 by 2	**15.** 909 by 9	**19.** 7709 by 7

20. Divide 1347 by 5.

When the divisor is not contained in an order of units, this order is changed to the next lower order and united with the units of that order. The same thing is done with any remainder that there may be after dividing any order. Thus:

For the hundreds, 5 in 13, **2** with 3 rem.; write **2.**

5)1347

For the tens, 5 in 34, **6** with 4 rem.; write **6.**

269, 2 rem.

For the units, 5 in 47, **9** with 2 rem.; write **9, 2 rem.**

The quotient is 269, and remainder, 2.

Test. 5 × 269, + 2 = 1347, the dividend.

When the divisor is so small that the work can be done mentally, and only the figures of the quotient are written, the process is **short division.**

Divide and test, timing yourself:

21. 174 by 2	**25.** 869 by 4	**29.** 2897 by 3	**33.** 5795 by 5
22. 648 by 3	**26.** 894 by 6	**30.** 7319 by 2	**34.** 3266 by 7
23. 399 by 5	**27.** 506 by 8	**31.** 9872 by 4	**35.** 4347 by 9
24. 952 by 7	**28.** 783 by 9	**32.** 4946 by 6	**36.** 9554 by 8

Express each result in the form best adapted:

37. 11)748   **38.** 4)$8.16   **39.** 4)8.16   **40.** 8)186.   **41.** 3).02
     68                $2.04   2.04   23.25   .006⅔

Divide by 11 and 12 just as by single-figure divisors.

Decimal expressions are divided as shown above, the decimal point being written in the quotient directly under that in the dividend.

Zeros may be annexed to the dividend (or assumed as in exercises **40** and **41**) and the quotient carried to any required number of decimal places.

42. $8.14 ÷ 2	**49.** 69.6 ÷ 4	**56.** 1535 ÷ 5	**63.** 86.07 ÷ 3
43. $5.25 ÷ 3	**50.** .973 ÷ 7	**57.** 7225 ÷ 4	**64.** .7535 ÷ 5
44. $7.32 ÷ 5	**51.** 7.04 ÷ 8	**58.** 8343 ÷ 6	**65.** 5.834 ÷ 4
45. $6.24 ÷ 6	**52.** 54.9 ÷ 2	**59.** 6908 ÷ 11	**66.** 93.57 ÷ 2
46. $9.68 ÷ 8	**53.** 6.37 ÷ 5	**60.** 5892 ÷ 12	**57.** 763.1 ÷ 8
47. $7.42 ÷ 7	**54.** 24.2 ÷ 8	**61.** 7985 ÷ 11	**68.** .0773 ÷ 5
48. $8.55 ÷ 9	**55.** .807 ÷ 6	**62.** 8515 ÷ 12	**69.** 655.6 ÷ 9

Written Exercises

1. How many weeks are there in 1736 days?

2. When 9 cows cost $330.75, find the cost of 1 cow.

3. If a train runs 251 miles in 6 hours, how many miles does it *average* per hour?

4. A man with his team constructed 6.25 miles of ditch for irrigation in 5 days. How many miles of ditch were constructed per day?

5. I have 1001 strawberry plants to set out in 11 rows. How many plants are there for each row?

6. If some ducks' food for 8 months cost $4.64, how much did it cost for one month?

7. A pen of White Leghorn hens laid 826 eggs in 7 months. Find the average number of eggs laid per month.

8. There are 8 quarts in one peck. How many pecks are there in 9136 quarts?

9. The cost of raising and marketing a crop of cotton from 11 acres was $167.53. Find the average cost per acre.

10. From a large rosebush 1819 roses were picked one year. How many dozen roses were picked?

11. On 9 acres of land a farmer sowed 19.125 bushels of barley. How many bushels did he sow per acre?

12. A motor boat covered 18,655 miles in 5 seasons. Find the average distance covered per season.

13. The 11 members of a football team weighed 1809.5 pounds. Find the average weight per member.

14. In 7 years a woman baked 229,915 pies for a restaurant. Find the average number of pies baked yearly.

Since a decimal point is assumed to follow every integer, every removal of the decimal point one place toward the left in an integral or decimal expression, corresponds to moving each figure one place toward the right. Hence,

To divide an integral or a decimal expression by 10 *move the decimal point toward the left one place; by* 100, *two places; by* 1000, *three places, etc.*

Thus, 25 ÷ 100 = .25. If necessary, prefix 0's as in 7.6 ÷ 1000 = .0076.

Written Exercises

1–12. Divide by 10: 46 ; 9 ; 3.2 ; $1.65 ; $476 ; 82.4 ; .75 ; .08 ; $24.75 ; .7 ; .125 ; $.60.

13–18. Divide by 100 : 24 ; 5 ; 66.4 ; $3.20 ; 4.18 ; .62.

19–24. Divide by 1000 : 125; 63; 436.5 ; 87.24 ; 4.85 ; .32.

25–29. Divide by 10,000 : 4000 ; 282.5 ; 42.3 ; 8.75 ; 140.

30. Divide 9384 by 400.

$$4)\overline{93.84}$$
$$23.46$$

Since 400 = 100 × 4, we divide by 100, obtaining 93.84; then we divide this quotient by 4, obtaining 23.46.

31. Divide 93.84 by 40.

$$4)\overline{9.384}$$
$$2.346$$

We move the decimal point *one* place toward the left, obtaining 9.384; then we divide this quotient by 4, obtaining 2.346.

Divide :

32. $5.24 by 20

33. $8.43 by 80

34. $3.50 by 100

35. $6.40 by 400

36. $24.80 by 40

37. $38.46 by 60

38. $46.00 by 200

39. $74.50 by 500

40. 5.96 by 40

41. .895 by 50

42. 6.44 by 70

43. .096 by 80

44. 167.7 by 300

45. 49.02 by 600

46. 573.6 by 3000

47. 93.73 by 7000

48. 8330 by 70

49. 7264 by 80

50. 9765 by 90

51. 38,400 by 400

52. 69,318 by 900

53. 97,624 by 800

54. 635,000 by 5000

55. 957,000 by 6000

1. How many dollars are there in 5675 ¢ ?

2. How many hours are there in 4320 minutes ?

3. How many lots costing $500 each can be bought for $16,000 ?

4. A certain railroad, 20 miles long, was built in 2160 hours. Find the average time required to build 1 mile.

5. A locomotive uses 1800 gallons of water in a run of 60 miles. How many gallons does it use per mile?

6. A prize cow in 10 months produced 23,981.4 pounds of milk. How many pounds did she average per month ?

7. How long will it take a fast freight train running 20 miles an hour to go from Charleston to New York, 804 miles ?

8. One year the yield of corn from 200 acres of land in Kansas was 10,000 bushels. Find the average yield per acre.

9. If the cost of producing 50 pounds of Cuban sugar by modern methods is $.625, what is the average cost per pound ?

10. A 40-acre vineyard in California produced grapes worth $4160. Find the value of the crop per acre.

11. Mr. Hoy bought a carload of grain. It weighed 34,000 pounds. How many tons did it weigh ? (1 ton = 2000 pounds.)

12. Find the cost, at $15 per thousand, of the burlap covers required for a shipment of 1506 barrels of cauliflower.

13. If the product of two numbers is 47,200 and one of the numbers is 800, what is the other number ?

14. A Boy Scout Club camped for 20 days at a cost of $7.80 apiece. Find the average cost per member per day.

15. A bushel of wheat weighs 60 pounds. How many bushels of wheat are there in a car that contains 37,560 pounds of wheat ?

1. Divide 56,249 by 68.

```
        827
68)56249
      544
      184
      136
      489
      476
       13, rem.
```

In practice, write the steps in division as shown in the process, dividing thus: 562 ÷ 68 = about 8; write **8** in the quotient directly over 2, *the last figure of the part of the dividend used;* 8 times 68 = 544; subtract and bring down 4, the next figure of the dividend. 184 ÷ 68 = about 2; write **2** in the quotient; 2 times 68 = 136; subtract and bring down 9. 489 ÷ 68 = about 7; write 7; 7 times 68 = 476; subtract. The quotient is 827 and the remainder 13.

Test. — Since 68 × 827, + 13 = 56,249, the dividend, the work is probably correct.

When the divisor is so large that each step in the work must be written the process is **long division.**

Find the quotient and test :

2. 672 ÷ 21	**6.** 899 ÷ 29	**10.** 2448 ÷ 51	**14.** 3304 ÷ 59
3. 722 ÷ 19	**7.** 943 ÷ 41	**11.** 2597 ÷ 49	**15.** 8875 ÷ 71
4. 837 ÷ 31	**8.** 897 ÷ 39	**12.** 3538 ÷ 61	**16.** 9177 ÷ 69
5. 904 ÷ 31	**9.** 985 ÷ 39	**13.** 4218 ÷ 61	**17.** 9547 ÷ 69

Divide as indicated :

18. 5427 ÷ 81	**27.** 28,348 ÷ 38	**36.** 400,752 ÷ 92
19. 5846 ÷ 79	**28.** 34,220 ÷ 52	**37.** 464,112 ÷ 88
20. 7100 ÷ 91	**29.** 66,192 ÷ 48	**38.** 510,472 ÷ 37
21. 8455 ÷ 89	**30.** 52,396 ÷ 62	**39.** 341,136 ÷ 46
22. 8025 ÷ 22	**31.** 83,346 ÷ 58	**40.** 816,050 ÷ 55
23. 7686 ÷ 18	**32.** 63,216 ÷ 72	**41.** 545,544 ÷ 64
24. 6058 ÷ 32	**33.** 65,552 ÷ 68	**42.** 572,028 ÷ 73
25. 9668 ÷ 28	**34.** 76,834 ÷ 82	**43.** 819,975 ÷ 87
26. 9996 ÷ 42	**35.** 98,992 ÷ 78	**44.** 916,055 ÷ 96

Find quotients:

$$\begin{array}{r} 206 \\ \textbf{45.}\ 47\overline{)9682} \\ 94 \\ \hline 282 \\ 282 \\ \hline \end{array}$$

$$\begin{array}{r} 5.07 \\ \textbf{46.}\ 78\overline{)395.46} \\ 390 \\ \hline 5\ 46 \\ 5\ 46 \\ \hline \end{array}$$

Bring down only one figure at a time and write a figure in the quotient (it may be 0) for each figure brought down.

In long division of a decimal expression, place the decimal point in the quotient directly over the decimal point in the dividend.

47.	$3888 + 36$	**54.**	$\$72.28 + 26$	**61.**	$932.24 + 43$
48.	$8778 + 43$	**55.**	$\$41.76 + 48$	**62.**	$70.686 + 54$
49.	$8151 + 39$	**56.**	$\$97.60 + 32$	**63.**	$.61332 + 76$
50.	$4220 + 54$	**57.**	$\$918.08 + 38$	**64.**	$936.215 + 35$
51.	$5460 + 65$	**58.**	$\$494.46 + 67$	**65.**	$4144.95 + 61$
52.	$8274 + 27$	**59.**	$\$474.36 + 59$	**66.**	$62.0928 + 77$
53.	$9835 + 45$	**60.**	$\$671.18 + 74$	**67.**	$865.364 + 94$

$$\begin{array}{r} .0268^{+} \\ \textbf{68.}\ 37\overline{).992} \\ 74 \\ \hline 252 \\ 222 \\ \hline 300 \\ 296 \\ \hline \end{array}$$

The quotient carried *four* decimal places is .0268, or .0268+, the sign + indicating that it is a little more than .0268.

The quotient carried *three* decimal places is .026, or .027⁻, the sign − indicating that it is a little less than .027.

The quotient to the "nearest thousandth" is .027; to the "nearest ten-thousandth," .0268.

	EXACT		To Nearest Hundredth		To Nearest Thousandth
69.	$38.08 + 28$	**74.**	$2749 + 18$	**79.**	$18,462 + 214$
70.	$9.065 + 37$	**75.**	$40.15 + 39$	**80.**	$30.695 + 428$
71.	$487.2 + 56$	**76.**	$5.731 + 42$	**81.**	$5903.8 + 563$
72.	$.8967 + 49$	**77.**	$97.09 + 65$	**82.**	$641.23 + 379$
73.	$63.92 + 68$	**78.**	$824.6 + 57$	**83.**	$4287.9 + 708$

Find the cost of each when

1. 36 alarm clocks cost $82.80.

2. 25 mantel clocks cost $78.75.

3. 24 cabinet clocks cost $87.60.

4. 21 opera glasses cost $186.90.

5. 33 velours couches cost $874.50.

6. 45 cooking stoves cost $1541.25.

7. 52 sewing machines cost $998.40.

8. 26 open-faced watches cost $903.50.

9. 71 nickeled parlor stoves cost $1370.30.

10. In one year the total yield of alfalfa from a 75-acre field was 450 tons. Find the average yield per acre.

11. A barge ferries 16 cars over the Hudson River in one trip. How many trips would it make in ferrying 640 cars?

12. If it cost $14.40 to keep 48 barrels of poultry in cold storage for 1 month, what was the cost per barrel per month?

13. A farmer plowed 11.2 acres of land in 14 days. Find the average number of acres plowed per day.

14. If there are 18,000 men in 24 battalions of soldiers, how many men are there in 1 battalion?

15. How many cars of 48 tons' capacity are required to receive the cargo of a Lake Erie vessel carrying 8400 tons of iron ore?

16. From 37 acres of meadow a farmer cut 64.38 tons of hay. Find the average yield per acre in tons.

17. A dealer in athletic goods bought 48 baseball gloves for which he paid $101.76. Find the wholesale price per glove.

1. Divide 619,770 by 146.

```
        4245
146)619770
    584
    ‾‾‾
    357
    292
    ‾‾‾
    657
    584
    ‾‾‾
    730
    730
```

Since 146 is only a little less than 150, the first figure of the quotient is found by dividing 619 by 150, or 61 by 15, which gives 4.

Find, timing yourself:

2. 476,654 ÷ 122

3. 570,688 ÷ 241

4. 468,835 ÷ 205

5. 8506.53 ÷ 423

6. 988.158 ÷ 314

7. 247,352 ÷ 392

8. 32.0264 ÷ 532

9. 2.98421 ÷ 653

10. 7,689,398 ÷ 454

11. 4265.728 ÷ 608

12. 1,409,562 ÷ 567

13. 743.8847 ÷ 389

14. 2,704,855 ÷ 865

15. 38.97115 ÷ 467

16. Divide $1424.64 by 265.

```
        $5.376
265)$1424.64
    1325
    ‾‾‾‾
     99 6
     79 5
     ‾‾‾‾
     20 14
     18 55
     ‾‾‾‾
      1 590
      1 590
```

The quotient, carried to mills by annexing a 0 to form the last partial dividend, is $5.376. To the *nearest cent* it is $5.38.

Find, to the nearest cent:

17. $8960.45 ÷ 201

18. $6789.24 ÷ 132

19. $3546.28 ÷ 519

20. $4672.83 ÷ 228

21. $2691.35 ÷ 435

22. $45,802.79 ÷ 612

23. $73,862.95 ÷ 353

24. $18,921.53 ÷ 734

25. $38,029.42 ÷ 468

26. $21,634.89 ÷ 247

27. $89,673.20 ÷ 1242

28. $17,672.36 ÷ 4341

29. $34,226.58 ÷ 2059

30. $42,630.87 ÷ 3418

Written Exercises

1. A steamship has 2450 tons of coal in her bunkers. If she burns 175 tons per day, how long will the coal last ?

2. One year California produced 706 ounces of refined platinum, valued at $13,414. Find its value per ounce.

3. A certain ore dock has 350 pockets with a combined capacity of 87,500 tons. Find the average capacity of each pocket.

4. A telephone company paid $5899.50 for 414 telephones. Find the cost of each instrument.

5. A large shipment of salt consisted of 4,200,000 pounds in 280-pound barrels. How many barrels of salt were there ?

6. A year's production of crude borax amounted to 2600 tons, valued at $91,000. Find the average value per ton.

7. A stockman sold a steer that weighed 1236 pounds for $83.43. How much did he receive per pound live weight ?

8. The operating expenses of a street railway for a year of 365 days were $125,092.80. Find the average daily expenses.

9. A rolling mill consumes about 3,500,000 cubic feet of natural gas per day. How many households would this amount supply, if each uses 875 cubic feet per-day ?

10. The total fall in the last 1180 miles of the Mississippi River is about 275.333 feet. Find, to the nearest hundredth, the average fall per mile.

11. A blast furnace in 1475 days produced 493,122 tons of pig iron. Find the average daily production.

12. A New York state road cost $6388.80 per mile (5280 feet). How much did it cost per foot ?

The value of the quotient depends upon that of the dividend and the divisor.   If one of these is changed while the other remains the same, the quotient will be changed.   If both are changed, the quotient may or may not be changed.

Consider the effect on the quotient of the following changes made in the dividend and the divisor:

1. $24 \div 6 = 4$ Multiplying the dividend by 2 *multiplies* the quo-
    $48 \div 6 = 8$ tient by 2.

2. $24 \div 6 = 4$ Dividing the divisor by 2 *multiplies* the quotient
    $24 \div 3 = 8$ by 2.

3. $24 \div 6 = 4$ Dividing the dividend by 2 *divides* the quotient
    $12 \div 6 = 2$ by 2.

4. $24 \div 6 = 4$ Multiplying the divisor by 2 *divides* the quotient
    $24 \div 12 = 2$ by 2.

5. $24 \div 6 = 4$ Multiplying both dividend and divisor by 2 *does*
    $48 \div 12 = 4$ *not change the quotient.*

6. $24 \div 6 = 4$ Dividing both dividend and divisor by 2 *does not*
    $12 \div 3 = 4$ *change the quotient.*

From the preceding illustrations the following *principles* may be deduced.   From 1 and 2, we have :

1. *Multiplying the dividend or dividing the divisor by any number multiplies the quotient by that number.*

From 3 and 4, we have :

2. *Dividing the dividend or multiplying the divisor by any number divides the quotient by that number.*

From 5 and 6, we have :

3. *Multiplying or dividing both dividend and divisor by the same number does not change the quotient.*

DIVISION BY DECIMALS

Written Exercises

1. Divide 7.6912 by 3.68.

7.6912 ÷ 3.68

$$\begin{array}{r} 2.09 \\ 368\overline{)769.12} \\ 736 \\ \hline 33\ 12 \\ 33\ 12 \\ \hline \end{array}$$

Before dividing we change the divisor to an *integer* by multiplying it by 100, but to avoid changing the quotient (p. 58, 3) we must also multiply the dividend by 100.

The work is thus changed to "division by an integer" and we proceed as in that case, placing the decimal point in the quotient directly over the decimal point in the dividend.

The method of placing the decimal point illustrated here is known as the **Austrian method**.

2. Divide 17.2 by .016.

$$\begin{array}{r} 1075 \\ 16\overline{)17200} \\ 16 \\ \hline 120 \\ 112 \\ \hline 80 \\ 80 \\ \hline \end{array}$$

We multiply both dividend and divisor by 1000.

3. Divide .0469 by 1.75.

$$\begin{array}{r} .0268 \\ 175\overline{)4.69} \\ 3\ 50 \\ \hline 1\ 190 \\ 1\ 050 \\ \hline 1400 \\ 1400 \\ \hline \end{array}$$

We multiply both dividend and divisor by 100.

Divide, carrying results as indicated:

EXACT	EXACT	TO NEAREST THOUSANDTH
4. 140 ÷ .032	**10.** $.959 ÷ $.035	**16.** 26.45 ÷ 14.6
5. 54.3 ÷ .075	**11.** $2.58 ÷ $3.44	**17.** 708.4 ÷ 2.79
6. .096 ÷ 2.56	**12.** $.042 ÷ $.875	**18.** .0436 ÷ .425
7. .9591 ÷ 34.5	**13.** $68.64 ÷ $1.43	**19.** 8720 ÷ 36.2
8. 6.1248 ÷ 1.74	**14.** $8.881 ÷ $1.07	**20.** 5.368 ÷ 64.4
9. .10057 ÷ 17.8	**15.** $98.89 ÷ $7.25	**21.** 62.45 ÷ 3.86

Find quotients :

22. 115 ÷ .2	**50.** $5.50 ÷ .22	**78.** $35.52 ÷ .005
23. 128 ÷ .5	**51.** $6.75 ÷ 4.5	**79.** $52.35 ÷ .075
24. 26.5 ÷ .4	**52.** $8.70 ÷ .29	**80.** $42.90 ÷ 1.32
25. .091 ÷ .07	**53.** 1400 ÷ 5.6	**81.** $61.75 ÷ 49.4
26. 15.3 ÷ .06	**54.** 2730 ÷ 7.5	**82.** 137.43 ÷ .009
27. .308 ÷ .08	**55.** 4.328 ÷ .32	**83.** 24.242 ÷ .016
28. 4.14 ÷ .09	**56.** .0513 ÷ 3.6	**84.** 41,808 ÷ 1.92
29. 20.7 ÷ 1.2	**57.** 368.8 ÷ 6.4	**85.** .00324 ÷ .144
30. .078 ÷ .15	**58.** 57.24 ÷ 7.2	**86.** 160.68 ÷ 31.2
31. 312 ÷ 2.6	**59.** .0972 ÷ .81	**87.** 5.8905 ÷ 25.5
32. 60.9 ÷ .29	**60.** $8.25 ÷ .008	**88.** 52.272 ÷ 3.52
33. .525 ÷ 4.2	**61.** $7.35 ÷ .024	**89.** 4500 ÷ .0288
34. 62.7 ÷ 3.3	**62.** $9.50 ÷ 1.52	**90.** 2280 ÷ .0384
35. 6.48 ÷ .54	**63.** 2142 ÷ .035	**91.** 630 ÷ .00504
36. .825 ÷ 7.5	**64.** 2.472 ÷ .048	**92.** $125.40 ÷ .048
37. 52.4 ÷ .64	**65.** .4131 ÷ 16.2	**93.** $327.78 ÷ 1.08
38. .788 ÷ 8.7	**66.** 5863 ÷ 1.76	**94.** $498.96 ÷ 69.3
39. 231 ÷ .016	**67.** .0396 ÷ 7.92	**95.** 133.056 ÷ .096
40. 574 ÷ .035	**68.** 36.72 ÷ 28.8	**96.** 2.24532 ÷ .072
41. 5.94 ÷ .027	**69.** 4.312 ÷ .385	**97.** .108056 ÷ 10.4
42. .833 ÷ .049	**70.** 784.5 ÷ 3.75	**98.** 2993.76 ÷ 13.5
43. 6.48 ÷ .096	**71.** 6500 ÷ 62.5	**99.** 44.9064 ÷ 2.16
44. .075 ÷ 12.5	**72.** .5733 ÷ .252	**100.** 54885.6 ÷ .264
45. 92.4 ÷ .231	**73.** 8.481 ÷ 13.2	**101.** 748,440 ÷ 40.5
46. 427 ÷ 24.4	**74.** 83.16 ÷ 8.25	**102.** 7.98336 ÷ 2.56
47. 630 ÷ 5.04	**75.** 904.5 ÷ .225	**103.** .049896 ÷ 39.6
48. .927 ÷ 6.18	**76.** 7560 ÷ 16.8	**104.** 349.272 ÷ 46.2
49. .693 ÷ 49.5	**77.** .6336 ÷ 35.2	**105.** 6.98544 ÷ 7.92

1. How many 8-pound baskets of cherries must a man pick in a day in order to earn $1.60, if he receives 1¢ per pound?

2. A flour mill turned out 250 barrels of flour in one day, which was .5 of its capacity. What was its capacity?

3. If a horse walks at the rate of .25 of a mile in 5 minutes, how long does it take him to walk 4.25 miles?

4. At 2.5 ¢ a box, how many boxes of oranges must a man pick per week in order to earn $15?

5. The school athletic association has $31.35 with which to buy baseball suits. If the suits cost $2.85 each, how many can be bought?

6. A press was constructed to bale 798 bales of cotton in 9.5 hours. What was its capacity in bales per hour?

7. A laborer moved 632.5 cubic feet of earth in one day by means of a wheelbarrow. If his average load was 2.5 cubic feet, how many loads did he handle?

8. An automobile traveled 153.9 miles in going from Buffalo to Syracuse. If the car averaged 22.8 miles per hour, how long did it take to make the trip?

9. A man received $81.75 for maple sugar that he sold at 12.5 ¢ a pound. How many pounds did he sell?

10. If a long ton (2240 pounds) of bituminous coal contains 29.47 bushels, find, to the nearest pound, the weight of a bushel.

11. A block of limestone containing 28.5 cubic feet weighs 4522.95 pounds. Find its weight per cubic foot.

12. If 4.48 tons of coal evaporate 76,900 pounds of water, how many pounds of water, to the nearest tenth of a pound, will 1 ton of coal evaporate?

Drill Exercises

Find quotients:

1.	$135 \div 2$	28.	\$7.36 ÷ 40	55.	\$73.20 ÷ 400
2.	$206 \div 4$	29.	\$6.72 ÷ 3.5	56.	\$84.60 ÷ 600
3.	$117 \div .5$	30.	\$.896 ÷ .64	57.	\$67.68 ÷ .009
4.	$324 \div .8$	31.	$9204 \div 39$	58.	\$93.40 ÷ \$.25
5.	\$5.73 ÷ 3	32.	$9570 \div 58$	59.	\$61.88 ÷ .034
6.	\$8.05 ÷ 7	33.	$3114 \div 4.5$	60.	$4.364 \div 2000$
7.	$576 \div 60$	34.	$22.95 \div 6.8$	61.	\$7.36 ÷ \$.046
8.	$375 \div .06$	35.	$.3708 \div .72$	62.	$46.31 \div .0088$
9.	$26.1 \div .09$	36.	$5.928 \div 5.7$	63.	$70,455 \div 231$
10.	$47.6 \div 40$	37.	$8.961 \div .87$	64.	$61.008 \div 372$
11.	$.075 \div .12$	38.	\$8.60 ÷ 200	65.	$26,800 \div 5.36$
12.	$5.85 \div 75$	39.	\$9.25 ÷ .008	66.	$206.08 \div 44.8$
13.	$660 \div 80$	40.	\$.056 ÷ \$.16	67.	\$377.41 ÷ .094
14.	$7.92 \div 36$	41.	$5824 \div 700$	68.	\$633.42 ÷ .162
15.	$.084 \div 2.4$	42.	$2108 \div .034$	69.	$167,700 \div 500$
16.	$345 \div .46$	43.	$42.64 \div 160$	70.	$431,766 \div 289$
17.	$.495 \div 5.5$	44.	$2470 \div 15.2$	71.	$275,324 \div 350$
18.	$736 \div 6.4$	45.	$19.95 \div 760$	72.	$.057324 \div .048$
19.	$.903 \div .42$	46.	\$7.20 ÷ \$.96	73.	$10.8936 \div 27.2$
20.	$.975 \div 78$	47.	$80.04 \div .087$	74.	$2070.12 \div 156$
21.	$250 \div 400$	48.	$342.4 \div .256$	75.	$441.518 \div 4.27$
22.	$.352 \div 160$	49.	$7050 \div 6.25$	76.	$335,025 \div 22.5$
23.	$.573 \div .006$	50.	$84.24 \div 57.6$	77.	$1.80205 \div .575$
24.	$4.88 \div .032$	51.	$9.106 \div 116$	78.	$6761.28 \div 3.84$
25.	$75.6 \div .084$	52.	$5.253 \div .408$	79.	$507.072 \div 60.8$
26.	$725 \div 125$	53.	$2848 \div 71.2$	80.	$35.8656 \div 5.12$
27.	$67.1 \div 2.44$	54.	$556.2 \div .864$	81.	$208.296 \div .792$

Miscellaneous Exercises

1. A fruit grower packed 3000 pears in boxes holding 60 pears each. How many boxes did he use?

2. In making 8 cubic yards of cement mortar, 13.6 barrels of cement were used. How many barrels of cement were required to make 1 cubic yard of cement mortar?

3. From one tree a fruit grower picked 26.5 bushels of apples for which he received $19.08. How much did he receive per bushel?

4. How many weeks are there in a year of 365 days and how many days over?

5. If 63.54 gallons of water are discharged from a pipe in 60 seconds, how many gallons are discharged per second?

6. If a pane of glass whose surface is 17 square feet weighs 29.75 pounds, what is its weight per square foot?.

7. A spool of barbed wire contains 1500 feet. How much does this length of wire weigh, if 12 feet weigh 1 pound?

8. If a boy makes .5¢ on each newspaper that he sells, how many must he sell in order to make $2.25?

9. If 92.5 pounds of white lead are used to make enough paint for the first coat on 500 square yards of surface, how much white lead is needed per square yard?

10. The monthly rent receipts from an apartment house amounted to $513. There were 19 apartments. What was the average rent of each?

11. One year only 94 ounces of platinum were produced in the United States. The estimated value at that time was $1814. Find, to the nearest cent, the value per ounce.

12. A piece of rolled iron rod 12 feet long weighs 31.848 pounds. Find its weight per foot.

13. If 72 cubic inches of rolled lead weigh 29.5272 pounds, what is the weight of rolled lead per cubic inch?

14. A stock raiser shipped a carload of 18 cattle weighing 23,868 pounds. Find the average weight per head.

15. The battleship *Georgia* took on 1779 tons of coal in 5.2 hours. How many tons, to the nearest tenth, were loaded per hour?

16. One year a tract of land produced 50,000 tons of sugar beets valued at $233,000. Find the average value per ton.

17. The English pound is equivalent to $4.8665 in United States money. To how many pounds are $389.32 equivalent?

18. A car that weighed 22,800 pounds when empty was loaded with wheat. It then weighed 56,460 pounds. How many bushels of wheat did it contain, 1 bushel weighing 60 pounds?

19. A road bed rises 3.2 feet for every 100 feet of length. Find the rise in 1 mile (5280 feet) of road.

20. The Welland Canal extending from Lake Erie to Lake Ontario has 27 locks, giving a total lift from Lake Ontario to Lake Erie of 326.75 feet. Find the average lift per lock, to the nearest tenth of a foot.

21. A train ran a mile (5280 feet) in 39.75 seconds. Find, to the nearest tenth, the number of feet traveled per second.

22. In New York state one year 52,560,000 bushels of potatoes were grown on 438,000 acres. Find the average yield per acre.

23. In making 250 revolutions a wheel traveled 3141.6 feet. Find the distance it traveled per revolution.

24. A steam shovel excavated 45,694 cubic yards of earth in 26 days. How many cubic yards, to the nearest hundredth, did it excavate per day?

COMBINATION OF PROCESSES

When $+$ and $-$ are the only signs in an expression, the operations are performed in order beginning at the left.

$$2 + 8 - 6 + 3 = 10 - 6 + 3 = 4 + 3 = 7.$$

When $\times$ or $\div$ occurs in an expression in connection with $+$, $-$, or both, the indicated *multiplications* or *divisions* must be performed first.

$$8 + 2 \times 3 - 9 = 8 + 6 - 9 = 5; \ 10 - 8 \div 2 + 5 = 10 - 4 + 5 = 11.$$

When $\times$ and $\div$ occur in succession in an expression, the operations are performed in order beginning at the left.

$$4 + 24 \div 8 \times 5 = 4 + 3 \times 5 = 4 + 15 = 19.$$

The parentheses $(\ )$, the brackets $[\ ]$, the braces $\{\ \}$, and the vinculum $\overline{\quad}$, called **signs of aggregation**, are used to group expressions, each group being regarded as a single number.

All operations within groups should be performed first.

$$10 \times \overline{6-2} + [9 \div (2+1)] = 10 \times 4 + 9 \div 3 = 40 + 3 = 43.$$

<div align="center">Written Exercises</div>

Find the value of :

1. $5 \times 10 - 7$
2. $5 \times (10 - 7)$
3. $32 \div 4 + 12$
4. $32 \div (4 + 12)$
5. $(25 - 13) \div 4 \times 2$
6. $2 \times 5 + 3 \times 10 - 5$
7. $16 - 2 \times 2 \times 12 \div 4$

8. $5 \times 2 - 3 \times 2$
9. $7 \times 3 + 12 \div 3$
10. $6 + 2 \times 8 \div 4 \div 2$
11. $(6 + 2) \times 8 - 4 \div 2$
12. $(6 + 2 \times 8 - 4) \div 2$
13. $6 + 2 \times (8 - 4) \div 2$
14. $6 + 2 \times (8 - 4 \div 2)$

15. $7 \times \overline{4 + 3} - 6 \times \overline{7 + 2} + 2$
16. $5 - \{18 - 3 \times 2 + 8 \div 4 - 5\} + 3$
17. $3 + [2 + \overline{10 - 8} + \overline{4 \times 3 - 2 \times 5}] \div 2$

Copy and add the numbers in each row:

	a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*
1.	5	4	36	144	5.76	345.6	4487.04
2.	6	8	24	72	3.6	324	6470.28
3.	7	8	40	320	9.6	768	1597.44
4.	8	7	56	392	15.68	1568	7420.952
5.	2	8	72	576	11.52	460.8	6480.576
6.	3	8	27	81	7.29	510.8	2835.081
7.	5	7	28	196	19.6	980	5306.7
8.	4	8	64	512	15.86	307.2	3932.16
9.	3	5	40	200	18	540	7862.4
10.	6	6	30	180	72	2880	6095.16
11.	9	8	48	384	7.68	230.4	6666.624
12.	3	6	36	216	8.64	345.6	8580.384
13.	8	8	56	448	22.4	448	4654.272

Add the numbers under:

14. *a*   **15.** *b*   **16.** *c*   **17.** *d*   **18.** *e*   **19.** *f*   **20.** *g*

Subtract each number under:

21–46. *c* from the number opposite it under *d*; under *f*.

47–72. *e* from the number opposite it under *f*; under *g*.

Multiply each number under:

73–98. *e* by the number opposite it under *b*; under *c*.

99–124. *f* by the number opposite it under *d*; under *e*.

Divide each number under:

125–150. *e* by the number opposite it under *b*; under *c*.

151–176. *g* by the number opposite it under *d*; under *e*.

Freight Problems

Freight rates depend upon the class of matter and the distance the freight is sent. The rates are usually quoted per 100 pounds (lb.).
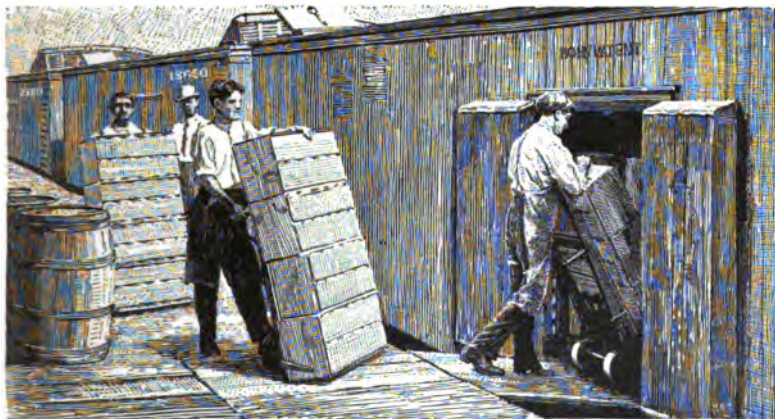
Find the freight charges on:

1. 16,400 lb. corn at 20¢.   4. 45,000 lb. sugar at 28¢.
2. 10,900 lb. rice at 38¢.   5. 36,300 lb. coffee at 35¢.
3. 23,200 lb. brick at 22¢.   6. 27,560 lb. raisins at 95¢.

7. 44,720 lb. oranges at $1.25.
8. 85,350 lb. machinery at 52¢.
9. 78,250 lb. cotton piece goods at 92¢.

10. Find the freight charges on 700 bags of coffee averaging 132 lb. each at 25¢ per 100 lb.

11. How much would the freight charges be on 1400 crates of strawberries at 65¢ per crate?

12. The freight and refrigeration charges on 640 boxes of cherries were $121.60. Find the cost of shipping per box.

Telegraph and Telephone Problems

1. Find the cost of sending a 22-word telegram at the day rate of 60¢ for the first 10 words and 4¢ for each additional word, or as it is more briefly written at the rate of 60–4.

2. Find the cost of sending the 22-word message at the night rate of 50–3.

3. How much cheaper is a 15-word telegram at a night rate of 40–3 than at a day rate of 50–3?

4. Find the cost of an 18-word telegram at the rate, 25–2.

At the day rate 40–3, find the cost of a telegram of:

5. 14 words	**7.** 16 words	**9.** 25 words
6. 23 words	**8.** 27 words	**10.** 19 words

11–16. In exercises **5–10**, find the cost at the night rate, 30–2.

17–20. Night letters of 50 words or less are sent at the day rate of a 10-word telegram, and $\frac{1}{5}$ more for each additional 10 words or part thereof. When the day rate on a 10-word telegram is 25¢, find the cost of a night letter of 45 words; of 60 words; of 55 words; of 75 words.

21. How much will it cost to talk over a long-distance telephone for 7 minutes at 25¢ for the first 3 minutes and 5¢ for each additional minute (25–5)?

22. Alfred talked with his father in a distant city from 7.55 to 8.05. Find the telephone charges at 50–15.

23. Find the charges for a 10-minute talk at 100–25.

Find the cost of telephoning for:

24. 10 minutes at 20–5		**27.** 6 minutes at 60–15
25. 11 minutes at 40–10		**28.** 8 minutes at 100–25
26. 12 minutes at 50–10		**29.** 5 minutes at 150–40

DOMESTIC RATES OF POSTAGE

All matter transmitted by the mails within the United States and her possessions (Tutuila, Porto Rico, Guam, Hawaii, the Philippines, and the " Canal Zone ") is divided into four classes :

First-class : Letters, all sealed matter, etc. . . 2 ¢ per oz. or part thereof
Post cards and postal cards . . . . 1 ¢ each
Second-class : Newspapers and periodicals . . 1 ¢ per 4 oz. or part thereof
Newspapers, periodicals sent by publishers . 1 ¢ per lb. or part thereof
Third-class : Miscellaneous printed matter (except books), photographs, maps, etc. (limit 4 lb.) . . . . 1 ¢ per 2 oz. or part thereof
Fourth-class (Parcel Post) :
Merchandise (limit 4 oz.) . . . . 1 ¢ per oz. or part thereof
Books, seeds, plants, etc. (limit 8 oz.) . . . 1 ¢ per 2 oz. or part thereof
Merchandise (over 4 oz.) . . . . ⎫
Books, seeds, etc. (over 8 oz.) . . . . . ⎬ According to distance (see zone rates below)
Miscellaneous printed matter (over 4 lb.) . ⎭

Domestic rates, with a few exceptions, apply also to mail for Canada, Mexico, Cuba, the Republic of Panama, and Shanghai, China.

PARCEL POST ZONE RATES

Packages sent by parcel post must not be greater than 84 inches in length and girth combined. On such parcels the limit of weight in the first and second zones is 50 pounds; in the other zones, 20 pounds. A fraction of a pound is considered one pound. The rates are fixed thus :

	FIRST POUND	EACH EXTRA POUND
1st zone — 1 to 50 miles	$.05	$.01
2d zone — 50 to 150 miles	.05	.01
3d zone — 150 to 300 miles	.06	.02
4th zone — 300 to 600 miles	.07	.04
5th zone — 600 to 1000 miles	.08	.06
6th zone — 1000 to 1400 miles	.09	.08
7th zone — 1400 to 1800 miles	.11	.10
8th zone — over 1800 miles	.12	.12

NOTE. — The rate for local rural and city delivery is 5 cents for the first pound and 1 cent for each additional 2 pounds or part thereof.

Written Exercises

Find the postage on each of the following packages :

1. Lace, 3 oz.
2. Seeds, 7 oz.
3. Letter, 1.5 oz.
4. Magazine, 18 oz.
5. Photographs, 8 oz.

6. Glass, 28 lb., 1st zone.
7. Cloth, 15 lb., 3d zone.
8. Coat, 9.5 lb., 8th zone.
9. Books, 17 lb., 5th zone.
10. Raisins, 6 lb., 7th zone.

11-13. Find the cost of sending a package weighing 20 lb. by parcel post a distance of 35 miles ; 527 miles ; 1250 miles.

14. Mr. Ford sent a 10-pound package by parcel post to a man in the same city. How much postage did he pay ?

15. May sent a letter weighing 1.75 oz. by special delivery. How much did it cost, the special delivery fee being 10 ¢ ?

16. Mrs. Lee sent a registered letter weighing 2 oz. to her son. Find the amount of postage paid (registry fee, 10 ¢).

17. Robert sent a letter weighing .5 oz. and a paper weighing 3 oz. to his brother. How much postage did he pay ?

18. One day a newspaper publisher mailed 50 copies of the paper to a certain town. If each paper weighed .25 lb., what was the amount of postage paid?

19. How much postage does a publisher pay on 50,000 copies of a magazine, if each copy weighs .75 lb. ?

20. One day a merchant mailed 50 letters weighing .5 oz. each and 175 separately addressed catalogues weighing 12 oz. each. How much postage did he pay ?

21. Christmas week Julia mailed 2 packages of books weighing 8 oz. each, 25 Christmas cards in sealed envelopes weighing 1 oz. each, and 2 packages of merchandise weighing 2.5 lb. and 5 lb., respectively, to go less than 50 miles by parcel post. How much postage did she pay ?

FACTORS, DIVISORS, MULTIPLES

(REVIEW)

A number that divides another number without a remainder is an **exact divisor** of that number.

2, 3, 4, and 6 are exact divisors of 12. *Divisible* means *exactly divisible.*

An exact divisor of a number is a **factor** of it.

A number that is divisible by another number is a **multiple** of that number.

Factors and multiples referred to in this chapter are *integers.*

A number that has no factors except itself and 1 is a **prime number.** Other numbers are **composite numbers.**

1, 2, 3, 5, 7, 11, 13, 17, 19, and 23 are prime numbers. Memorize them.

A number that is divisible by 2 is an **even number.** A number that is not divisible by 2 is an **odd number.**

0 is regarded as an even number; 1 as an odd number.

DIVISIBILITY OF NUMBERS

The figures that are used to represent a number are its **digits.**

The digits of 358 are 3, 5, and 8.

Illustrate with numbers each of these **tests of divisibility.**
A number is divisible by

2, *if the units' figure is* 2, 4, 6, 8, *or* 0.
5, *if the units' figure is* 5 *or* 0.
3, *if the sum of the digits is divisible by* 3.
9, *if the sum of the digits is divisible by* 9.
10, *if the units' figure is* 0.

By applying the preceding tests tell which of the numbers 2, 5, 3, 9, 10, are exact divisors of:

1. 24	**4.** 225	**7.** 567	**10.** 3705	**13.** 7964
2. 35	**5.** 374	**8.** 654	**11.** 4839	**14.** 8730
3. 72	**6.** 460	**9.** 864	**12.** 7080	**15.** 9828

Illustrate these additional tests; a number is divisible:

16. By 6, if it is *even* and the sum of its digits is divisible by 3.

17. By 4, if its *two* right-hand digits are 0's or if the number expressed by them is divisible by 4.

18. By 25, if its *two* right-hand digits are 0's or if the number expressed by them is divisible by 25.

19. By 8, if its *three* right-hand digits are 0's or if the number expressed by them is divisible by 8.

20. By 11, if the *difference* between the *sums* of its *alternate* digits is zero or is divisible by 11.

Find which of the numbers 2, 3, 4, 5, 6, 8, 9, 10, 11, 25, are exact divisors of:

21. 17,418	**24.** 23,512	**27.** 210,705	**30.** 568,392
22. 97,317	**25.** 58,914	**28.** 105,372	**31.** 890,550
23. 77,670	**26.** 97,450	**29.** 953,000	**32.** 5,949,900

Illustrate the following with numbers:

33. If an *even* number is divisible by an *odd* number, it is divisible by *twice* that number.

34. An exact divisor of a number is an exact divisor of any number of times that number.

35. An exact divisor of each of two numbers is an exact divisor of their *sum* and of their *difference*.

FACTORING

Factors that are prime numbers are **prime factors**.

2, 2, and 8 are the prime factors of 12.

When numbers have no common factor except 1 they are **prime to each other**.

4 and 21 are prime to each other, though neither is prime.

The process of separating a number into its factors is **factoring**.

In separating a number into two factors, any exact divisor may be taken for one factor and the quotient for the other.

In $16 = 2^4$, the 4 is an **exponent**, indicating that 2 occurs 4 times as a factor of 16; that is, 2^4 means $2 \times 2 \times 2 \times 2$.

Written Exercises

1. Find the prime factors of 2295.

$$\begin{array}{r|r} 5 & 2295 \\ \hline 9 & 459 \\ \hline 3 & 51 \\ \hline & 17 \end{array}$$

$2295 = 5 \times 3^3 \times 17$

Since the units' figure of 2295 is 5, by what number is 2295 divisible? Dividing by 5, we find the other factor of 2295 to be 459.

Since the sum of the digits of 459 is divisible by 9, we divide by 9 and find the other factor, 51.

The sum of the digits of 51 is divisible by 3; then, 3 is one factor of 51 and 17 is the other.

Hence, the factors of 2295 found are 5, 9, 3, and 17, but the *prime factors* are 5, 3, 3, 3, and 17; that is, $2295 = 5 \times 3^3 \times 17$.

2. Find the prime factors of all the composite numbers from 1 to 100 and make a list of the prime numbers.

Find the prime factors of:

3. 144	**8.** 576	**13.** 624	**18.** 1050	**23.** 64,640
4. 260	**9.** 891	**14.** 594	**19.** 9702	**24.** 30,888
5. 315	**10.** 672	**15.** 768	**20.** 4620	**25.** 44,000
6. 480	**11.** 756	**16.** 936	**21.** 6250	**26.** 18,612
7. 875	**12.** 484	**17.** 864	**22.** 9360	**27.** 57,330

CANCELLATION

The process of shortening work in division by rejecting equal factors from both dividend and divisor is **cancellation**.

Written Exercises

1. Divide $8 \times 77 \times 15$ by $4 \times 44 \times 20$.

$$\frac{\overset{2}{\cancel{8}} \times \overset{7}{\cancel{77}} \times \overset{3}{\cancel{15}}}{\underset{\underset{2}{4}}{\cancel{4}} \times \cancel{44} \times \underset{4}{\cancel{20}}} = \frac{21}{8} = 2\tfrac{5}{8}$$

We indicate the division by writing $8 \times 77 \times 15$ above a line and $4 \times 44 \times 20$ below it (p. 47).

Since (p. 58, 3) dividing both dividend and divisor by 4 does not change the quotient, the factor 4 is canceled from 8 and 4, leaving 2 in the dividend and 1 (not written) in the divisor.

Similarly, 11 is canceled from 77 and 44; 5 from 15 and 20; and 2 from 2, left in the dividend, and from one of the 4's left in the divisor.

Since the factors now left in the dividend are prime to those left in the divisor, we have 7×3 divided by 2×4, or $\tfrac{21}{8}$. The quotient is $2\tfrac{5}{8}$.

Divide, using cancellation :

2. 4×99 by 11×12

3. 6×98 by 14×21

4. 9×56 by 48×12

5. 8×84 by 16×35

6. 99×96 by 88×12

7. 95×81 by 27×15

8. 63×48 by 32×54

9. 72×88 by 33×96

10. 36×84 by 108×10

11. 99×55 by 135×22

12. 72×80 by 144×10

13. 45×96 by 288×15

14. 14×900 by 56×75

15. 68×243 by 54×90

16. 60×156 by 52×36

17. 78×231 by 66×26

18. $14 \times 27 \times 24 \times 80$ by $32 \times 63 \times 45$

19. $25 \times 42 \times 18 \times 54$ by $12 \times 70 \times 30 \times 9$

20. $11 \times 81 \times 26 \times 100 \times 12$ by $39 \times 15 \times 90 \times 22$

21. $5000 \times 810 \times 1750$ by $625 \times 45 \times 30 \times 50 \times 36$

22. $243 \times 210 \times 147 \times 78 \times 88$ by $162 \times 63 \times 231 \times 32 \times 130$

Find the cost of:

23. 4 plows when 3 plows cost $31.50.

24. 6 wagons when 2 wagons cost $164.

25. 12 sleighs when 15 sleighs cost $525.

26. 10 saddles when 6 saddles cost $145.50.

27. 15 harrows when 10 harrows cost $152.50.

28. 24 wrenches when 15 wrenches cost $18.75.

29. 18 incubators when 14 incubators cost $262.50.

30. 33 grain drills when 27 grain drills cost $877.50.

31. 28 force pumps when 25 force pumps cost $187.50.

32. 49 tool cabinets when 14 tool cabinets cost $230.30.

33. 48 horse blankets when 30 horse blankets cost $127.50.

34. What number multiplied by 15 × 35 will give 14,700?

35. What number multiplied by 24 × 28 will give 18,144?

GREATEST COMMON DIVISOR

A number that is an exact divisor of two or more numbers is **a common divisor** of those numbers.

3 is a common divisor of 9 and 12.

The *largest* number that is an exact divisor of two or more numbers is their **greatest common divisor (g. c. d.)**.

6 is the g. c. d. of 12 and 18, though they have other common divisors.

Oral Exercises

Tell by inspection the greatest common divisor of:

1. 4 and 6	**5.** 12 and 16	**9.** 4, 6, and 8
2. 6 and 9	**6.** 10 and 15	**10.** 6, 9, and 12
3. 8 and 12	**7.** 14 and 18	**11.** 12, 15, and 18
4. 6 and 10	**8.** 16 and 20	**12.** 16, 24, and 40

Written Exercises

1. Find the greatest common divisor of 63, 105, and 231.

SOLUTION. — Factoring, we find that:

$$63 = 3 \times 3 \times 7,$$
$$105 = 3 \times 5 \times 7,$$
$$231 = 3 \times 7 \times 11.$$

Hence, the *common* divisors of 63, 105, and 231 are 3, 7, and 21 ($= 3 \times 7$); therefore, the *greatest* common divisor of these numbers is 21.

3	63	105	231
7	21	35	77
	3	5	11

g. c. d. $= 3 \times 7 = 21$

The work may be conveniently arranged as in the margin, the common prime factors being taken out successively until the quotients contain no common factor.

Find the product of all the common prime factors.

Find the greatest common divisor of:

2. 24 and 54	**15.** 78 and 90	**28.** 25, 40, and 75
3. 36 and 60	**16.** 64 and 84	**29.** 18, 45, and 99
4. 56 and 72	**17.** 72 and 96	**30.** 36, 72, and 90
5. 44 and 66	**18.** 84 and 98	**31.** 42, 54, and 96
6. 45 and 60	**19.** 52 and 78	**32.** 32, 72, and 120
7. 36 and 63	**20.** 56 and 84	**33.** 33, 55, and 132
8. 32 and 48	**21.** 63 and 108	**34.** 24, 72, and 180
9. 42 and 63	**22.** 48 and 112	**35.** 28, 48, and 128
10. 36 and 54	**23.** 60 and 144	**36.** 36, 54, 72, and 99
11. 28 and 70	**24.** 45 and 120	**37.** 30, 60, 75, and 90
12. 50 and 75	**25.** 64 and 144	**38.** 42, 56, 84, and 98
13. 48 and 72	**26.** 81 and 216	**39.** 32, 48, 80, and 96
14. 56 and 98	**27.** 72 and 288	**40.** 36, 54, 72, and 90

LEAST COMMON MULTIPLE

A number that is divisible by each of several numbers is a common multiple of those numbers.

24 is a common multiple of 2, 3, and 4.

The *least* number that is divisible by each of several numbers is their least common multiple (l. c. m.).

12 is the least common multiple of 2, 3, and 4.

Oral Exercises

Tell by inspection the least common multiple of:

1. 4 and 6　　　4. 4 and 10　　　7. 4, 6, and 8
2. 6 and 8　　　5. 6 and 15　　　8. 4, 8, and 10
3. 9 and 6　　　6. 8 and 12　　　9. 6, 8, and 12

Written Exercises

1. Find the least common multiple of 20, 28, and 55.

SOLUTION. — Factoring, we find that:

$$20 = 2 \times 2 \times 5,$$
$$28 = 2 \times 2 \times 7,$$
$$55 = 5 \times 11.$$

The factors of the l. c. m. are 2, 2 (the greatest number of 2's found in any number) and 5, 7, 11 (the only factors of any of them not already taken).

Hence, the l.c.m. of 20, 28, and 55 is $2 \times 2 \times 5 \times 7 \times 11$, or 1540.

2	20	28	55
2	10	14	55
5	5	7	55
	1	7	11

$2 \times 2 \times 5 \times 7 \times 11 = 1540$

The work may be conveniently arranged as in the margin, the l.c.m. being obtained by dividing the given numbers by any prime factor common to two or more of them, until the quotients are prime to each other and then finding the product of these divisors and the last quotients.

Find the product of all the different prime factors, each used the greatest number of times it occurs in any of the numbers.

Find the least common multiple of:

2. 12 and 18	**6.** 18 and 45	**10.** 42 and 56	**14.** 63 and 45
3. 15 and 25	**7.** 24 and 32	**11.** 48 and 54	**15.** 72 and 80
4. 16 and 24	**8.** 30 and 35	**12.** 32 and 72	**16.** 49 and 84
5. 21 and 28	**9.** 36 and 48	**13.** 60 and 42	**17.** 96 and 64

18. 8, 16, 20	**25.** 7, 15, 28, 35	**32.** 5, 14, 21, 25, 28
19. 4, 18, 21	**26.** 8, 27, 36, 42	**33.** 6, 16, 28, 32, 35
20. 9, 20, 24	**27.** 5, 28, 35, 55	**34.** 4, 18, 25, 36, 27
21. 18, 27, 30	**28.** 26, 12, 39, 18	**35.** 24, 36, 42, 48, 54
22. 25, 35, 42	**29.** 12, 63, 50, 36	**36.** 30, 45, 50, 63, 100
23. 12, 44, 24	**30.** 32, 40, 48, 50	**37.** 33, 36, 45, 66, 135
24. 64, 36, 54	**31.** 49, 56, 70, 98	**38.** 32, 40, 56, 35, 112

Find the greatest common divisor and the least common multiple of:

39. 24 and 60	**47.** 16, 40, 64	**55.** 8, 16, 24, 32
40. 28 and 42	**48.** 25, 50, 75	**56.** 12, 20, 36, 40
41. 30 and 45	**49.** 24, 60, 72	**57.** 15, 25, 45, 60
42. 48 and 64	**50.** 35, 56, 84	**58.** 21, 42, 56, 84
43. 45 and 54	**51.** 27, 63, 81	**59.** 27, 45, 54, 90
44. 72 and 84	**52.** 36, 48, 72	**60.** 24, 48, 96, 144
45. 66 and 88	**53.** 32, 80, 96	**61.** 22, 66, 99, 132
46. 42 and 63	**54.** 30, 45, 75	**62.** 32, 48, 80, 160

NOTE. — The *greatest common divisor* and the *least common multiple* of numbers that are large and difficult to factor may be found by a method given in the Appendix on pages 465 and 466. In practice, however, it is rarely necessary to resort to this method.

FRACTIONS

1. If any one thing is divided into 8 *equal* parts, what is one part called? What are 3 parts called? 5 parts? 7 parts?

2. One eighth is written ⅛; three fourths, ¾. Write: three eighths; five eighths; seven twelfths.

3. In the fraction ⅚, what does 6, the number below the line, show? 5, the number above the line?

One or more of the equal parts of a unit is a **fraction.**

The number that shows into how many equal parts the unit is divided is the **denominator.**

The number that shows how many parts form the fraction is the **numerator.**

The numerator and denominator of a fraction are its **terms.**

A fraction is read by reading the numerator (the number of parts) and then the denominator (telling the kind of parts).

Thus, ⅘ is read, "four fifths"; ²⁷⁄₂₀ is read, "twenty-seven twentieths."

A fraction that expresses tenths, or hundredths, or thousandths, etc., is a **decimal fraction,** or a **decimal.** All other fractions are **common fractions,** or simply **fractions.**

.5, .72, .625 are decimal fractions; ⅘, ¹¹⁄₁₂ are common fractions. The denominator of a decimal fraction is not usually written.

A number expressed by an integer and a fraction is a **mixed number.**

The mixed number 8⅘ is read, "eight and four fifths"; $4.33⅓ is read, "four dollars and thirty-three and one third cents."

79

READING AND WRITING FRACTIONS

Oral Exercises

Read :

1. $\frac{11}{12}$.
2. $\frac{14}{15}$
3. $\frac{17}{20}$
4. $\frac{18}{25}$

5. $\frac{87}{100}$
6. $\frac{184}{231}$
7. $\frac{881}{1000}$
8. $\frac{111}{1728}$

9. $15\frac{3}{4}$ feet
10. $36\frac{3}{8}$ inches
11. $25\frac{7}{12}$ yards
12. $278\frac{11}{16}$ tons

13. 10.12\frac{1}{2}$
14. 24.16\frac{2}{3}$
15. 15.28\frac{1}{4}$
16. 48.08\frac{3}{8}$

Written Exercises

Write with figures in common fractional form :

1. Three fortieths.
2. Eight twenty-fifths.
3. Nineteen thousandths.
4. Thirty-two thirty-fifths.
5. Seventy-nine hundredths.
6. Seven and seven twelfths.
7. Twenty-seven sixty-fourths.
8. Twelve and nineteen twentieths.
9. Seventy-seven one-hundred-fortieths.
10. Ten dollars and twelve and one half cents.
11. Six hundred forty and forty-three fiftieths.
12. Three hundred seventy-eight two-thousandths.
13. Twenty dollars and sixty-six and two thirds cents.
14. Two hundred one and thirty-one two-hundred-sixths.

Write in words :

15. $\frac{25}{48}$
16. $\frac{55}{88}$
17. $\frac{24}{75}$

18. $4\frac{3}{4}$
19. $7\frac{5}{8}$
20. $9\frac{7}{8}$

21. $\frac{87}{144}$
22. $\frac{188}{200}$
23. $\frac{71}{320}$

24. $\frac{127}{2500}$
25. $\frac{249}{4800}$
26. $57\frac{5}{12}$

27. $46.62\frac{1}{2}$
28. $25.33\frac{1}{2}$
29. $76.24\frac{5}{8}$

REDUCTION OF FRACTIONS

The process of changing the form of *any* number without changing its value is reduction.

Reduction of fractions to higher or lower terms.

1. Draw a line and mark it into halves ; then into fourths ; then into eighths.

$\frac{1}{2}$ = —— fourths $\frac{1}{2}$ = —— eighths

$\frac{2}{4}$ = —— half $\frac{4}{8}$ = —— half

2. How may the terms of $\frac{2}{4}$ be obtained from those of $\frac{1}{2}$? from those of $\frac{4}{8}$?

3. Which has the larger, or *higher*, terms, $\frac{1}{2}$ or $\frac{2}{4}$? the *lower* terms, $\frac{2}{4}$ or $\frac{4}{8}$? the *lowest* terms, $\frac{4}{8}$, $\frac{2}{4}$, or $\frac{1}{2}$?

Multiplying or dividing both terms of a fraction by the same number does not change its value.

When the terms of a fraction are prime to each other, the fraction is expressed in its **lowest terms.**

Oral Exercises

1–6. Change to twelfths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{5}{6}$.

7–13. Change to sixteenths: $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{8}$; $\frac{5}{8}$; $\frac{6}{8}$; $\frac{7}{8}$.

14–19. Change to twenty-fourths: $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{7}{8}$; $\frac{11}{12}$.

20–22. Reduce $\frac{8}{10}$ to halves ; $\frac{8}{12}$ to thirds; $\frac{12}{16}$ to fourths.

23–28. Reduce to lower terms : $\frac{2}{3}$; $\frac{6}{10}$; $\frac{4}{12}$; $\frac{8}{12}$; $\frac{10}{20}$; $\frac{12}{16}$.

29–34. Reduce to lowest terms : $\frac{6}{8}$; $\frac{6}{12}$; $\frac{8}{16}$; $\frac{12}{18}$; $\frac{15}{20}$; $\frac{16}{24}$.

At sight, express each of the following in lowest terms :

35–42. $\frac{6}{25}$ $\frac{8}{32}$ $\frac{9}{45}$ $\frac{7}{35}$ $\frac{8}{80}$ $\frac{6}{36}$ $\frac{9}{36}$ $\frac{8}{40}$

43–50. $\frac{4}{48}$ $\frac{8}{48}$ $\frac{6}{48}$ $\frac{8}{64}$ $\frac{7}{56}$ $\frac{9}{54}$ $\frac{6}{72}$ $\frac{9}{72}$

51–58. $\frac{16}{80}$ $\frac{16}{32}$ $\frac{18}{72}$ $\frac{24}{76}$ $\frac{24}{80}$ $\frac{12}{48}$ $\frac{16}{48}$ $\frac{12}{60}$

Written Exercises

1. Change ⅗ to seventy-fifths.

$$\frac{3 \times 15}{5 \times 15} = \frac{45}{75}$$

To produce a denominator 75, the given denominator must be multiplied by 75 ÷ 5, or 15.

Then, in order that the value of the fraction shall not be changed (p. 81) we multiply *both* terms of the fraction by 15, giving 45/75.

2–8. Reduce to forty-eighths : ½; ⅔; ¾; ⅙; ⅞; 5/12; 9/16.

9–16. Reduce to sixtieths : ⅓; ¼; ⅖; ⅚; 7/10; 7/12; 8/15; 11/20.

Reduce :

17. ½ to 36ths	**20.** ⅚ to 72ds	**23.** 9/20 to 100ths
18. ⅔ to 75ths	**21.** ⅘ to 85ths	**24.** 11/12 to 144ths
19. ¾ to 64ths	**22.** 7/8 to 96ths	**25.** 13/16 to 320ths

26. Reduce 45/60 to its lowest terms.

$$\frac{5)45}{5)60} = \frac{9}{12}$$

$$\frac{3)\,9}{3)12} = \frac{3}{4}$$

Or

$$\frac{15)45}{15)60} = \frac{3}{4}$$

In order that the value of the fraction shall not be changed (p. 81), we divide *both* terms by the same number. Dividing the terms by 5 and then by 3, we obtain ¾.

Since the terms of ¾ are prime to each other, 45/60 reduced to lowest terms is ¾.

We may obtain the same result directly by dividing both terms of the fraction 45/60 by 15, their greatest common divisor (p. 76).

Reduce to lowest terms :

27. 32/60	**32.** 35/42	**37.** 32/96	**42.** 75/100	**47.** 300/360
28. 24/30	**33.** 36/54	**38.** 56/80	**43.** 60/160	**48.** 154/231
29. 42/75	**34.** 56/64	**39.** 84/96	**44.** 72/144	**49.** 480/640
30. 45/72	**35.** 15/80	**40.** 70/84	**45.** 80/128	**50.** 256/480
31. 44/66	**36.** 54/72	**41.** 32/72	**46.** 64/320	**51.** 275/625

52–59. Reduce to fortieths: ½; ¾; ⅖; ⅜; 7/10; 34/40; 95/200; 378/560.

60–66. Reduce to hundredths: ¼; ⅖; 9/10; 11/20; 14/25; 130/400; 252/800.

Reduction of integers and mixed numbers to fractions.

1. How many half dollars are there in $1? in $2? in $2½?

2. How many quarters are there in $1? in $2? in $2¼?

3. How many fourths are there in 1? in 3? in 3¼? in 3¾?

<center>Oral Exercises</center>

1-6. Express as thirds: 1; 2⅓; 3⅔; 8; 6⅓; 9.

7-13. Reduce to fifths: 1; 3; 2⅕; 4⅖; 6; 5⅗; 7⅘.

Read these mixed numbers rapidly as fractions:

14-20. 4½	3¾	4⅓	2⅛	3⅙	5¼	3⅞
21-27. 2⅗	7¼	5⅜	3⅖	8¾	6⅚	4⅗
28-34. 4⅝	6⅕	8⅜	6⅞	3⅔	7⅘	5¾

<center>Written Exercises</center>

1. Reduce 48¾ to fourths.

48¾
  4
───
192   Since 1 unit = 4 fourths, 48 = 48 × 4 fourths, or 192
               fourths.
  3   192 fourths and 3 fourths are 195 fourths, or ¹⁹⁵⁄₄.
───
195; ¹⁹⁵⁄₄

Reduce to a fraction:

2. 17½	**8.** 18⅔	**14.** 37¾	**20.** 75¼	**26.** 34⁸⁄₂₀
3. 26¼	**9.** 35⅗	**15.** 58⅝	**21.** 63⅗	**27.** 57⁵⁄₁₂
4. 45⅔	**10.** 46⅜	**16.** 75⅚	**22.** 49⅝	**28.** 73⁴⁄₁₅
5. 37⅕	**11.** 67¼	**17.** 66¾	**23.** 36⁹⁄₁₀	**29.** 68⁷⁄₁₆
6. 66¾	**12.** 98⅔	**18.** 83⅞	**24.** 78¹¹⁄₁₂	**30.** 85¹¹⁄₂₄
7. 89⅙	**13.** 35⅞	**19.** 94⅘	**25.** 82¹⁵⁄₁₆	**31.** 97¹⁸⁄₂₅

84 FRACTIONS

Reduction of improper fractions to integers or to mixed numbers.

A fraction *indicates division*, and its value is the quotient of the numerator divided by the denominator.

A fraction whose numerator is less than its denominator is a **proper fraction**. The value of a proper fraction is less than 1.

A fraction whose numerator equals or exceeds its denominator is an **improper fraction**. The value of an improper fraction is 1 or more than 1.

Oral Exercises

1–8. Find the value of: $\frac{8}{8}$; $\frac{6}{3}$; $\frac{12}{4}$; $\frac{17}{4}$; $\frac{10}{5}$; $\frac{14}{5}$; $\frac{12}{8}$; $\frac{15}{8}$.

9–14. Express as a mixed number: $\frac{7}{2}$; $\frac{9}{4}$; $\frac{11}{3}$; $\frac{14}{5}$; $\frac{13}{6}$; $\frac{16}{8}$.

Read as an integer or as a mixed number:

15–22. $\frac{12}{2}$	$\frac{13}{3}$	$\frac{16}{4}$	$\frac{18}{5}$	$\frac{22}{6}$	$\frac{25}{5}$	$\frac{42}{6}$	$\frac{37}{14}$
23–30. $\frac{15}{4}$	$\frac{24}{6}$	$\frac{31}{8}$	$\frac{22}{3}$	$\frac{57}{8}$	$\frac{40}{4}$	$\frac{22}{3}$	$\frac{48}{12}$
31–38. $\frac{53}{6}$	$\frac{67}{8}$	$\frac{28}{2}$	$\frac{35}{6}$	$\frac{47}{5}$	$\frac{30}{6}$	$\frac{25}{4}$	$\frac{68}{16}$

Written Exercises

1. Reduce $\frac{254}{8}$ to a mixed number.

$\frac{254}{8} = 254 \div 8 = 31\frac{6}{8} = 31\frac{3}{4}$ How many eighths equal 1? How many times does 254 contain 8? How many units, then, are there in 254 eighths? how many eighths over? Express $\frac{6}{8}$ in its lowest terms. Then, $\frac{254}{8} = 31\frac{3}{4}$.

Reduce to an integer or to a mixed number:

2. $\frac{32}{2}$	**9.** $\frac{78}{4}$	**16.** $\frac{98}{20}$	**23.** $\frac{178}{8}$	**30.** $\frac{708}{8}$	**37.** $\frac{656}{12}$
3. $\frac{68}{4}$	**10.** $\frac{98}{5}$	**17.** $\frac{99}{18}$	**24.** $\frac{814}{6}$	**31.** $\frac{584}{6}$	**38.** $\frac{475}{25}$
4. $\frac{56}{8}$	**11.** $\frac{99}{6}$	**18.** $\frac{53}{24}$	**25.** $\frac{265}{4}$	**32.** $\frac{804}{9}$	**39.** $\frac{760}{16}$
5. $\frac{82}{6}$	**12.** $\frac{75}{9}$	**19.** $\frac{85}{26}$	**26.** $\frac{472}{16}$	**33.** $\frac{128}{16}$	**40.** $\frac{660}{36}$
6. $\frac{95}{5}$	**13.** $\frac{93}{12}$	**20.** $\frac{98}{16}$	**27.** $\frac{756}{6}$	**34.** $\frac{186}{12}$	**41.** $\frac{220}{50}$
7. $\frac{98}{8}$	**14.** $\frac{95}{16}$	**21.** $\frac{93}{24}$	**28.** $\frac{634}{4}$	**35.** $\frac{885}{20}$	**42.** $\frac{268}{48}$
8. $\frac{91}{7}$	**15.** $\frac{76}{16}$	**22.** $\frac{84}{36}$	**29.** $\frac{245}{7}$	**36.** $\frac{336}{24}$	**43.** $\frac{975}{100}$

Reduction of decimals to common fractions.

1. How many halves are there in .5, that is, in $\frac{5}{10}$?

2. How many fifths are there in .4, that is, in $\frac{4}{10}$?

3. How many fourths are there in .75, that is, in $\frac{75}{100}$?

Written Exercises

1. Reduce .075 to a common fraction.

Writing the decimal as a common fraction,
$.075 = \frac{75}{1000} = \frac{3}{40}$. we have $\frac{75}{1000}$, which reduced to its lowest terms equals $\frac{3}{40}$.

Omit the decimal point, supply the proper denominator, and reduce the resulting fraction to its lowest terms.

Reduce to a common fraction or to a mixed number:

2. .25	9. .125	16. 4.50	23. 60.28	30. 742.44
3. 5.6	10. 45.5	17. 39.2	24. 2.075	31. 25.025
4. .50	11. 5.40	18. 2.75	25. 76.85	32. 508.65
5. .08	12. 32.8	19. .6875	26. 8.0625	33. 4.5625
6. .16	13. .375	20. 4.875	27. .21875	34. 7.96875
7. .80	14. 8.35	21. .8125	28. 7.8125	35. 26.9375
8. 7.4	15. .625	22. .1875	29. .53125	36. 6.09375

37. Reduce $.37\frac{1}{2}$ to a common fraction.

$$.37\frac{1}{2} = \frac{37\frac{1}{2}}{100} = \frac{\frac{75}{2}}{100} = \frac{75}{200} = \frac{3}{8}$$

Reduce to a common fraction or to a mixed number:

38. $.33\frac{1}{3}$	42. $.62\frac{1}{2}$	46. $.06\frac{1}{4}$	50. $5.37\frac{1}{2}$	54. $3.91\frac{2}{3}$
39. $.12\frac{1}{2}$	43. $.31\frac{1}{4}$	47. $.41\frac{2}{3}$	51. $7.18\frac{3}{4}$	55. $6.68\frac{2}{3}$
40. $.16\frac{2}{3}$	44. $.66\frac{2}{3}$	48. $.83\frac{1}{3}$	52. $4.87\frac{1}{2}$	56. $9.81\frac{1}{4}$
41. $.08\frac{1}{3}$	45. $.58\frac{1}{3}$	49. $.56\frac{1}{4}$	53. $6.43\frac{3}{4}$	57. $8.93\frac{3}{4}$

86 FRACTIONS

Reduction of common fractions to decimals.

Written Exercises

1. Reduce $\frac{2}{3}$ to a decimal.

$$3)\overline{2.00}$$
$$\overline{.66\frac{2}{3}}$$

Since $\frac{2}{3}$ means $2 \div 3$, we may annex decimal zeros to the numerator and divide by the denominator as in the process.

It is seen that the division is not exact, but such a result as .66$\frac{2}{3}$ is often a convenient one to use when the terms of the fractional part are small.

2. Reduce $\frac{20}{27}$ to a five-place decimal.

$$.74074^+$$
$$27)\overline{20.0000}$$
$$\underline{18\ 9}$$
$$1\ 10$$
$$\underline{1\ 08}$$
$$200$$

After finding the first three figures of the quotient, it is seen that the new dividend is like the original dividend. Then, the next three figures will be like the first three, and so on.

Hence, $\frac{20}{27}$ reduced to a five-place decimal is .74074+.

NOTE. — A common fraction in its *lowest terms* cannot be reduced to an exact decimal, if its denominator contains any prime factor besides 2 and 5; for annexing a 0 to the numerator multiplies it by 10, introducing *only* the factors 2 and 5.

Place a decimal point after the numerator, annex zeros, and divide by the denominator.

One decimal zero is annexed for each decimal place required.

Reduce to decimals, expressing each in the form best adapted, and carrying none farther than five places:

3. $\frac{1}{4}$	11. $\frac{4}{6}$	19. $\frac{3}{16}$	27. $\frac{11}{12}$	35. $62\frac{1}{4}$
4. $\frac{3}{5}$	12. $\frac{4}{6}$	20. $\frac{7}{20}$	28. $\frac{15}{16}$	36. $58\frac{2}{5}$
5. $\frac{1}{8}$	13. $\frac{6}{7}$	21. $\frac{5}{16}$	29. $\frac{13}{25}$	37. $3.4\frac{1}{2}$
6. $\frac{1}{3}$	14. $\frac{5}{8}$	22. $\frac{6}{25}$	30. $\frac{12}{80}$	38. $.46\frac{1}{4}$
7. $\frac{1}{6}$	15. $\frac{8}{9}$	23. $\frac{7}{32}$	31. $\frac{21}{11}$	39. $.84\frac{1}{8}$
8. $\frac{3}{8}$	16. $\frac{7}{9}$	24. $\frac{9}{16}$	32. $\frac{27}{40}$	40. $.76\frac{2}{4}$
9. $\frac{5}{6}$	17. $\frac{9}{10}$	25. $\frac{5}{24}$	33. $\frac{25}{36}$	41. $.45\frac{3}{8}$
10. $\frac{4}{7}$	18. $\frac{5}{12}$	26. $\frac{8}{25}$	34. $\frac{19}{32}$	42. $.0\frac{5}{16}$

Reduction to least common denominator.

Fractions that have the same denominator are said to have a **common denominator**, and are called **similar fractions**.

Fractions that have the smallest common denominator possible are said to have the **least common denominator** (l. c. d.).

1. Reduce $\frac{5}{6}$, $\frac{3}{8}$, and $\frac{7}{12}$ to fractions having the l. c. d.

$$\frac{5}{6} \times \frac{4}{4} = \frac{20}{24}$$

Either by inspection or by the method on p. 77, we may discover that 24 is the least common multiple of the denominators of the given fractions; consequently,

$$\frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$$

we reduce the fractions to fractions whose common denominator is 24, as on p. 82.

Hence, by multiplying the terms of $\frac{5}{6}$ by 4, of $\frac{3}{8}$ by 3,

$$\frac{7}{12} \times \frac{2}{2} = \frac{14}{24}$$

and of $\frac{7}{12}$ by 2, we reduce the given fractions to the similar fractions $\frac{20}{24}$, $\frac{9}{24}$, and $\frac{14}{24}$, respectively.

NOTE. — *Fractions should first be reduced to their lowest terms.*

Reduce to fractions having the least common denominator:

2. $\frac{4}{5}, \frac{9}{10}$	**14.** $\frac{3}{10}, \frac{11}{15}$	**26.** $\frac{3}{4}, \frac{1}{2}, \frac{5}{8}$	**38.** $\frac{7}{10}, \frac{9}{20}, \frac{17}{30}$
3. $\frac{2}{4}, \frac{7}{10}$	**15.** $\frac{5}{12}, \frac{7}{10}$	**27.** $\frac{2}{3}, \frac{1}{2}, \frac{3}{4}$	**39.** $\frac{7}{12}, \frac{7}{16}, \frac{11}{24}$
4. $\frac{1}{8}, \frac{1}{12}$	**16.** $\frac{7}{15}, \frac{11}{12}$	**28.** $\frac{3}{4}, \frac{5}{6}, \frac{2}{3}$	**40.** $\frac{4}{15}, \frac{23}{30}, \frac{16}{20}$
5. $\frac{5}{6}, \frac{7}{10}$	**17.** $\frac{9}{10}, \frac{9}{16}$	**29.** $\frac{4}{5}, \frac{3}{8}, \frac{5}{6}$	**41.** $\frac{5}{36}, \frac{15}{16}, \frac{11}{24}$
6. $\frac{3}{8}, \frac{5}{12}$	**18.** $\frac{9}{20}, \frac{7}{30}$	**30.** $\frac{1}{3}, \frac{7}{5}, \frac{1}{4}$	**42.** $\frac{4}{21}, \frac{11}{12}, \frac{13}{14}$
7. $\frac{3}{4}, \frac{11}{14}$	**19.** $\frac{9}{16}, \frac{13}{20}$	**31.** $\frac{5}{8}, \frac{4}{5}, \frac{3}{4}$	**43.** $\frac{11}{16}, \frac{15}{18}, \frac{17}{24}$
8. $\frac{1}{3}, \frac{7}{15}$	**20.** $\frac{11}{12}, \frac{15}{18}$	**32.** $\frac{5}{8}, \frac{5}{6}, \frac{5}{12}$	**44.** $\frac{1}{8}, \frac{3}{4}, \frac{5}{6}, \frac{4}{15}$
9. $\frac{2}{3}, \frac{5}{14}$	**21.** $\frac{13}{18}, \frac{17}{24}$	**33.** $\frac{2}{3}, \frac{5}{6}, \frac{9}{16}$	**45.** $\frac{1}{2}, \frac{2}{4}, \frac{5}{6}, \frac{7}{16}$
10. $\frac{1}{4}, \frac{5}{18}$	**22.** $\frac{11}{12}, \frac{23}{30}$	**34.** $\frac{7}{8}, \frac{7}{16}, \frac{7}{24}$	**46.** $\frac{3}{8}, \frac{5}{16}, \frac{3}{4}, \frac{7}{32}$
11. $\frac{7}{8}, \frac{7}{20}$	**23.** $\frac{5}{24}, \frac{5}{16}$	**35.** $\frac{2}{3}, \frac{8}{15}, \frac{4}{25}$	**47.** $\frac{5}{12}, \frac{7}{8}, \frac{3}{4}, \frac{5}{36}$
12. $\frac{5}{6}, \frac{11}{18}$	**24.** $\frac{7}{32}, \frac{7}{12}$	**36.** $\frac{4}{5}, \frac{7}{12}, \frac{13}{20}$	**48.** $\frac{1}{2}, \frac{3}{5}, \frac{4}{15}, \frac{13}{20}$
13. $\frac{5}{8}, \frac{13}{18}$	**25.** $\frac{11}{14}, \frac{27}{20}$	**37.** $\frac{4}{5}, \frac{11}{24}, \frac{23}{36}$	**49.** $\frac{3}{8}, \frac{5}{6}, \frac{11}{36}, \frac{23}{24}$

ADDITION AND SUBTRACTION OF FRACTIONS

1. How many fourths are 3 fourths and 1 fourth? 3 fourths less 1 fourth?

How many units are 4 fourths? How many halves are 2 fourths?

$$\tfrac{3}{4} + \tfrac{1}{4} = \text{———.} \qquad\qquad \tfrac{3}{4} - \tfrac{1}{4} = \text{———.}$$

2. Can you add the fractions $\tfrac{3}{4}$ and $\tfrac{2}{3}$ without changing them? Can you subtract $\tfrac{2}{3}$ from $\tfrac{3}{4}$?

Reduce $\tfrac{3}{4}$ and $\tfrac{2}{3}$ to similar fractions.

$$\tfrac{3}{4} + \tfrac{2}{3} = \tfrac{9}{12} + \tfrac{8}{12} = \text{———.} \qquad \tfrac{3}{4} - \tfrac{2}{3} = \text{———} - \text{———} = \text{———.}$$

Fractions must be made similar before they can be added or subtracted.

Oral Exercises

In fractions, unless otherwise specified, express the answer as an integer, as a proper fraction in its lowest terms, or as a mixed number with the fraction in its lowest terms, as the case may be.

Add or subtract, as the signs indicate:

1. $\tfrac{2}{5} + \tfrac{2}{5}$	**9.** $\tfrac{1}{2} + \tfrac{2}{3}$	**17.** $\tfrac{3}{10} + \tfrac{2}{5}$	**25.** $\tfrac{1}{2} - \tfrac{3}{10}$
2. $\tfrac{7}{8} - \tfrac{3}{8}$	**10.** $\tfrac{3}{5} + \tfrac{1}{2}$	**18.** $\tfrac{1}{12} + \tfrac{2}{3}$	**26.** $\tfrac{4}{5} + \tfrac{7}{10}$
3. $\tfrac{5}{6} - \tfrac{1}{2}$	**11.** $\tfrac{3}{4} - \tfrac{1}{8}$	**19.** $\tfrac{9}{10} - \tfrac{3}{5}$	**27.** $\tfrac{3}{4} - \tfrac{1}{12}$
4. $\tfrac{1}{2} + \tfrac{2}{4}$	**12.** $\tfrac{1}{6} + \tfrac{1}{4}$	**20.** $\tfrac{5}{12} + \tfrac{1}{4}$	**28.** $\tfrac{5}{6} + \tfrac{7}{12}$
5. $\tfrac{3}{4} - \tfrac{3}{8}$	**13.** $\tfrac{4}{5} - \tfrac{1}{5}$	**21.** $\tfrac{7}{12} - \tfrac{1}{2}$	**29.** $\tfrac{3}{5} - \tfrac{1}{10}$
6. $\tfrac{5}{6} + \tfrac{1}{2}$	**14.** $\tfrac{3}{4} - \tfrac{2}{5}$	**22.** $\tfrac{3}{16} + \tfrac{3}{4}$	**30.** $\tfrac{2}{3} + \tfrac{4}{15}$
7. $\tfrac{7}{8} - \tfrac{1}{4}$	**15.** $\tfrac{3}{8} - \tfrac{1}{3}$	**23.** $1\tfrac{1}{2} - \tfrac{5}{6}$	**31.** $\tfrac{3}{4} + \tfrac{7}{16}$
8. $\tfrac{3}{4} + \tfrac{5}{8}$	**16.** $\tfrac{3}{5} - \tfrac{1}{6}$	**24.** $1\tfrac{5}{16} - \tfrac{3}{8}$	**32.** $\tfrac{7}{8} - \tfrac{11}{16}$

33. What is the sum of $\$\tfrac{1}{2}$ and $\$\tfrac{3}{4}$?

34. A newsboy earned $\$\tfrac{3}{4}$ and gave his mother $\$\tfrac{1}{2}$. What part of a dollar did he have left?

35. Mary spent $\tfrac{1}{8}$ and $\tfrac{1}{4}$ of her money. What part of her money had she left?

Written Exercises

1. Find the sum of $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{11}{12}$.

$$\frac{5}{6} = \frac{20}{24}$$
$$\frac{7}{8} = \frac{21}{24}$$
$$\frac{11}{12} = \frac{22}{24}$$
$$\frac{63}{24} = 2\frac{5}{8}$$

We have learned (p. 88) that the fractions must be changed to similar fractions before they can be added. The l. c. d. of the given fractions is 24; then, $\frac{5}{6} = \frac{20}{24}$, $\frac{7}{8} = \frac{21}{24}$, and $\frac{11}{12} = \frac{22}{24}$. Adding these similar fractions, we find the sum to be $\frac{63}{24}$, or $2\frac{15}{24}$, or $2\frac{5}{8}$.

Find the sum of:

2. $\frac{7}{8}, \frac{1}{6}$	**11.** $\frac{3}{8}, \frac{5}{16}$	**20.** $\frac{11}{12}, \frac{3}{8}$	**29.** $\frac{7}{10}, \frac{4}{15}$	**38.** $\frac{23}{24}, \frac{11}{32}$
3. $\frac{5}{6}, \frac{3}{8}$	**12.** $\frac{1}{4}, \frac{7}{18}$	**21.** $\frac{7}{10}, \frac{1}{6}$	**30.** $\frac{11}{12}, \frac{7}{10}$	**39.** $\frac{13}{18}, \frac{17}{36}$
4. $\frac{3}{8}, \frac{8}{10}$	**13.** $\frac{5}{6}, \frac{11}{18}$	**22.** $\frac{5}{12}, \frac{1}{6}$	**31.** $\frac{9}{10}, \frac{7}{16}$	**40.** $\frac{23}{36}, \frac{23}{24}$
5. $\frac{1}{4}, \frac{7}{10}$	**14.** $\frac{5}{6}, \frac{7}{12}$	**23.** $\frac{9}{10}, \frac{3}{4}$	**32.** $\frac{11}{16}, \frac{7}{12}$	**41.** $\frac{17}{18}, \frac{11}{36}$
6. $\frac{3}{8}, \frac{1}{12}$	**15.** $\frac{5}{8}, \frac{7}{18}$	**24.** $\frac{11}{16}, \frac{5}{8}$	**33.** $\frac{7}{12}, \frac{7}{30}$	**42.** $\frac{23}{36}, \frac{7}{16}$
7. $\frac{1}{8}, \frac{1}{10}$	**16.** $\frac{2}{3}, \frac{9}{20}$	**25.** $\frac{17}{18}, \frac{3}{4}$	**34.** $\frac{17}{24}, \frac{9}{16}$	**43.** $\frac{27}{40}, \frac{13}{36}$
8. $\frac{5}{6}, \frac{1}{15}$	**17.** $\frac{3}{8}, \frac{9}{14}$	**26.** $\frac{11}{16}, \frac{1}{6}$	**35.** $\frac{13}{18}, \frac{11}{24}$	**44.** $\frac{23}{48}, \frac{11}{36}$
9. $\frac{7}{8}, \frac{9}{20}$	**18.** $\frac{7}{8}, \frac{13}{18}$	**27.** $\frac{23}{24}, \frac{5}{8}$	**36.** $\frac{23}{24}, \frac{5}{12}$	**45.** $\frac{5}{48}, \frac{23}{40}$
10. $\frac{5}{8}, \frac{8}{15}$	**19.** $\frac{5}{8}, \frac{5}{24}$	**28.** $\frac{13}{12}, \frac{3}{8}$	**37.** $\frac{11}{16}, \frac{23}{40}$	**46.** $\frac{11}{24}, \frac{13}{40}$

47. Find the difference between $\frac{7}{8}$ and $\frac{5}{12}$.

$$\frac{7}{8} = \frac{21}{24}$$
$$\frac{5}{12} = \frac{10}{24}$$
$$\overline{\frac{11}{24}}$$

We have learned that the fractions must be changed to similar fractions before one can be subtracted from the other. On reducing them to their least common denominator, $\frac{7}{8}$ becomes $\frac{21}{24}$ and $\frac{5}{12}$ becomes $\frac{10}{24}$. The difference between $\frac{21}{24}$ and $\frac{10}{24}$ is seen to be $\frac{11}{24}$.

48–92. Find the difference of the fractions in exercises **2–46**.

Add:

93. $\frac{1}{8}, \frac{3}{4}, \frac{5}{12}$	**97.** $\frac{3}{4}, \frac{4}{5}, \frac{7}{10}$	**101.** $\frac{3}{8}, \frac{2}{5}, \frac{1}{2}, \frac{5}{6}$
94. $\frac{1}{4}, \frac{5}{6}, \frac{7}{12}$	**98.** $\frac{5}{6}, \frac{3}{8}, \frac{9}{16}$	**102.** $\frac{3}{4}, \frac{5}{6}, \frac{1}{3}, \frac{1}{2}$
95. $\frac{3}{8}, \frac{5}{6}, \frac{11}{16}$	**99.** $\frac{5}{8}, \frac{3}{4}, \frac{17}{16}$	**103.** $\frac{3}{8}, \frac{5}{6}, \frac{5}{8}, \frac{3}{4}$
96. $\frac{3}{4}, \frac{7}{8}, \frac{13}{18}$	**100.** $\frac{7}{8}, \frac{5}{16}, \frac{11}{12}$	**104.** $\frac{5}{6}, \frac{7}{8}, \frac{11}{12}, \frac{13}{36}$

ADDITION AND SUBTRACTION OF MIXED NUMBERS

Oral Exercises

Find the sum :

Add the fractions and integers separately; then combine the sums.

| 1. $5\frac{1}{8}$ | 4. $9\frac{5}{8}$ | 7. $5\frac{3}{4}$ | 10. $4\frac{5}{8}$ | 13. $7\frac{3}{4}$ |
| $2\frac{1}{6}$ | $4\frac{1}{2}$ | $3\frac{1}{4}$ | $2\frac{2}{3}$ | $4\frac{5}{12}$ |

| 2. $6\frac{4}{5}$ | 5. $8\frac{1}{4}$ | 8. $5\frac{3}{8}$ | 11. $8\frac{1}{2}$ | 14. $9\frac{1}{4}$ |
| $1\frac{1}{2}$ | $3\frac{1}{8}$ | $2\frac{1}{2}$ | $4\frac{1}{8}$ | $6\frac{3}{10}$ |

| 3. $8\frac{5}{6}$ | 6. $5\frac{7}{8}$ | 9. $6\frac{5}{8}$ | 12. $9\frac{3}{4}$ | 15. $5\frac{3}{8}$ |
| $6\frac{3}{4}$ | $4\frac{1}{4}$ | $3\frac{1}{4}$ | $5\frac{1}{6}$ | $2\frac{7}{12}$ |

16–30. Find the difference in exercises **1–15** by subtracting the fractions and integers separately.

31. A $3\frac{1}{2}$-inch nail was driven through a $1\frac{1}{4}$-inch board into a post. How far did it extend into the post ?

32. If the price of pineapples per crate was \$$2\frac{1}{4}$ in season and \$$3\frac{1}{4}$ more at Christmas, what was the Christmas price ?

33. A steamship crossed the Atlantic in $5\frac{1}{2}$ days. A yacht took $6\frac{5}{8}$ days longer. How long did the yacht take ?·

34. If the average cost per pound to raise rice and get it ready for market in the United States is $2\frac{1}{16}$ ¢, what is the profit per pound when the market price is $4\frac{3}{4}$ ¢ ? $6\frac{1}{2}$ ¢ ?

35. The United States military rifle weighs $9\frac{1}{4}$ pounds, the cartridges to go with it $6\frac{4}{5}$ pounds. When a soldier carries rifle and cartridges, how many pounds does he carry ?

36. How much does it cost per bushel to transport wheat from Chicago to New York by water, if the charge to Buffalo is $1\frac{3}{4}$ ¢, from there to New York $2\frac{1}{2}$ ¢, and the elevator charge is $\frac{1}{2}$ ¢ ?

Written Exercises

1. Add $23\frac{1}{2}$, $49\frac{4}{5}$, and $36\frac{3}{4}$.

$23\frac{1}{2} = 23\frac{10}{20}$ Reducing the fractions to fractions having the l. c. d.,

$49\frac{4}{5} = 49\frac{16}{20}$ we have $\frac{10}{20}$, $\frac{16}{20}$, and $\frac{15}{20}$, whose sum is $\frac{41}{20}$, or $2\frac{1}{20}$. Writ-

$36\frac{3}{4} = \underline{36\frac{15}{20}}$ ing $\frac{1}{20}$ under the fractions and adding the 2 to the in-

$\phantom{36\frac{3}{4} = } 110\frac{1}{20}$ tegers, we find the whole sum to be $110\frac{1}{20}$.

Add:

2. $47\frac{3}{4}$	**5.** $43\frac{1}{8}$	**8.** $54\frac{3}{8}$	**11.** $7.3\frac{2}{5}$	**14.** $.40\frac{1}{6}$
$\underline{26\frac{1}{2}}$	$\underline{25\frac{3}{4}}$	$\underline{16\frac{1}{2}}$	$\underline{5.6\frac{1}{4}}$	$\underline{.29\frac{2}{3}}$
3. $38\frac{5}{6}$	**6.** $72\frac{2}{3}$	**9.** $85\frac{1}{6}$	**12.** $8.7\frac{3}{5}$	**15.** $.95\frac{3}{4}$
$\underline{12\frac{2}{3}}$	$\underline{43\frac{5}{6}}$	$\underline{55\frac{2}{4}}$	$\underline{4.5\frac{3}{4}}$	$\underline{.48\frac{5}{6}}$
4. $54\frac{3}{8}$	**7.** $65\frac{1}{4}$	**10.** $73\frac{3}{4}$	**13.** $6.4\frac{3}{8}$	**16.** $.63\frac{5}{8}$
$\underline{16\frac{1}{2}}$	$\underline{34\frac{3}{4}}$	$\underline{32\frac{1}{6}}$	$\underline{3.7\frac{7}{8}}$	$\underline{.25\frac{5}{8}}$

17. From $84\frac{3}{8}$ subtract $27\frac{3}{4}$.

 Reducing the fractions to similar fractions,

$84\frac{3}{8} = 84\frac{3}{8} = 83\frac{11}{8}$ we see that $\frac{6}{8}$ cannot be subtracted from $\frac{3}{8}$; hence

$27\frac{3}{4} = 27\frac{6}{8} = \underline{27\frac{6}{8}}$ 1 is taken from 84, changed to eighths, and com-

$\phantom{27\frac{3}{4} = 27\frac{6}{8} = } 56\frac{5}{8}$ bined with $\frac{3}{8}$; then, $\frac{6}{8}$ is subtracted from $\frac{11}{8}$ and

 27 from 83, giving the remainder $56\frac{5}{8}$.

18-32. Subtract in exercises 2-16.

Add:

33. $19\frac{1}{8}$	**35.** $25\frac{1}{6}$	**37.** $44\frac{1}{2}$	**39.** $3.7\frac{1}{8}$	**41.** $.56\frac{2}{3}$
$28\frac{1}{2}$	$38\frac{5}{8}$	$68\frac{2}{3}$	$9.1\frac{1}{2}$	$.75\frac{3}{4}$
$\underline{32\frac{1}{6}}$	$\underline{74\frac{1}{2}}$	$\underline{47\frac{3}{4}}$	$\underline{8.4\frac{1}{4}}$	$\underline{.94\frac{5}{6}}$
34. $24\frac{1}{2}$	**36.** $41\frac{1}{3}$	**38.** $96\frac{1}{4}$	**40.** $7.7\frac{2}{3}$	**42.** $.48\frac{7}{10}$
$85\frac{3}{4}$	$9\frac{1}{2}$	$74\frac{5}{6}$	$1.3\frac{3}{4}$	$.64\frac{7}{12}$
$\underline{62\frac{1}{3}}$	$\underline{37\frac{3}{8}}$	$\underline{42\frac{3}{8}}$	$\underline{.5\frac{7}{8}}$	$\underline{.58\frac{7}{20}}$

Written Exercises

1. When the average load of freight per train was 140⅘ tons in Great Britain, it was 287⅘ tons in the United States. How much more did a freight train load in this country weigh?

2. Find the difference in height between a locomotive wheel 6⅛ feet high and a freight car wheel 2⅔ feet high.

3. It took a steam locomotive 3⅘ minutes and an electric locomotive 2⅓ minutes to attain a speed of fifty miles per hour. How much more quickly did the latter attain that speed?

4. The United States 12-inch naval gun weighs 53¾ tons, the German gun of the same bore, 48⅞ tons. How much less does the German gun weigh than the American gun?

5. Find the cost of this miner's outfit: drilling machine for coal, $8¼; drilling machine for rock, $8⅝; pick, $¾; shovel, $½; drill, $1½; needle, $¼; scraper, $½; ax, $1; saw, $¾.

6. Add seventy-five and seven eighths, forty-three and five sixths, and ninety-one and five twelfths.

7. The cost per mile of running a train by steam was: coal, 14¼¢; water, ½¢; crew, 12¼¢; maintenance, 6¼¢; supplies, ⅝¢. The cost by electricity was: power, 10⅜¢; crew, 6⅝¢; maintenance, 4¢; supplies, ¼¢. Find the saving per mile with electricity.

8. The equipment of an American soldier in war time weighs 55¼ lb. If the British soldier carries 3⅛ lb. less and the German soldier 3½ lb. more, what is the weight of each one's equipment?

9. Following is the amount of the principal animal foods annually consumed per person in a certain town: 19½ lb. butter, 3¼ lb. ready-cooked meat, 34⅞ lb. fish, 154¾ lb. beef, 25½ lb. pork, 27⅚ lb. fowl and game. Find the total amount.

MULTIPLICATION OF FRACTIONS

Multiplication of a fraction by an integer.

1. How many tenths are $\frac{3}{10} + \frac{3}{10} + \frac{3}{10}$, or 3 times $\frac{3}{10}$?

In multiplying $\frac{3}{10}$ by 3 we do not change the denomination, tenths, but *multiply* the number of tenths, or the *numerator*, by 3.

2. Multiply $\frac{3}{10}$ by 5 without changing the denomination. What additional step must be taken to express the product $\frac{15}{10}$ in the *form*, $\frac{3}{2}$?

3. Since 5 times $\frac{1}{10} = \frac{5}{10}$, or $\frac{1}{2}$, how many *halves* are there in 5 times $\frac{3}{10}$?

In multiplying $\frac{3}{10}$ by 5 we do not change the number of parts, but we increase the size of each part by *dividing* the *denominator* by 5.

Multiplying the numerator or dividing the denominator of a fraction by an integer multiplies the fraction by that integer.

The second method should be used whenever the denominator exactly contains the multiplier.

Oral Exercises

Multiply, and explain each multiplication :

1. $\frac{2}{5}$ by 2	**4.** $\frac{1}{8}$ by 4	**7.** $\frac{7}{10}$ by 5	**10.** $\frac{11}{15}$ by 5
2. $\frac{3}{4}$ by 2	**5.** $\frac{2}{3}$ by 2	**8.** $\frac{5}{12}$ by 6	**11.** $\frac{7}{18}$ by 6
3. $\frac{1}{6}$ by 3	**6.** $\frac{6}{8}$ by 7	**9.** $\frac{3}{16}$ by 4	**12.** $\frac{5}{12}$ by 5

13. Find 2 times $\frac{1}{4}$; 2 times 3 ; 2 times $3\frac{1}{4}$.

14. Multiply $4\frac{2}{3}$ by 6 by finding the sum of 6 times $\frac{2}{3}$ and 6 times 4.

Multiply :

15. $4\frac{1}{2}$ by 2	**18.** $5\frac{1}{8}$ by 9	**21.** $4\frac{3}{4}$ by 8	**24.** $9\frac{1}{10}$ by 5
16. $3\frac{1}{2}$ by 4	**19.** $6\frac{1}{4}$ by 8	**22.** $5\frac{1}{6}$ by 6	**25.** $6\frac{1}{12}$ by 4
17. $2\frac{1}{8}$ by 3	**20.** $7\frac{2}{5}$ by 5	**23.** $6\frac{2}{8}$ by 4	**26.** $4\frac{5}{12}$ by 6

Written Exercises

1. Find 8 times $\frac{13}{24}$.

$$8 \times \frac{13}{24} = \frac{\cancel{8} \times 13}{\cancel{24}\atop 3} = \frac{13}{3} = 4\frac{1}{3}$$

We have learned (p. 93) that to find 8 times $\frac{13}{24}$ we may multiply the numerator 13 by 8. Indicating this as in the process, canceling, and reducing, we find the result to be $4\frac{1}{3}$.

Find :   Multiply, timing yourself :

2. $9 \times \frac{11}{12}$	**8.** $4 \times \frac{21}{32}$	**14.** $\frac{9}{10}$ by 25	**20.** $\frac{19}{36}$ by 45
3. $4 \times \frac{19}{24}$	**9.** $8 \times \frac{17}{36}$	**15.** $\frac{11}{12}$ by 32	**21.** $\frac{53}{60}$ by 24
4. $5 \times \frac{11}{25}$	**10.** $7 \times \frac{27}{42}$	**16.** $\frac{9}{16}$ by 40	**22.** $\frac{41}{48}$ by 64
5. $6 \times \frac{15}{18}$	**11.** $6 \times \frac{15}{32}$	**17.** $\frac{13}{24}$ by 36	**23.** $\frac{35}{48}$ by 72
6. $9 \times \frac{7}{24}$	**12.** $9 \times \frac{41}{60}$	**18.** $\frac{24}{35}$ by 28	**24.** $\frac{19}{100}$ by 75
7. $7 \times \frac{17}{28}$	**13.** $8 \times \frac{65}{72}$	**19.** $\frac{17}{32}$ by 48	**25.** $\frac{67}{132}$ by 88

26. Multiply $45\frac{3}{8}$ by 26.

$\begin{array}{r} 45\frac{3}{8} \\ 26 \\ \hline 9\frac{3}{4} \\ 270 \\ 90 \\ \hline 1179\frac{3}{4} \end{array}$ Multiplying the fraction $\frac{3}{8}$ by 26, we find the product to be $9\frac{3}{4}$.

Multiplying the integer 45 by 6 and by 20 (2 tens), writing these partial products in their proper places and adding, we find the product of $45\frac{3}{8}$ multiplied by 26 to be $1179\frac{3}{4}$.

Multiply :

27. $8\frac{3}{4}$ by 12	**33.** $12\frac{1}{4}$ by 16	**39.** $94\frac{3}{8}$ by 55
28. $6\frac{7}{8}$ by 20	**34.** $41\frac{2}{8}$ by 24	**40.** $36\frac{1}{2}$ by 27
29. $5\frac{5}{6}$ by 15	**35.** $42\frac{3}{4}$ by 12	**41.** $35\frac{1}{4}$ by 28
30. $24\frac{1}{8}$ by 6	**36.** $51\frac{1}{4}$ by 25	**42.** $8.4\frac{7}{8}$ by 78
31. $16\frac{3}{8}$ by 4	**37.** $56\frac{2}{3}$ by 36	**43.** $5.6\frac{1}{6}$ by 49
32. $34\frac{1}{6}$ by 8	**38.** $68\frac{1}{8}$ by 18	**44.** $8.5\frac{4}{5}$ by 96

Find the cost of:

1. 4 plows at $7¾ each.

2. 5 wagons at 48\frac{9}{10}$ each.

3. 7 tons of hay at $19½ per ton.

4. 6 barrels of potatoes at $1⅞ per barrel.

5. Dr. Jones pays $3½ a month for his telephone. How much does it cost him per year?

6. How far can James walk in 4 hours, if he walks 2⅞ miles an hour?

7. Mr. Seymour bought a set of 6 books for his library. How much did they cost, if the price was $1¾ each?

8. George earned 2\frac{7}{10}$ a week delivering papers. How much did he earn in 8 weeks?

9. One of the first locomotives built weighed 4½ tons. Find the weight of a modern locomotive that is 22 times as heavy.

10. A net for catching salmon has meshes 6¼ inches wide. If the net has 24 meshes in its width, how wide is it?

11. The highest pay of a U. S. private soldier is 83⅛¢ per day. How much does he receive per month of 30 days?

12. An automobile used 3¼ gallons of gasoline per 100 miles. How much gasoline did it use on a trip of 600 miles?

13. The mills of a New England city make 1⅛ miles of cotton cloth per minute. How many miles of this cloth are made in an 8-hour day?

14. According to the *Scientific American*, a large railroad company imported hardwood ties at a cost of $1⅔ each. What was the cost of ties for one mile of track, if 2600 were used?

Multiplication of an integer by a fraction.

1. $12 = 3 + 3 + 3 + 3.$ $12 = 2 + 2 + 2 + 2 + 2 + 2.$

What is 1 of the 4 equal parts of 12, or $\frac{1}{4}$ of 12?

Find $\frac{2}{4}$ of 12.   Find $\frac{1}{6}$ of 12, then $\frac{5}{6}$ of 12.

2. Find $\frac{2}{5}$ of 50 ; $\frac{1}{5}$ of 100.   Compare the results.

How does 2 times $\frac{1}{5}$ of 50 compare with $\frac{1}{5}$ of 2 times 50?

Tell two ways of finding $\frac{2}{5}$ of 50, or of *multiplying* 50 by $\frac{2}{5}$.

3. Find $\frac{2}{5}$ of 8 by finding $\frac{1}{5}$ of 2 times 8.   Find $\frac{2}{5}$ of 35 by finding $\frac{1}{5}$ of 2 times 35 ; also find $\frac{2}{5}$ of 35 *in a shorter way*.

4. Find $\frac{2}{3}$ of 11 ; $\frac{2}{3}$ of 21 ; $\frac{4}{5}$ of 7 ; $\frac{2}{5}$ of 45.

Finding a fractional part of a number is **multiplying by a fraction.**

In using a fraction as a multiplier, we *multiply by the numerator and divide by the denominator.*

We may perform these two operations in either order, but it is better to divide first when the multiplicand exactly contains the denominator.

Oral Exercises

Find :

1. $\frac{1}{3}$ of 12	**5.** $\frac{1}{4}$ of 20	**9.** $\frac{1}{6}$ of 30	**13.** $\frac{1}{5}$ of 15
2. $\frac{2}{3}$ of 12	**6.** $\frac{2}{4}$ of 20	**10.** $\frac{2}{3}$ of 30	**14.** $\frac{4}{5}$ of 11
3. $\frac{4}{3}$ of 12	**7.** $\frac{5}{4}$ of 20	**11.** $\frac{3}{6}$ of 12	**15.** $\frac{3}{4}$ of 13
4. $\frac{5}{3}$ of 12	**8.** $\frac{9}{4}$ of 20	**12.** $\frac{1}{6}$ of 18	**16.** $\frac{7}{5}$ of 16

Multiply :

17. 24 by $\frac{1}{6}$	**22.** 12 by $\frac{3}{8}$	**27.** 36 by $\frac{3}{4}$	**32.** 90 by $\frac{3}{10}$
18. 20 by $\frac{1}{5}$	**23.** 33 by $\frac{1}{3}$	**28.** 54 by $\frac{5}{6}$	**33.** 12 by $\frac{7}{10}$
19. 18 by $\frac{5}{6}$	**24.** 40 by $\frac{5}{8}$	**29.** 14 by $\frac{5}{2}$	**34.** 36 by $\frac{1}{12}$
20. 10 by $\frac{4}{5}$	**25.** 11 by $\frac{3}{8}$	**30.** 15 by $\frac{5}{6}$	**35.** 24 by $\frac{7}{12}$
21. 10 by $\frac{4}{7}$	**26.** 96 by $\frac{7}{8}$	**31.** 12 by $\frac{2}{6}$	**36.** 84 by $\frac{5}{12}$

<center>**Written Exercises**</center>

1. Find $\frac{5}{6}$ of 42 ;   also $\frac{5}{6}$ of 22.

$\frac{5}{6}$ of 42 = 5 times $\frac{1}{6}$ of 42 $\frac{5}{6}$ of 22 = $\frac{1}{6}$ of 5 times 22

$$6)\overline{42}$$
$$7$$
$$\underline{5}$$
$$35$$

$$\begin{array}{c} 22 \\ \underline{5} \\ 6)\overline{110} \\ 18\frac{1}{3} \end{array}$$ Or, $\dfrac{5 \times \overset{11}{\cancel{22}}}{\underset{3}{\cancel{6}}} = \dfrac{55}{3} = 18\frac{1}{3}$

Since 6 is exactly contained in 42, in finding $\frac{5}{6}$ of 42 it is easier and shorter to divide by the denominator before multiplying by the numerator, obtaining $\frac{1}{6}$ of 42, then $\frac{5}{6}$ of 42.

Since 6 is not exactly contained in 22, it is easier and shorter to multiply before dividing. When the integer and the denominator contain a common factor, cancel it.

Find in the shortest way: Multiply, timing yourself:

2. $\frac{2}{3}$ of 87	**10.** $\frac{2}{3}$ of $1.20	**18.** 45 by $\frac{5}{6}$	**26.** 14.6 by $\frac{2}{3}$
3. $\frac{2}{3}$ of 97	**11.** $\frac{1}{4}$ of $1.12	**19.** 50 by $\frac{5}{6}$	**27.** 1.32 by $\frac{3}{8}$
4. $\frac{3}{4}$ of 76	**12.** $\frac{5}{6}$ of $1.08	**20.** 84 by $\frac{3}{8}$	**28.** .244 by $\frac{1}{8}$
5. $\frac{4}{5}$ of 65	**13.** $\frac{3}{8}$ of $2.16	**21.** 68 by $\frac{1}{4}$	**29.** 4.08 by $\frac{1}{6}$
6. $\frac{5}{8}$ of 46	**14.** $\frac{5}{6}$ of $1.00	**22.** 88 by $\frac{3}{4}$	**30.** 295 by $\frac{7}{10}$
7. $\frac{3}{4}$ of 84	**15.** $\frac{2}{3}$ of $3.25	**23.** 77 by $\frac{7}{8}$	**31.** 507 by $\frac{5}{12}$
8. $\frac{7}{8}$ of 75	**16.** $\frac{2}{3}$ of $4.27	**24.** 85 by $\frac{3}{10}$	**32.** 348 by $\frac{15}{18}$
9. $\frac{4}{5}$ of 89	**17.** $\frac{7}{8}$ of $5.84	**25.** 92 by $\frac{5}{12}$	**33.** 864 by $\frac{13}{20}$

34. Multiply 285 by $27\frac{5}{6}$.

$$\begin{array}{r} 285 \\ \underline{27\frac{5}{6}} \\ 6)\overline{1425} \\ \hline 237\frac{1}{2}, \text{ product by } \frac{5}{6} \\ 1995 \text{ , product by } 7 \\ \underline{570 \text{ , product by } 20} \\ 7932\frac{1}{2}, \text{ product by } 27\frac{5}{6} \end{array}$$

Multiplying 285 by the numerator 5 and dividing the product by 6, we find that $237\frac{1}{2}$ is the product by $\frac{5}{6}$.

Multiplying by 7 and by 20 (2 tens), writing these partial products in their proper places as in the process, and then adding, we find that the product of 285 multiplied by $27\frac{5}{6}$ is $7932\frac{1}{2}$.

Multiply:

35. 27 by $4\frac{1}{2}$

36. 34 by $6\frac{3}{4}$

37. 45 by $5\frac{3}{4}$.

38. 72 by $3\frac{5}{8}$

39. 64 by $2\frac{1}{6}$

40. 94 by $3\frac{7}{8}$

41. 37 by $1.7\frac{3}{4}$

42. 46 by $.25\frac{1}{2}$

43. 54 by $4.2\frac{5}{8}$

44. 6.2 by $18\frac{3}{4}$

45. .79 by $38\frac{1}{2}$

46. .75 by $26\frac{5}{8}$

47. \$1.34 by $36\frac{1}{2}$

48. \$3.41 by $91\frac{1}{4}$

49. \$6.04 by $43\frac{3}{4}$

50. \$4.67 by $56\frac{3}{4}$

51. \$5.28 by $87\frac{3}{4}$

52. \$8.42 by $74\frac{1}{4}$

Note.— While in some instances the seller collects a whole cent for any part of a cent, in others anything less than half a cent is discarded.

In this book when directed to give results "to the nearest cent," exactly $\frac{1}{2}$ cent is to be counted as a whole cent.

Find, to the nearest cent, the cost of:

53. $1\frac{1}{2}$ dozen oranges at $35\cancel{c}$ a dozen.

54. $2\frac{1}{2}$ dozen pineapples at $15\cancel{c}$ each.

55. $3\frac{3}{8}$ dozen bananas at $20\cancel{c}$ a dozen.

56. $1\frac{1}{4}$ pounds of meat at $25\cancel{c}$ a pound.

57. $\frac{3}{4}$ of a pound of tea at $50\cancel{c}$ a pound.

58. $2\frac{1}{2}$ pounds of coffee at $30\cancel{c}$ a pound.

59. $1\frac{3}{4}$ pounds of ginger at $28\cancel{c}$ a pound.

60. $4\frac{1}{2}$ pounds of cheese at $21\cancel{c}$ a pound.

61. $3\frac{7}{8}$ bushels of apples at $75\cancel{c}$ a bushel.

62. An ordinary stock car will hold 75 hogs or $1\frac{3}{8}$ times as many sheep. How many sheep will it hold?

63. If $\frac{5}{8}$ of a magazine of 312 pages consists of advertisements, how many pages are left for reading matter?

64. A horse can carry a load of 250 pounds and a camel can carry $\frac{8}{5}$ as much. How many pounds can a camel carry?

65. The velocity of sound in air at a certain temperature is 1112 feet per second, and in copper it is $11\frac{1}{6}$ times as great. Find the velocity of sound in copper.

Multiplication of a fraction by a fraction.

1. A boy who had $\frac{1}{2}$ of a dollar gave $\frac{1}{2}$ of his money to his sister. What part of a dollar did he give her? $\frac{1}{2}$ of $\frac{1}{2} = ?$

2. Find the value of $\frac{1}{3}$ of $\frac{1}{2}$; of $\frac{1}{3} \times \frac{1}{2}$.

Since $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$, how many sixths are there in $\frac{2}{3} \times \frac{1}{2}$? how many thirds?

3. How many twelfths are there in $\frac{1}{3}$ of $\frac{1}{4}$? in $\frac{2}{3}$ of $\frac{1}{4}$? in $\frac{2}{3}$ of $\frac{3}{4}$? in $\frac{1}{3} \times \frac{3}{4}$? how many halves?

4. Compare $\dfrac{2}{3} \times \dfrac{1}{2}$ with $\dfrac{2 \times 1}{3 \times 2}$; $\dfrac{2}{3} \times \dfrac{3}{4}$ with $\dfrac{2 \times 3}{3 \times 4}$.

Finding a fractional part of a fraction is **multiplying a fraction by a fraction.**

The word "of" between two fractions signifies multiplication, and when the sign × is used between two fractions, it may be read "of."

Such expressions are sometimes called **compound fractions.**

To multiply a fraction by a fraction, *multiply the numerators together for the numerator of the product and the denominators for the denominator of the product.*

Oral Exercises

Find quickly:

1. $\frac{1}{2}$ of $\frac{1}{6}$	**6.** $\frac{4}{5} \times \frac{1}{3}$	**11.** $\frac{3}{4}$ of $\frac{1}{6}$	**16.** $\frac{4}{5} \times \frac{5}{6}$
2. $\frac{1}{6}$ of $\frac{1}{3}$	**7.** $\frac{1}{2} \times \frac{2}{7}$	**12.** $\frac{2}{3}$ of $\frac{4}{5}$	**17.** $\frac{5}{8} \times \frac{2}{3}$
3. $\frac{1}{6}$ of $\frac{1}{4}$	**8.** $\frac{1}{4} \times \frac{4}{5}$	**13.** $\frac{1}{3}$ of $\frac{6}{7}$	**18.** $\frac{3}{4} \times \frac{5}{6}$
4. $\frac{1}{2}$ of $\frac{4}{5}$	**9.** $\frac{3}{5} \times \frac{1}{2}$	**14.** $\frac{5}{6}$ of $\frac{2}{3}$	**19.** $\frac{3}{8} \times \frac{2}{4}$
5. $\frac{1}{5}$ of $\frac{3}{8}$	**10.** $\frac{2}{3} \times \frac{1}{6}$	**15.** $\frac{3}{4}$ of $\frac{7}{8}$	**20.** $\frac{3}{4} \times \frac{4}{7}$

21. Find the cost of $\frac{4}{5}$ of a yard of ribbon at $\$\frac{3}{5}$ a yard.

22. Henry had $\$\frac{3}{4}$. If he spent $\frac{2}{3}$ of his money for fire-crackers, how much did he spend?

Written Exercises

1. Find the value of $\frac{5}{6} \times \frac{24}{25}$.

$$\frac{5}{6} \times \frac{24}{25} = \frac{\cancel{5} \times \overset{4}{\cancel{24}}}{\cancel{6} \times \underset{5}{\cancel{25}}} = \frac{4}{5} \qquad \text{Or, } \frac{\cancel{5}}{\cancel{6}} \times \frac{\overset{4}{\cancel{24}}}{\underset{5}{\cancel{25}}} = \frac{4}{5}$$

In practice it is not necessary to rewrite the fractions as in the first process. We may simply cancel as in the second process.

Find :   Timing yourself, find :

2. $\frac{5}{6}$ of $\frac{3}{5}$ **8.** $\frac{9}{7} \times \frac{14}{16}$ **14.** $\frac{5}{12}$ of $\frac{8}{15}$ **20.** $\frac{3}{4} \times \frac{5}{6} \times \frac{2}{5}$

3. $\frac{5}{8}$ of $\frac{3}{10}$ **9.** $\frac{4}{5} \times \frac{18}{18}$ **15.** $\frac{9}{10}$ of $\frac{17}{18}$ **21.** $\frac{4}{5} \times \frac{5}{8} \times \frac{2}{3}$

4. $\frac{2}{3}$ of $\frac{5}{12}$ **10.** $\frac{3}{4} \times \frac{11}{16}$ **16.** $\frac{7}{20}$ of $\frac{5}{21}$ **22.** $\frac{5}{6} \times \frac{7}{8} \times \frac{12}{25}$

5. $\frac{1}{6}$ of $\frac{9}{10}$ **11.** $\frac{5}{6} \times \frac{9}{20}$ **17.** $\frac{5}{18}$ of $\frac{27}{50}$ **23.** $\frac{7}{8} \times \frac{5}{14} \times \frac{6}{15}$

6. $\frac{3}{4}$ of $\frac{1}{12}$ **12.** $\frac{5}{8} \times \frac{16}{25}$ **18.** $\frac{7}{40}$ of $\frac{25}{28}$ **24.** $\frac{3}{4} \times \frac{7}{12} \times \frac{16}{35}$

7. $\frac{3}{8}$ of $\frac{9}{16}$ **13.** $\frac{5}{8} \times \frac{16}{32}$ **19.** $\frac{9}{20}$ of $\frac{25}{36}$ **25.** $\frac{5}{8} \times \frac{14}{16} \times \frac{9}{16}$

26. Find the value of $4\frac{1}{2} \times \frac{5}{12} \times 6 \times 1\frac{2}{3}$.

$$4\frac{1}{2} \times \frac{5}{12} \times 6 \times 1\frac{2}{3} = \frac{\overset{9}{\cancel{9}}}{2} \times \frac{5}{\underset{2}{\cancel{12}}} \times \frac{\overset{3}{\cancel{6}}}{1} \times \frac{5}{\cancel{3}} = \frac{75}{4} = 18\frac{3}{4}$$

Reducing the mixed numbers to improper fractions, regarding the integer 6 as $\frac{6}{1}$, and canceling, we find the product to be $\frac{75}{4}$, or $18\frac{3}{4}$.

Find the value of:

27. $\frac{3}{5} \times 8\frac{1}{3}$ **33.** $\frac{3}{5} \times 3\frac{1}{3} \times 2\frac{1}{2}$ **39.** $\frac{5}{6} \times \frac{1}{5} \times 12 \times 3\frac{3}{8}$

28. $\frac{7}{8} \times 6\frac{2}{5}$ **34.** $1\frac{4}{7} \times \frac{2}{4} \times 14$ **40.** $2\frac{1}{2} \times \frac{3}{8} \times 9\frac{3}{5} \times 1\frac{1}{4}$

29. $\frac{2}{3} \times 12\frac{3}{4}$ **35.** $6\frac{1}{4} \times 8 \times 4\frac{1}{5}$ **41.** $1\frac{7}{8} \times 3\frac{1}{3} \times 8 \times 2\frac{4}{5}$

30. $\frac{7}{4} \times 13\frac{3}{5}$ **36.** $6\frac{2}{3} \times 7\frac{1}{2} \times 8\frac{1}{4}$ **42.** $2\frac{11}{12} \times \frac{9}{14} \times \frac{2}{3} \times 6\frac{2}{3}$

31. $5\frac{2}{8} \times 7\frac{4}{5}$ **37.** $1\frac{4}{15} \times 6\frac{1}{4} \times 21$ **43.** $4\frac{7}{8} \times 5\frac{1}{3} \times 3\frac{3}{4} \times \frac{5}{13}$

32. $9\frac{3}{4} \times 2\frac{4}{3}$ **38.** $4\frac{1}{6} \times 9\frac{7}{8} \times 5\frac{2}{3}$ **44.** $6\frac{1}{4} \times 5\frac{2}{5} \times \frac{11}{14} \times 8\frac{3}{4}$

Find the cost in dollars and cents, to the nearest cent, of:

45. $2\frac{1}{4}$ yards of lawn at $\$\frac{1}{4}$ per yard.

46. $6\frac{1}{4}$ yards of silk at $\$\frac{7}{8}$ per yard.

47. $3\frac{1}{2}$ yards of lace at $\$\frac{1}{8}$ per yard.

48. $5\frac{1}{2}$ yards of crêpe at $\$\frac{2}{3}$ per yard.

49. $4\frac{3}{8}$ yards of linen at $\$\frac{3}{8}$ per yard.

50. $3\frac{1}{8}$ yards of ribbon at $\$\frac{3}{4}$ per yard.

51. $4\frac{5}{8}$ yards of satin at $\$2\frac{1}{2}$ per yard.

52. $6\frac{3}{4}$ yards of serge at $\$1\frac{1}{4}$ per yard.

53. $8\frac{1}{2}$ yards of flannel at $\$\frac{1}{4}$ per yard.

54. $1\frac{7}{8}$ yards of velvet at $\$1\frac{3}{4}$ per yard.

55. How many acres of grain can a man reap by hand in $8\frac{1}{2}$ days, if he reaps $\frac{3}{8}$ of an acre each day?

56. If a man plows $\frac{4}{5}$ of an acre per day, how many acres can he plow in $12\frac{1}{2}$ days?

57. During a feeding test, a lamb gained $40\frac{1}{2}$ pounds at an average cost of $2\frac{5}{8}$ cents per pound. Find the total cost.

58. An automobile traveled at the rate of $49\frac{5}{8}$ miles per hour for $7\frac{1}{4}$ consecutive hours. How many miles did it go?

59. If butter fat will make $1\frac{1}{8}$ times its weight of butter, and milk is $\frac{1}{24}$ butter fat, how many pounds of butter can be made from 720 pounds of milk?

60. During a contest, a balloon remained in the air $35\frac{3}{4}$ hours and traveled $33\frac{1}{8}$ miles per hour. How far did it travel?

61. If an electric plow travels $353\frac{1}{4}$ feet per minute, how far will it travel in $12\frac{1}{4}$ minutes?

62. At $\$\frac{1}{4}$ per pound, what is the value of the cocoa obtained from $\frac{3}{4}$ of an acre of cacao trees, planted 408 trees to the acre, if $1\frac{1}{2}$ pounds of salable cocoa are obtained per tree?

DIVISION OF FRACTIONS

Division of a fraction by an integer.

1. If $\frac{4}{5}$ of an orange is divided equally between 2 girls, what part of the orange will each receive? $\frac{4}{5} \div 2 = ?$

In dividing $\frac{4}{5}$ by 2, which term of the fraction is divided?

2. Multiply the denominator of $\frac{4}{5}$ by 2 and express the result in its lowest terms.   Compare your answer with that in **1.**

3. In what two ways, then, may $\frac{4}{5}$ be divided by 2?

4. Which of these two ways should you use to divide $\frac{4}{5}$ by 2? $\frac{8}{9}$ by 2? $\frac{9}{10}$ by 3? $\frac{7}{10}$ by 3?

Dividing the numerator or multiplying the denominator of a fraction by an integer divides the fraction by that integer.

The first method should be used whenever the numerator exactly contains the divisor.

Oral Exercises

Divide :

1. $\frac{2}{3}$ by 3	**4.** $\frac{3}{5}$ by 7	**7.** $\frac{12}{13}$ by 6	**10.** $\frac{24}{11}$ by 12
2. $\frac{3}{5}$ by 4	**5.** $\frac{5}{6}$ by 2	**8.** $\frac{16}{8}$ by 8	**11.** $\frac{13}{2}$ by 15
3. $\frac{5}{8}$ by 5	**6.** $\frac{6}{8}$ by 3	**9.** $\frac{14}{5}$ by 7	**12.** $\frac{80}{21}$ by 20

13. Divide 4 by 2; $\frac{2}{3}$ by 2; $4\frac{2}{3}$ by 2.

14. It is not so easy to divide $3\frac{2}{3}$ by 2 as to divide $4\frac{2}{3}$ by 2. How many thirds are there in $3\frac{2}{3}$?   Then $3\frac{2}{3} \div 2 = ?$

Divide:

15. $6\frac{3}{5}$ by 3	**17.** $25\frac{5}{6}$ by 5	**19.** $1\frac{1}{5}$ by 3	**21.** $5\frac{1}{4}$ by 7
16. $8\frac{4}{7}$ by 4	**18.** $49\frac{7}{9}$ by 7	**20.** $3\frac{1}{4}$ by 2	**22.** $3\frac{3}{8}$ by 9

23. If it is $16\frac{1}{5}$ feet around a square, how long is each side?

24. A boy sawed a board $13\frac{1}{2}$ feet long into 3 equal pieces to make shelves for a cupboard.   How long was each shelf?

1. Divide $\frac{14}{15}$ by 7; divide $\frac{5}{9}$ by 2; divide $\frac{15}{7}$ by 6.

$$\frac{14}{15} \div 7 = \frac{14 \div 7}{15} = \frac{2}{15}$$

$$\frac{5}{9} \div 2 = \frac{5}{2 \times 9} = \frac{5}{18}$$

$$\frac{15}{7} \div 6 = \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{6}} \times 7} = \frac{5}{14}$$

In the first process the division is performed by *dividing the numerator* by the integer.

In the second process the division is performed by *multiplying the denominator* by the integer.

In the third process the work is indicated and the result simplified by cancellation.

Find :

2. $\frac{7}{16} \div 2$

3. $\frac{56}{5} \div 4$

4. $\frac{9}{20} \div 6$

5. $\frac{45}{24} \div 3$

6. $\frac{85}{16} \div 5$

7. $\frac{11}{16} \div 4$

8. $\frac{15}{16} \div 6$

9. $\frac{36}{40} \div 8$

10. $\frac{42}{24} \div 4$

11. $\frac{24}{30} \div 9$

Divide :

12. $\frac{25}{6}$ by 10

13. $\frac{5}{18}$ by 25

14. $\frac{21}{2}$ by 15

15. $\frac{24}{5}$ by 16

16. $\frac{44}{3}$ by 33

17. $\frac{18}{16}$ by 26

18. $\frac{38}{8}$ by 14

19. $\frac{18}{25}$ by 36

20. $\frac{45}{60}$ by 15

21. $\frac{72}{16}$ by 24

22. Divide $13\frac{2}{4}$ by 5;   divide $213\frac{3}{4}$ by 5.

$$13\frac{2}{4} = \frac{54}{4}$$

$$\frac{54}{4} \div 5 = \frac{11}{4} = 2\frac{3}{4}$$

$$\begin{array}{r} 5)213\frac{3}{4} \\ \hline 42\frac{3}{4} \end{array}$$

The integral part of the dividend is *small*. In such a case the mixed number may be reduced to an improper fraction and the division may be performed as before.

The integral part of the dividend is *large* and the divisor is small. In this case short division may be used. Thus, 5 is contained in $213\frac{3}{4}$, 42 times with $3\frac{3}{4}$, or $\frac{15}{4}$, undivided; $\frac{15}{4} \div 5 = \frac{3}{4}$.

Divide, as indicated, timing yourself on exercises **33-42**:

23. $5\frac{1}{2} \div 2$

24. $4\frac{4}{5} \div 4$

25. $7\frac{1}{3} \div 3$

26. $6\frac{2}{4} \div 6$

27. $9\frac{1}{3} \div 8$

28. $14\frac{2}{3} \div 4$

29. $17\frac{2}{7} \div 3$

30. $16\frac{1}{4} \div 6$

31. $29\frac{1}{6} \div 7$

32. $44\frac{1}{3} \div 8$

33. $321\frac{1}{3} \div 2$

34. $264\frac{1}{4} \div 4$

35. $146\frac{7}{8} \div 5$

36. $285\frac{1}{4} \div 7$

37. $426\frac{1}{2} \div 8$

38. $420\frac{5}{6} \div 5$

39. $575\frac{1}{4} \div 3$

40. $518\frac{2}{3} \div 4$

41. $416\frac{2}{5} \div 6$

42. $815\frac{5}{8} \div 9$

43. Divide $1006\frac{1}{2}$ by 22.

$22)1006\frac{1}{2}$
$2 \quad 2$
$44)2013$

Here it is necessary to use long division. The work may be simplified by changing both divisor and dividend to *halves*, obtaining 44 (halves) and 2013 (halves), respectively.

The student may complete the solution, dividing as in integers.

Divide:

44. $760\frac{1}{2}$ by 18	**48.** $899\frac{1}{2}$ by 21	**52.** $5215\frac{7}{8}$ by 42
45. $751\frac{1}{2}$ by 24	**49.** $616\frac{4}{5}$ by 36	**53.** $7321\frac{1}{2}$ by 45
46. $773\frac{1}{2}$ by 32	**50.** $657\frac{1}{2}$ by 25	**54.** $5466\frac{2}{3}$ by 64
47. $990\frac{1}{2}$ by 28	**51.** $635\frac{2}{5}$ by 35	**55.** $5156\frac{3}{4}$ by 56

56. A flywheel made $12\frac{1}{2}$ revolutions in 5 seconds. How many revolutions did it make per second?

57. Four cars contained $168\frac{1}{2}$ tons of coal. Find the average weight of coal per car.

58. A retail meat dealer used $3\frac{3}{4}$ tons of ice in his refrigerators in 5 days. Find the daily consumption of ice.

59. A workman's wages amount to $\$568\frac{2}{5}$ per year. If he works 313 days, what are his average wages per day?

60. If $38\frac{1}{4}$ ounces of gold were extracted from 68 tons of ore, what was the amount of gold per ton of ore?

61. Mr. Wright drew $458\frac{1}{2}$ bushels of corn to the crib in 14 loads. How many bushels, on the average, did he draw per load?

62. From 50,000 pounds of cod $68\frac{3}{4}$ gallons of cod-liver oil were obtained. Find the yield of oil per 1000 pounds of fish.

63. It cost $\$7\frac{1}{2}$ to store 30 cases of eggs in Philadelphia. Find the storage rate per case.

64. The receipts of an excursion were $\$3307\frac{1}{2}$. If 945 tickets were sold, how much was the fare?

Division of an integer by a fraction.

1. How many fourths are there in 1 ? in 2 ? in 8 ?

$$1 \div \tfrac{1}{4} = ? \qquad 2 \div \tfrac{1}{4} = ? \qquad 3 \div \tfrac{1}{4} = ?$$

2. Since 3 contains $\tfrac{1}{4}$ 12 times, how many times does 3 contain $\tfrac{2}{4}$? $3 \div \tfrac{2}{4} = ?$

3. How does $1 \div \tfrac{1}{4}$ compare with $1 \times \tfrac{4}{1}$? $2 \div \tfrac{1}{4}$ with $2 \times \tfrac{4}{1}$? $3 \div \tfrac{1}{4}$ with $3 \times \tfrac{4}{1}$? $3 \div \tfrac{2}{4}$ with $3 \times \tfrac{4}{2}$; that is, with 3 multiplied by $\tfrac{2}{4}$ *inverted* (turned upside down)?

To divide an integer by a fraction, *multiply the integer by the fraction inverted.*

Oral Exercises

Divide :   Find :

1. 5 by $\tfrac{1}{2}$ **7.** 6 by $\tfrac{2}{4}$ **13.** $9 \div \tfrac{2}{4}$ **19.** $4 \div \tfrac{2}{4}$

2. 4 by $\tfrac{1}{8}$ **8.** 8 by $\tfrac{1}{6}$ **14.** $5 \div \tfrac{5}{6}$ **20.** $2 \div \tfrac{3}{8}$

3. 4 by $\tfrac{2}{3}$ **9.** 8 by $\tfrac{2}{5}$ **15.** $9 \div \tfrac{3}{5}$ **21.** $3 \div \tfrac{4}{7}$

4. 3 by $\tfrac{1}{5}$ **10.** 8 by $\tfrac{4}{5}$ **16.** $8 \div \tfrac{2}{3}$ **22.** $8 \div \tfrac{5}{6}$

5. 3 by $\tfrac{2}{3}$ **11.** 7 by $\tfrac{1}{4}$ **17.** $6 \div \tfrac{3}{5}$ **23.** $7 \div \tfrac{2}{3}$

6. 6 by $\tfrac{1}{4}$ **12.** 7 by $\tfrac{7}{8}$ **18.** $9 \div \tfrac{3}{8}$ **24.** $6 \div \tfrac{4}{5}$

25. How many times does 2 feet contain $\tfrac{1}{3}$ of a foot ?

26. Robert paid \$2 for phonograph records. If the cost of each was \$$\tfrac{1}{4}$, how many did he purchase ?

27. Mary's mother gave her \$3 with which to buy lace at \$$\tfrac{1}{8}$ a yard. How many yards could she buy ?

28. If it costs \$$\tfrac{4}{5}$ to store 1 ton of goods, how many tons can be stored for \$4?

29. If a laborer on a sugar-cane plantation earns \$$\tfrac{3}{4}$ per day, in how many days will he earn \$12 ?

30. Mr. Fox paid \$14 for some apples at \$$\tfrac{7}{8}$ per bushel. How many bushels of apples did he buy ?

1. Divide 21 by $\frac{9}{10}$.

$$21 \div \frac{9}{10} = \overset{7}{\cancel{21}} \times \frac{10}{\underset{3}{\cancel{9}}} = \frac{70}{3} = 23\frac{1}{3}$$

Indicate the work and cancel wherever possible.

Find quotients :   Timing yourself, find :

2. $16 \div \frac{2}{3}$ **8.** $35 \div \frac{5}{6}$ **14.** $45 \div \frac{3}{10}$ **20.** $125 \div \frac{5}{6}$

3. $28 \div \frac{2}{5}$ **9.** $27 \div \frac{6}{7}$ **15.** $81 \div \frac{5}{12}$ **21.** $230 \div \frac{4}{5}$

4. $25 \div \frac{5}{8}$ **10.** $62 \div \frac{3}{5}$ **16.** $78 \div \frac{9}{10}$ **22.** $315 \div \frac{7}{8}$

5. $20 \div \frac{8}{6}$ **11.** $57 \div \frac{5}{8}$ **17.** $85 \div \frac{7}{12}$ **23.** $245 \div \frac{14}{15}$

6. $30 \div \frac{3}{4}$ **12.** $68 \div \frac{8}{9}$ **18.** $95 \div \frac{15}{16}$ **24.** $517 \div \frac{11}{12}$

7. $34 \div \frac{4}{5}$ **13.** $98 \div \frac{7}{8}$ **19.** $63 \div \frac{18}{25}$ **25.** $567 \div \frac{15}{16}$

Reduce mixed numbers to improper fractions and divide :

26. 30 by $2\frac{1}{2}$ **31.** 44 by $3\frac{1}{5}$ **36.** 77 by $4\frac{2}{5}$ **41.** 133 by $3\frac{1}{2}$

27. 36 by $2\frac{2}{3}$ **32.** 63 by $6\frac{3}{4}$ **37.** 80 by $8\frac{1}{3}$ **42.** 175 by $4\frac{3}{8}$

28. 21 by $4\frac{1}{2}$ **33.** 65 by $4\frac{1}{6}$ **38.** 91 by $8\frac{3}{4}$ **43.** 351 by $2\frac{3}{5}$

29. 32 by $5\frac{1}{3}$ **34.** 98 by $5\frac{5}{6}$ **39.** 87 by $4\frac{5}{8}$ **44.** 126 by $6\frac{3}{4}$

30. 42 by $2\frac{2}{5}$ **35.** 90 by $1\frac{7}{8}$ **40.** 50 by $9\frac{3}{8}$ **45.** 330 by $8\frac{4}{5}$

46. Divide 225 by $18\frac{3}{4}$.

SUGGESTION. — Reduce both dividend and divisor to *fourths* and divide the numerator of the dividend by the numerator of the divisor.

Divide :

47. 275 by $12\frac{1}{2}$ **52.** 415 by $27\frac{2}{3}$ **57.** 2800 by $33\frac{1}{3}$

48. 345 by $26\frac{2}{3}$ **53.** 786 by $32\frac{3}{4}$ **58.** 7200 by $37\frac{1}{2}$

49. 180 by $19\frac{1}{5}$ **54.** 528 by $38\frac{2}{3}$ **59.** 2216 by $69\frac{1}{4}$

50. 340 by $21\frac{1}{4}$ **55.** 910 by $56\frac{7}{8}$ **60.** 4112 by $42\frac{5}{8}$

51. 954 by $19\frac{7}{8}$ **56.** 800 by $31\frac{1}{3}$ **61.** 4710 by $62\frac{4}{5}$

1. At $1¼ each, how many chairs can be bought for $48?

2. A door is 8 feet high and its height is 2⅔ times its width. How wide is it?

3. Frederick earned $12 in 7½ days. How much did he earn per day?

4. How long will it take a wind storm, moving at the rate of 37½ miles per hour, to travel from Bismarck, North Dakota, to Chicago, about 750 miles?

5. If a woman can reel 5⅝ pounds of silk in 6 days of 10 hours each, in how many hours can she reel 1 pound of silk?

6. If a glacier moves at an average rate of 1⅔ feet per day, how long will it take to move 660 feet, or an eighth of a mile?

7. The steel rails for 2¾ miles of railroad track cost $5170. What was the cost per mile?

8. The expense for labor in laying 24 7/10 miles of railroad track was $12,350. Find the expense per mile.

9. How many pounds of cotton must a man pick to earn $1, if he receives $28¼ for picking 7200 pounds?

10. If the weight of 1 gallon of kerosene oil is 6½ pounds, how many gallons does a tank car contain whose capacity is 41,600 pounds?

11. A man took a trip in a gasoline launch, traveling 6 hours a day. If the launch averaged 8⅝ miles an hour, covering a distance of 1311 miles, how many days did the trip last?

12. How long does it take a train to go from San Francisco to New York, if the average rate per hour is 33¾ miles for the 2565 miles to Chicago, and 54⅙ miles for the remaining distance of 975 miles?

Division of a fraction by a fraction.

Reduce to a common denominator and find :

1. $\frac{1}{2} + \frac{1}{4}$
3. $\frac{1}{2} + \frac{1}{6}$
5. $\frac{2}{3} + \frac{5}{6}$
7. $\frac{1}{8} + \frac{1}{2}$

2. $\frac{2}{3} + \frac{1}{6}$
4. $\frac{3}{4} + \frac{1}{4}$
6. $\frac{2}{3} + \frac{2}{3}$
8. $\frac{1}{4} + \frac{1}{8}$

9. Compare $\frac{4}{5} + \frac{2}{5}$ with $\frac{4}{5} \times \frac{5}{2}$, that is, with $\frac{4}{5}$ multiplied by $\frac{2}{5}$ inverted.

10-17. Do exercises **1-8** by inverting the divisor and mul tiplying.

To divide a fraction by a fraction, *multiply the dividend by the divisor inverted*.

1. Divide $\frac{15}{16}$ by $\frac{5}{6}$.

$$\frac{15}{16} \div \frac{5}{6} = \frac{\overset{3}{\cancel{15}}}{\underset{8}{\cancel{16}}} \times \frac{\cancel{6}}{\cancel{5}} = \frac{9}{8} = 1\frac{1}{8}$$

To divide $\frac{15}{16}$ by $\frac{5}{6}$, we may multiply $\frac{15}{16}$ by $\frac{5}{6}$ inverted.
Indicating the work and *canceling*, we find the result to be $1\frac{1}{8}$.

Divide :

Divide, timing yourself :

2. $\frac{5}{6}$ by $\frac{1}{12}$
7. $\frac{3}{4}$ by $\frac{24}{30}$
12. $\frac{9}{10}$ by $\frac{3}{5}$
17. $\frac{15}{16}$ by $\frac{5}{12}$

3. $\frac{3}{5}$ by $\frac{6}{15}$
8. $\frac{5}{6}$ by $\frac{25}{36}$
13. $\frac{7}{12}$ by $\frac{5}{6}$
18. $\frac{25}{48}$ by $\frac{5}{16}$

4. $\frac{7}{8}$ by $\frac{7}{16}$
9. $\frac{7}{8}$ by $\frac{21}{48}$
14. $\frac{11}{16}$ by $\frac{3}{4}$
19. $\frac{13}{16}$ by $\frac{39}{40}$

5. $\frac{3}{4}$ by $\frac{9}{24}$
10. $\frac{5}{6}$ by $\frac{15}{26}$
15. $\frac{11}{15}$ by $\frac{2}{3}$
20. $\frac{11}{25}$ by $\frac{27}{45}$

6. $\frac{5}{8}$ by $\frac{3}{40}$
11. $\frac{4}{5}$ by $\frac{14}{15}$
16. $\frac{49}{50}$ by $\frac{7}{8}$
21. $\frac{21}{30}$ by $\frac{35}{36}$

Reduce mixed numbers to improper fractions and divide :

22. $3\frac{3}{4}$ by $\frac{5}{6}$
27. $16\frac{2}{3}$ by $\frac{5}{6}$
32. $2\frac{1}{2}$ by $3\frac{3}{4}$
37. $12\frac{1}{4}$ by $1\frac{3}{8}$

23. $4\frac{3}{8}$ by $\frac{2}{3}$
28. $22\frac{1}{2}$ by $\frac{3}{4}$
33. $6\frac{3}{4}$ by $4\frac{1}{2}$
38. $16\frac{1}{4}$ by $6\frac{3}{4}$

24. $6\frac{5}{8}$ by $\frac{3}{4}$
29. $43\frac{3}{4}$ by $\frac{5}{6}$
34. $5\frac{1}{4}$ by $1\frac{1}{8}$
39. $25\frac{1}{2}$ by $2\frac{1}{4}$

25. $6\frac{3}{8}$ by $\frac{2}{3}$
30. $35\frac{2}{3}$ by $\frac{5}{6}$
35. $5\frac{4}{5}$ by $6\frac{2}{3}$
40. $20\frac{5}{6}$ by $9\frac{2}{3}$

26. $9\frac{1}{6}$ by $\frac{7}{8}$
31. $54\frac{4}{8}$ by $\frac{5}{6}$
36. $9\frac{4}{8}$ by $6\frac{1}{4}$
41. $40\frac{7}{8}$ by $2\frac{1}{4}$

42. Find the value of $1\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} + \frac{3}{8} + 3 + \frac{1}{6}$.

$$1\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} + \frac{3}{8} + 3 + \frac{1}{6} = \frac{5}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{8}{3} \times \frac{1}{3} \times \frac{\overset{2}{6}}{1} = \frac{16}{3} = 5\frac{1}{3}$$

After reducing all integers and mixed numbers to improper fractions, we invert the fractions that are divisors and write them as multipliers. Canceling and reducing, we find the result to be $5\frac{1}{3}$.

Find the value of:

43. $\frac{3}{4} \times \frac{3}{5} + \frac{1}{6}$

44. $\frac{5}{6} \times 9 + \frac{3}{4}$

45. $\frac{7}{8} + \frac{2}{4} \times 8$

46. $6 \times \frac{3}{8} + \frac{1}{6}$

47. $\frac{3}{8} \times \frac{5}{6} + \frac{2}{3} + \frac{3}{4}$

48. $\frac{1}{4} \times 7 + \frac{1}{4} + \frac{7}{8}$

49. $\frac{5}{8} \times 6 + \frac{3}{4} + 5$

50. $9 + \frac{5}{8} \times \frac{1}{4} \times \frac{5}{6}$

51. $1\frac{2}{3} + \frac{1}{6} \times \frac{1}{4} + \frac{6}{7}$

52. $\frac{5}{6} + 2\frac{2}{3} \times 14 + \frac{5}{8}$

53. $\frac{1}{6} + \frac{3}{10} + 1\frac{2}{6} \times 3\frac{1}{2}$

54. $\frac{2}{3} \times 2\frac{5}{8} + 1\frac{1}{6} + 4\frac{1}{2}$

55. Divide $\frac{1}{2}$ of $\frac{3}{8}$ of $\frac{5}{6}$ by $\frac{1}{4}$ of $\frac{5}{12}$.

SUGGESTION. — $\frac{1}{2}$ of $\frac{3}{8}$ of $\frac{5}{6}$ divided by $\frac{1}{4}$ of $\frac{5}{12} = \frac{1}{2} \times \frac{3}{8} \times \frac{5}{6} \times \frac{4}{1} \times \frac{12}{5}$.

56. Divide $\frac{2}{3}$ of $\frac{4}{7}$ of $2\frac{1}{2\frac{2}{2}}$ by $\frac{3}{8}$ of $\frac{2}{11}$ of $\frac{4}{5}$.

57. Divide $\frac{7}{12}$ of $2\frac{3}{4}$ by $\frac{1}{2}$ of $\frac{7}{10}$ of $\frac{1}{8}$ of $3\frac{3}{8}$.

58. Divide $\frac{4}{5}$ of $1\frac{5}{28}$ of $4\frac{1}{6}$ by $\frac{2}{3}$ of $\frac{1}{8}$ of $\frac{9}{10}$ of $6\frac{1}{4}$.

59. Divide $\frac{7}{8}$ of $1\frac{1}{12}$ of $\frac{4}{5}$ of 75 by $\frac{1}{4}$ of $\frac{9}{16}$ of $\frac{5}{8}$ of $1\frac{1}{24}$ of 32.

60. A street vender has $2\frac{3}{4}$ pecks of chestnuts. How many times will they fill a measure that holds $\frac{1}{8}$ of a peck?

61. A steam shovel handled $45\frac{1}{2}$ cubic yards of earth in $4\frac{1}{8}$ minutes. What was its rate per minute?

62. In $2\frac{1}{2}$ days a machine in a paper mill turned out $43\frac{3}{4}$ tons of paper. How many tons did it make per day?

63. If the yield from $1\frac{1}{4}$ acres of barley was $90\frac{3}{4}$ bushels, what was the average yield per acre?

64. A gallon of naphtha weighs $5\frac{3}{4}$ pounds. How many gallons of naphtha are there in a barrel containing $287\frac{1}{2}$ pounds?

FRACTIONAL RELATIONS

Finding a fractional part of a number.

You have already learned how to find a *fractional part* of an *integer* or of a *fraction* and that it is the same as *multiplying by a fraction*.

Oral Exercises

In the following give the parts indicated:

$\frac{1}{3}$ of	$\frac{1}{4}$ of	$\frac{1}{5}$ of	$\frac{1}{6}$ of	$\frac{1}{6}$ of	$\frac{1}{2}$ of	$\frac{1}{8}$ of
1. 21	**6.** 24	**11.** 35	**16.** 27	**21.** 30	**26.** 36	**31.** 40
2. 27	**7.** 18	**12.** 22	**17.** 42	**22.** 48	**27.** 56	**32.** 64
3. 19	**8.** 32	**13.** 50	**18.** 38	**23.** 24	**28.** 50	**33.** 32
4. 33	**9.** 29	**14.** 41	**19.** 54	**24.** 36	**29.** 72	**34.** 80
5. 26	**10.** 43	**15.** 59	**20.** 64	**25.** 72	**30.** 86	**35.** 96

Written Exercises

Find:

1. $\frac{2}{3}$ of 48

2. $\frac{3}{4}$ of 72

3. $\frac{5}{6}$ of 81

4. $\frac{4}{5}$ of 95

5. $\frac{7}{8}$ of 98

6. $\frac{1}{2}$ of 158

7. $\frac{3}{5}$ of 370

8. $\frac{5}{6}$ of 452

9. $\frac{8}{10}$ of 535

10. $\frac{7}{12}$ of 708

11. $\frac{7}{8}$ of $\frac{4}{5}$

12. $\frac{8}{9}$ of $\frac{3}{4}$

13. $\frac{7}{8}$ of $\frac{2}{3}$

14. $\frac{4}{5}$ of $\frac{7}{12}$

15. $\frac{9}{10}$ of $\frac{5}{16}$

16. $\frac{2}{3}$ of $7\frac{1}{2}$

17. $\frac{4}{5}$ of $8\frac{3}{4}$

18. $\frac{5}{6}$ of $6\frac{2}{3}$

19. $\frac{3}{8}$ of $5\frac{5}{6}$

20. $\frac{7}{8}$ of $9\frac{4}{7}$

Find the cost of:

21. $\frac{3}{4}$ of a yard of ribbon at $\$\frac{2}{5}$ per yard.

22. $\frac{4}{5}$ of a ton of coal at $\$6\frac{3}{4}$ per ton.

23. $\frac{3}{8}$ of an acre of land at $\$132$ per acre.

24. Pauline bought 2 packages of gilt tacks, 100 in each package. She used $\frac{7}{8}$ of them. How many did she use?

25. A farmer stored 384 bushels of potatoes and $\frac{5}{12}$ of them decayed. How many bushels decayed?

Finding what fractional part one number is of another.

1. What part of 8 is 3 ?

SOLUTION. — Since 1 is $\frac{1}{8}$ of 8, 3 is 3 times $\frac{1}{8}$ of 8, or $\frac{3}{8}$ of 8.
The same result may be found by dividing 3 by 8.

2. What part of 15 is 10 ?

SOLUTION. 10 is $\frac{10}{15}$, or $\frac{2}{3}$, of 15.

3. What part of 5 is $2\frac{1}{2}$?

SOLUTION. $2\frac{1}{2} \div 5 = \frac{5}{2} \div 5$, or $\frac{1}{2}$;
therefore, $2\frac{1}{2}$ is $\frac{1}{2}$ of 5.

4. What part of $\frac{2}{3}$ is $\frac{1}{6}$?

SOLUTION. $\frac{1}{6} \div \frac{2}{3} = \frac{1}{6} \times \frac{3}{2}$, or $\frac{1}{4}$;
therefore, $\frac{1}{6}$ is $\frac{1}{4}$ of $\frac{2}{3}$.

What part of

5.	6 is 2 ?	**11.**	10 is 2 ?	**17.**	16 is 4 ?	**23.**	$\frac{3}{4}$ is $\frac{1}{4}$?
6.	8 is 4 ?	**12.**	12 is 4 ?	**18.**	21 is 7 ?	**24.**	$\frac{1}{6}$ is $\frac{1}{6}$?
7.	5 is 3 ?	**13.**	12 is 6 ?	**19.**	10 is 6 ?	**25.**	$\frac{2}{3}$ is $\frac{1}{3}$?
8.	8 is 5 ?	**14.**	15 is 6 ?	**20.**	25 is 5 ?	**26.**	$\frac{4}{5}$ is $\frac{2}{5}$?
9.	9 is 3 ?	**15.**	20 is 5 ?	**21.**	15 is 9 ?	**27.**	$\frac{5}{8}$ is $\frac{1}{8}$?
10.	9 is 6 ?	**16.**	30 is 9 ?	**22.**	20 is 8 ?	**28.**	$\frac{5}{7}$ is $\frac{4}{7}$?

29. What part of a foot is 6 inches? 9 inches ?

30. What part of an hour is 15 minutes? 20 minutes ?

31. Of 12 boys camping at Moose Lake, 4 went fishing and the rest went hunting. What part of the party went hunting?

32. One boy caught 12 fish, another 8, and another 4. What part of all the fish did each of the three boys catch ?

33. Mr. Boyd feeds his horse 12 quarts of oats per day. What part of a bushel (32 quarts) does he feed him each day?

34. A grocer's boy solicited orders at 30 houses and received 18 orders. What part of his calls resulted in orders ?

35. Of the 36 boys in the sixth grade, 27 were promoted. What part of the boys in this grade was promoted ?

Written Exercises

Tell what part the second number is of the first :

1. 36, 12
6. $\frac{3}{4}$, $\frac{1}{4}$
11. 12, $3\frac{3}{4}$
16. $1\frac{1}{4}$, $\frac{1}{2}$
21. 16, $2\frac{2}{3}$

2. 25, 15
7. $\frac{3}{8}$, $\frac{1}{6}$
12. 25, $6\frac{2}{3}$
17. $2\frac{2}{3}$, $\frac{4}{5}$
22. 20, $4\frac{1}{2}$

3. 64, 40
8. $\frac{3}{4}$, $\frac{2}{5}$
13. 36, $3\frac{3}{5}$
18. $4\frac{1}{2}$, $\frac{3}{4}$
23. 35, $5\frac{1}{4}$

4. 72, 24
9. $\frac{5}{6}$, $\frac{3}{4}$
14. 28, $5\frac{3}{5}$
19. $6\frac{2}{3}$, $\frac{5}{6}$
24. 55, $6\frac{3}{5}$

5. 84, 36
10. $\frac{3}{5}$, $\frac{3}{8}$
15. 78, $4\frac{7}{8}$
20. $3\frac{3}{4}$, $\frac{5}{8}$
25. 63, $3\frac{3}{8}$

Tell what part of 100 each of the following numbers is :

26. 50
29. 20
32. 80
35. 5
38. $8\frac{1}{8}$
41. $62\frac{1}{2}$
44. $66\frac{2}{3}$

27. 25
30. 40
33. 30
36. 90
39. $12\frac{1}{2}$
42. $87\frac{1}{2}$
45. $16\frac{2}{3}$

28. 75
31. 60
34. 70
37. $6\frac{1}{4}$
40. $37\frac{1}{2}$
43. $33\frac{1}{3}$
46. $83\frac{1}{3}$

47. What part of $1 is 50¢ ? 75¢ ? $33\frac{1}{3}$¢ ? $37\frac{1}{2}$¢ ?

48. What part of a ton (2000 pounds) is 1200 pounds ?

49. What part of a mile (320 rods) is 80 rods ? 120 rods ?

50. Crater Lake, Oregon, is 6 miles by $4\frac{1}{2}$ miles. What part of the number of miles in its length is the number of miles in its width ?

51. A farmer raised 64 tons of hay and sold 24 tons. What part of his crop did he sell ?

52. When an acre of land yields $18\frac{3}{4}$ bushels of grain, what part of an acre will yield $12\frac{1}{2}$ bushels ?

53. If $26\frac{2}{3}$ pounds of cream produced $6\frac{2}{3}$ pounds of butter fat, what part of the cream was butter fat ?

54. The cost of raising an acre of celery was $$87\frac{1}{2}$. If it sold for $$218\frac{3}{4}$, what part of the selling price was the cost ?

55. The height of the Statue of Liberty is about 150 feet, and the length of the forefinger holding the torch, $8\frac{1}{4}$ feet. What part of the whole height is the length of the forefinger ?

Finding the whole when a fractional part is given.

1. If $\frac{2}{3}$ of a number is 20, what is $\frac{1}{3}$ of it? $\frac{2}{3}$ of it?

2. If $\frac{1}{4}$ of a number is 16, what is the number?

Oral Exercises

1. Our football team won $\frac{3}{4}$ of the games that it played. It won 12 games. How many games did it play?

SOLUTION. $\frac{3}{4}$ of the number of games played = 12.
$\frac{1}{4}$ of the number of games played = $\frac{1}{3}$ of 12, or 4.
The number of games played = 4×4, or 16.

Find the number of which

2. 10 is $\frac{2}{3}$	7. 27 is $\frac{3}{8}$	12. 60 is $\frac{5}{8}$	17. 30 is $\frac{8}{10}$
3. 15 is $\frac{3}{4}$	8. 24 is $\frac{2}{3}$	13. 36 is $\frac{3}{7}$	18. 63 is $\frac{7}{10}$
4. 16 is $\frac{2}{5}$	9. 30 is $\frac{3}{4}$	14. 44 is $\frac{4}{4}$	19. 45 is $\frac{5}{12}$
5. 20 is $\frac{5}{6}$	10. 36 is $\frac{3}{8}$	15. 56 is $\frac{7}{4}$	20. 72 is $\frac{9}{10}$
6. 28 is $\frac{4}{5}$	11. 45 is $\frac{5}{8}$	16. 96 is $\frac{6}{8}$	21. 56 is $\frac{7}{12}$

22. If $\frac{2}{5}$ of the cost of a ball is 80 ¢, what is the whole cost?

23. Helen paid 12 ¢ for $\frac{3}{4}$ of a pound of mixed nuts. At that rate how much would a pound cost?

24. Our baseball team won $\frac{2}{3}$ of the games that it played. If it won 14 games, how many games did it play?

25. How deep is a well that goes through 18 feet of rock, if $\frac{3}{8}$ of its depth is through rock?

26. If $\frac{5}{8}$ of the passengers on a car were men, and there were 35 men, how many passengers were there on the car?

27. A football player ran $\frac{4}{5}$ of the length of the field. If he ran 80 yards, how long was the field?

28. Mr. Hayes bought $\frac{5}{6}$ of a bushel of corn for 35 ¢. At that rate how much would a bushel cost?

29. The balcony of a music hall holds 400 people. If this is $\frac{1}{4}$ of the hall's capacity, how many people will the hall hold?

Written Exercises

1. Find the number of which 679 is $\frac{7}{8}$; $\frac{5}{6}$ is $\frac{2}{3}$.

SOLUTION	SOLUTION
$\frac{7}{8}$ of the number = 679.	$\frac{2}{3}$ of the number = $\frac{5}{6}$.
$\frac{1}{8}$ of the number = $\frac{1}{7}$ of 679, or 97.	$\frac{1}{3}$ of the number = $\frac{1}{2}$ of $\frac{5}{6}$, or $\frac{5}{12}$.
$\frac{8}{8}$ of the number = 8 × 97, or 776.	The number = $3 \times \frac{5}{12}$, or $\frac{5}{4}$, or $1\frac{1}{4}$.
Or, 679 ÷ $\frac{7}{8}$ = 776.	Or, $\frac{5}{6} ÷ \frac{2}{3} = 1\frac{1}{4}$.

To find the whole when a fractional part is given, *divide the given part by the fraction.*

Find the number of which

2. 38 is $\frac{2}{3}$	**7.** $\frac{1}{4}$ is $\frac{2}{3}$	**12.** 225 is $\frac{3}{4}$	**17.** $\frac{7}{12}$ is $\frac{5}{6}$
3. 54 is $\frac{3}{4}$	**8.** $\frac{5}{6}$ is $\frac{3}{4}$	**13.** 452 is $\frac{4}{5}$	**18.** $\frac{9}{16}$ is $\frac{3}{8}$
4. 70 is $\frac{5}{6}$	**9.** $\frac{3}{4}$ is $\frac{5}{6}$	**14.** 822 is $\frac{6}{8}$	**19.** $7\frac{1}{2}$ is $\frac{3}{4}$
5. 92 is $\frac{4}{5}$	**10.** $\frac{4}{7}$ is $\frac{2}{3}$	**15.** 1158 is $\frac{2}{3}$	**20.** $6\frac{2}{3}$ is $\frac{5}{6}$
6. 98 is $\frac{7}{8}$	**11.** $\frac{7}{8}$ is $\frac{3}{4}$	**16.** 2335 is $\frac{5}{6}$	**21.** $5\frac{5}{8}$ is $\frac{3}{4}$

22. If $\frac{7}{8}$ of an acre of land costs $126, find the price per acre.

23. The weight of a bushel of barley is 48 pounds or $\frac{4}{5}$ that of a bushel of wheat. Find the weight of a bushel of wheat.

24. When a balloon had traveled 800 miles it had completed $\frac{2}{3}$ of its journey. How far did it travel?

25. The Suez Canal is 180 feet wide at the bottom or $\frac{4}{9}$ as wide as at the water's surface. Find the width at the surface.

26. One section of an anchor chain is 180 feet long. If this section is $\frac{2}{11}$ of the chain, how long is the chain?

27. The cruiser *Charleston* usually has a coal supply of 650 tons, but this is only $\frac{13}{30}$ of her capacity. Find her capacity.

28. Forest fires in the Adirondacks one year destroyed buildings to the value of $50,000, which was $\frac{2}{7}$ of the value of the lumber destroyed. Find the value of the lumber destroyed.

Drill Exercises

1–20. Find the prime factors of each number under c and i :

a.	b.	c.	d.	e.	f.	g.	h.	i.
6	$\frac{1}{4}$	18	$\frac{1}{2}$	$\frac{12}{6}$	$8\frac{1}{3}$	$\frac{26}{64}$	$25\frac{1}{2}$	120
7	$\frac{1}{8}$	21	$\frac{3}{4}$	$\frac{45}{4}$	$6\frac{3}{4}$	$\frac{60}{72}$	$42\frac{3}{4}$	126
4	$\frac{1}{2}$	22	$\frac{2}{3}$	$\frac{48}{5}$	$8\frac{3}{4}$	$\frac{36}{45}$	$81\frac{3}{8}$	616
5	$\frac{3}{8}$	45	$\frac{5}{8}$	$\frac{57}{3}$	$6\frac{1}{4}$	$\frac{42}{63}$	$56\frac{1}{4}$	660
8	$\frac{2}{3}$	72	$\frac{1}{4}$	$\frac{71}{8}$	$6\frac{7}{8}$	$\frac{75}{90}$	$86\frac{3}{4}$	792
7	$\frac{3}{4}$	84	$\frac{7}{8}$	$\frac{75}{6}$	$9\frac{5}{8}$	$\frac{38}{96}$	$65\frac{5}{8}$	770
9	$\frac{2}{3}$	36	$\frac{2}{3}$	$\frac{88}{9}$	$5\frac{1}{4}$	$\frac{44}{60}$	$63\frac{5}{8}$	486
6	$\frac{5}{6}$	60	$\frac{5}{6}$	$\frac{90}{4}$	$7\frac{3}{8}$	$\frac{54}{72}$	$75\frac{3}{8}$	690
8	$\frac{5}{6}$	64	$\frac{3}{8}$	$\frac{78}{5}$	$5\frac{5}{8}$	$\frac{84}{96}$	$72\frac{3}{4}$	672
9	$\frac{2}{5}$	48	$\frac{3}{4}$	$\frac{91}{7}$	$9\frac{4}{5}$	$\frac{44}{99}$	$98\frac{3}{8}$	784

21–60. Reduce to its lowest terms, each fraction under g ; to an improper fraction, each number under f ; under h ; to an integer or a mixed number, each fraction under e.

61–80. Find the g. c. d. of the numbers in each row under c and i ; the l. c. m. of the numbers in each row under a, c, and i.

81–120. Find the sum of and then the difference between the numbers in each row under b and d ; under f and h.

Multiply each number under :

121–140. i by the number opposite it under b ; under f.

141–160. b by the number opposite it under c ; under h.

Divide each number under :

161–180. d by the number opposite it under a ; under b.

181–200. i by the number opposite it under d ; under f.

201–210. Tell what fractional part each number under a is of the number opposite it under c.

116　　　　　　　　　FRACTIONS

Written Exercises

1. What part of a yard (36 inches) is 18 inches? 24 inches?

2. Find the cost of $9\frac{3}{4}$ yards of carpet at $\$\frac{5}{6}$ per yard.

3. If a man earns $\$11\frac{1}{4}$ in 5 days, what are his daily wages?

4. When 15 dozen eggs cost $\$3\frac{3}{4}$, what is the price per dozen?

5. If $\frac{3}{4}$ of a pound of tea costs 45¢, what is the price per pound?

6. If a duck flying $\frac{5}{6}$ as fast as a hawk flies 90 miles in an hour, how fast does the hawk fly?

7. Find the annual cost of fuel to a manufacturer who uses 1800 tons of coal per year at $\$2\frac{3}{4}$ per ton.

8. A boy bought 90 coconuts at $\$\frac{1}{2}$ per dozen, and sold them at $\$\frac{1}{20}$ apiece. How much money did he gain?

9. At a certain gold mine it costs $\$\frac{3}{5}$ a ton to mine the ore. How many tons of gold ore can be mined for $\$61\frac{1}{4}$?

10. Find the cost of crushing 75 tons of ore at $\$\frac{9}{10}$ per ton.

11. At $\$6\frac{3}{4}$ per barrel, how many barrels of flour can be bought for $\$74\frac{1}{4}$?

12. At $\$\frac{9}{10}$ a foot, how much will it cost to drill an oil well 2256 feet deep?

13. A grocer bought a bunch of bananas for $\$1\frac{1}{4}$. He sold $3\frac{1}{3}$ dozen at $\$\frac{3}{20}$ per dozen, and the rest, $6\frac{1}{4}$ dozen, at $\$\frac{1}{5}$ per dozen. How much did he gain?

14. In building $9\frac{3}{16}$ miles of railroad track, 2640 ties were used per mile. Find the cost of the ties used at $\$\frac{5}{6}$ each.

15. If a hog gains $13\frac{1}{2}$ pounds for each bushel of wheat fed, how many bushels of wheat are required to increase the hog's weight from $225\frac{3}{4}$ pounds to $397\frac{7}{8}$ pounds?

Celery. **1.** In a field of celery, 21,000 square feet of boards were used for blanching. Find the cost of the lumber at $2.75 per 100 square feet.

2. A 5-acre field of celery was watered 4 times in a season at a cost of $1.60 per acre each time. Find the total cost.

3. A celery grower had 8 acres of celery to which he applied 700 pounds of salt per acre. Find the cost of the salt used at $2.10 per barrel of 280 pounds.

4. How many celery plants are required for a row 91 feet long, if $\frac{1}{4}$ of a foot is left at each end of the row and the plants are set $\frac{5}{12}$ of a foot apart?

5. The yield from $1\frac{3}{4}$ acres of celery was 1500 dozen bunches per acre. Find its value at 25 ¢ per dozen bunches.

6. If the cost of producing a field of celery was $500 and the gross income from it was $750, what part of the cost was the gain?

Oats.   7. If a farmer plows 1¼ acres per day, how long will it take him to plow a 15-acre oat field ? Find the cost at $2.50 per day.

8. A 14-acre field was harrowed 4 times before being sown to oats. If 1¼ acres were harrowed per hour, what was the cost of harrowing the field at 30 ¢ per hour?

9. On each of 8 acres 3 bushels of oats were sown. Find the cost of the seed oats at 65 ¢ per bushel.

10. Some oats were sown with a drill, 1 acre being drilled in per hour. Find the cost of sowing 15 acres at 40 ¢ per hour.

11. A farmer had 45 bushels of seed oats which he treated with a formalin solution to prevent smut. Find the cost of the treatment at 1⅘ ¢ per bushel.

12. To kill the weeds in 12 acres of young oats, they were sprayed with an iron-sulphate solution at a cost of $1.25 per acre. Find the total cost of the spraying.

13. If 6 acres of oats were cut per day with a reaper, how long did it take to cut a 20-acre field of oats? Find, to the nearest cent, the cost at $4.60 per day.

14. The yield from a 16-acre field of oats was 47 bushels per acre. Find the total yield ; the value of the yield at 48 ¢ per bushel.

15. In one half of a 12-acre field, oats were sown broadcast, in the other half the seed was drilled in. The yield from the first half was 50¾ bushels per acre and from the second, 56 bushels. Find the gain in yield due to drilling in the seed.

16. A Minnesota farmer had 1688 bushels of oats thrashed from the shock at a cost of 4½ ¢ per bushel, and 824 bushels from the stack at 5¼ ¢ per bushel. Find the total cost.

DENOMINATE NUMBERS

To ascertain the quantity of anything, or to *measure* it, is to find how many times it contains some established unit called the **unit of measure.**

To measure the corn in a bin is to find how many times the whole quantity of corn contains some *unit measure*, as 1 bushel, or 1 hundredweight.

While 1 book is a *unit* it is not regarded as a *unit of measure.*

A concrete number in which the unit of measure is established by law or custom is a **denominate number.**

16 bushels is a denominate number; also 16 bushels 3 pecks.

A denominate number that is composed of units of one denomination only is a **simple denominate number.**

16 bushels is a simple denominate number.

A denominate number that is composed of units of two or more related denominations is a **compound denominate number.**

16 bushels 3 pecks is a compound denominate number.

The pupil should obtain accurate concepts of the various measuring units of denominate numbers by practical experience. He should actually use liquid and dry measures, length and weight measures. He should walk a mile, lay out a plot that contains an acre; practice estimating capacity, length, extent, and weight and then test the accuracy of his estimates.

Tables of denominate numbers will be found in the Appendix. Those marked with an asterisk (*) are given for reference only. The others are important and should be *memorized.*

The student should read the notes given in connection with each table.

119

REDUCTION OF DENOMINATE NUMBERS

LINEAR MEASURES

A **line** has only one dimension, **length.**
The measures that are used in measuring length are **linear measures.**

Learn the table and read the accompanying notes on page 496.

· Oral Exercises

Change to the next lower unit :

1. 3 ft.	**4.** 8 yd.	**7.** ¾ ft.	**10.** 12 yd.	**13.** 2¼ ft.
2. 5 yd.	**5.** 7 ft.	**8.** ⅔ yd.	**11.** 10 ft.	**14.** 5½ yd.
3. 6 ft.	**6.** 9 yd.	**9.** ¼ mi.	**12.** 11 yd.	**15.** 6¼ ft.

16–18. What part of a foot is 3 in.? 6 in. ? 9 in. ?

19–21. What part of a mile is 80 rd.? 160 rd.? 240 rd.?

Change to the next higher unit :

22. 18 ft.	**24.** 60 in.	**26.** 45 ft.	**28.** 20 ft.	**30.** 320 rd.
23. 48 in.	**25.** 39 ft.	**27.** 21 in.	**29.** 40 in.	**31.** 640 rd.

32–36. Change to feet: 10 yd. ; 96 in. ; 15 yd. ; 108 in.; 1 rd.

37–41. Change to yards: 36 ft. ; 2 rd.; 48 ft.; 4 rd. ; 60 ft.

Change :

42. 18 yd. to feet	**46.** 10 mi. to rods
43. 22 yd. to rods	**47.** 12 rd. to yards
44. 32 ft. to yards	**48.** 132 in. to feet
45. 4⅛ ft. to inches	**49.** 480 rd. to miles

The process of changing a denominate number from one denomination (unit) to another without changing its value is **reduction of denominate numbers.**

Written Exercises

1. Reduce 24 ft. 8 in. to inches.

$$\begin{array}{r} 24 \\ \underline{12} \\ 288 \\ \underline{8} \\ 296 \end{array}$$

Since there are 12 inches in 1 foot, in 24 feet there are 24 times 12 inches (p. 33), or 288 inches; and in 24 ft. 8 in. there are 288 in. + 8 in., or 296 in.

Hence, 24 ft. 8 in. = 296 in.

2. Reduce ⅜ mi. to feet.

3. Reduce .625 yd. to feet and inches.

SOLUTION

⅜ mi. = ⅜ of 320 rd. = 120 rd.
120 rd. = 120 × 16½ ft. = 1980 ft.
Hence, ⅜ mi. = 1980 ft.

SOLUTION

.625 yd. = .625 × 3 ft. = 1.875 ft.
.875 ft. = .875 × 12 in. = 10.5 in.
Hence, .625 yd. = 1 ft. 10.5 in.

Reduction to a *lower* unit is **reduction descending**.

Reduce to the next lower unit:

4. 27 yd.   **9.** 45 ft.   **14.** 1⅝ mi.   **19.** 22½ yd.

5. 14 ft.   **10.** 28 mi.   **15.** 44⅔ yd.   **20.** 45.5 ft.

6. 12 mi.   **11.** 69 yd.   **16.** .125 ft.   **21.** 24¼ mi.

7. 35 ft.   **12.** 55 ft.   **17.** 2.75 mi.   **22.** .875 ft.

8. 42 yd.   **13.** 76 yd.   **18.** 37.5 ft.   **23.** 45.7 mi.

Reduce:

24. 18 yd. 2 ft. to feet   **30.** .875 mi. to feet

25. 26 rd. 12 ft. to feet   **31.** 6 rd. 2 yd. to feet

26. 15 mi. 25 rd. to rods   **32.** .1875 yd. to inches

27. 27 ft. 6 in. to inches   **33.** ⅔ rd. to yards and feet

28. 32 rd. 3 yd. to yards   **34.** ⅞ yd. to feet and inches

29. 14 yd. 1 ft. to inches   **35.** 15⅔ mi. to rods and feet

1. Express 450 in. in yards, feet, and inches.

$$12 | 450$$
$$8 \overline{| 37, + 6 \text{ in.}}$$
$$\overline{12, + 1 \text{ ft.}}$$
12 yd. 1 ft. 6 in.

Since 12 in. = 1 ft.,
450 in. = $\frac{450}{12}$ ft. = 37 ft. + 6 in.
Since 3 ft. = 1 yd.,
37 ft. = $\frac{37}{3}$ yd. = 12 yd. + 1 ft.
Hence, 450 in. = 12 yd. 1 ft. 6 in.

2. What fractional part of a rod is 8 ft. 3 in.?

3. Express 440 yd. as the decimal part of a mile.

SOLUTION

1 rd. = 198 in.; and 8 ft. 3 in. = 99 in.
Hence, 8 ft. 3 in. = $\frac{99}{198}$ rd., or $\frac{1}{2}$ rd.

SOLUTION

1 mi. = 320 × 5½ yd., or 1760 yd.
Hence, 440 yd. = $\frac{440}{1760}$ mi., or .25 mi.

Reduction to a *higher* unit is **reduction ascending**.

Express in the next higher unit or in that and the given unit:

4. 72 ft.	**8.** 168 in.	**12.** 287 ft.	**16.** 1680 rd.
5. 44 yd.	**9.** 960 rd.	**13.** 328 in.	**17.** 1454 yd.
6. 84 ft.	**10.** 114 ft.	**14.** 685 rd.	**18.** 5319 in.
7. 66 yd.	**11.** 216 in.	**15.** 224 yd.	**19.** 3950 rd.

20–21. What fractional part of a yard is 24 in.? 2 ft. 6 in.?

22–23. What decimal part of a mile is 880 yd.? 274 rd. 33 yd.?

Express:

24. 98 in. in yards, feet, and inches.

25. 7152 yd. in miles, rods, and yards.

26. 4⅛ ft. as the fractional part of a rod.

27. 66 in. as the fractional part of a rod.

28. 2 ft. 3 in. as the decimal part of a yard.

29. 29,002 ft. (height of Mt. Everest) in miles and feet.

SQUARE MEASURES

A flat **surface** has only two dimensions, **length** and **breadth** (width).

This page, the floor, and the ceiling are surfaces.

The difference in the direction of two lines that meet in a point, called the *vertex*, is an **angle**.

A flat surface that is bounded by *four equal* straight lines and has *four equal* angles is a **square**.

A square, each of whose sides is *one inch*, is a **square inch**.

Describe a square foot; other square units.

Learn the table of square measures on page 496.

ONE
SQUARE INCH
1 SQ. IN.

Written Exercises

Reduce to the next lower unit :

1. 121 sq. yd.	**8.** 1280 A.	**15.** 3840 A.	**22.** .125 A.
2. 126 sq. ft.	**9.** 3200 A.	**16.** 5120 A.	**23.** 24½ A.
3. 320 sq. rd.	**10.** 288 sq. ft.	**17.** 484 sq. yd.	**24.** ¼ sq. ft.
4. 162 sq. ft.	**11.** 640 sq. rd.	**18.** 960 sq. rd.	**25.** ⅚ sq. mi.
5. 480 sq. rd.	**12.** 863 sq. yd.	**19.** 378 sq. ft.	**26.** 4.4 sq. rd.
6. 216 sq. ft.	**13.** 800 sq. rd.	**20.** 726 sq. yd.	**27.** 7¾ sq. ft.
7. 242 sq. yd.	**14.** 342 sq. ft.	**21.** 432 sq. ft.	**28.** 9⅞ sq. mi.

29-49. In exercises **1-21** reduce to the next higher unit.

Reduce to the lower of the given units :

50. 7 sq. yd. 5 sq. ft.	**54.** 5 A. 15 sq. rd.	**58.** 35 sq. yd. 8 sq. ft.
51. 5 sq. ft. 8 sq. in.	**55.** 9 A. 24 sq. rd.	**59.** 28 sq. rd. 6 sq. yd.
52. 8 sq. rd. 7 sq. yd.	**56.** 16 sq. mi. 8 A.	**60.** 17 sq. ft. 9 sq. in.
53. 9 sq. ft. 6 sq. in.	**57.** 25 sq. mi. 9 A.	**61.** 65 sq. yd. 7 sq. ft.

Express in the next higher and the given unit:

62. 114 sq. ft.   **65.** 735 sq. in.   **68.** 825 sq. rd.   **71.** 1290 A.

63. 295 sq. in.   **66.** 259 sq. yd.   **69.** 870 sq. in.   **72.** 3910 A.

64. 490 sq. rd.   **67.** 428 sq. ft.   **70.** 365 sq. yd.   **73.** 5175 A.

Express:

74. 320 A. as the decimal part of a square mile.

75. 120 sq. rd. as the fractional part of an acre.

76. 15 sq. yd. 12 sq. ft. 25 sq. in. in square inches.

77. 126 sq. in. as the fractional part of a square foot.

AREA OF RECTANGLES

A flat surface that is bounded by four straight lines and has four *equal* angles is a **rectangle**.

A square is a rectangle whose sides are equal.
The angles of a rectangle are right angles.
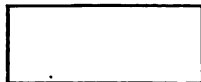
RECTANGLE

The number of square units that a surface contains is its **area**.

If each square in this rectangle represents some square unit, each row represents 5 of those units, and since there are 3 rows, the rectangle contains 3 × 5 square units, that is, its *area* is 15 square units.
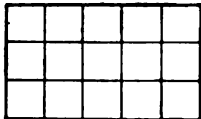
If the length and the breadth of a rectangle are expressed in *inches*, the area is found by multiplying the *number* of inches in the length by the *number* of inches in the breadth, calling the result *square inches*. If both are in *feet*, the area is found by finding the product of the *numbers* that measure the dimensions, calling the result *square feet*. Stated more briefly:

The area of a rectangle is equal to the product of its length and breadth, expressed in like units.

For brevity we speak of the *product of lines* when we mean the product of the *numbers* that measure them. A rectangle that is 5 feet long and 3 feet wide, or more briefly 5 ft. by 3 ft., has an area of 15 sq. ft.

5 ft. by 3 ft. may be written $5' \times 3'$; and 5 in. by 3 in., $5'' \times 3''$.

Find the area of a rectangle :

1. 16 in. by 11 in.   **4.** 14 ft. by 13 ft.   **7.** 17 yd. by 14 yd.

2. 18 in. by 15 in.   **5.** 32 ft. by 28 ft.   **8.** 28 yd. by 24 yd.

3. 26 in. by 22 in.   **6.** 58 ft. by 48 ft.   **9.** 42 yd. by 37 vd.

10. Find the area of a rectangle 16 ft. by 5 ft. 3 in.

SOLUTION

Length = 16 ft.; breadth = $5\frac{1}{4}$ ft.

Area = $(5\frac{1}{4} \times 16)$ sq. ft., or 84 sq. ft.

Since the length and breadth must be expressed in *like units*, 5 ft. 3 in. is first changed to *feet*. Then, the *number* of *square feet* in the area is found by multiplying 16 by $5\frac{1}{4}$.

Find the area of rectangles of the following dimensions :

11. $16' \times 7'$ **14.** $120' \times 40''$ **17.** $24' \times 3'\ 6''$

12. $28' \times 4\frac{1}{2}'$ **15.** $280' \times 50''$ **18.** $54' \times 7'\ 2''$

13. $39' \times 6\frac{1}{3}'$ **16.** $110' \times 4$ yd.   **19.** $32' \times 4'\ 9''$

20. Find the area of a square flower bed 14 ft. on a side.

21. A large electric sign is 300 ft. long and 44 ft. wide. What is its area ?

22. A room, said to be the largest in the world, is 620 ft. long and 150 ft. wide.   Find the area of the floor.

23. A rug in the Hall of Records in New York is 56 ft. long and 12 ft. wide.   How many square yards does it contain ?

24. A large American flag recently made is 95 ft. by 50 ft. Find its area in square yards.

25. A piece of silk made from spiders' webs was 18 yd. long and 18 in. wide.   How many square yards did it contain?

The method of computing the area of figures that are not rectangular is given in the chapter on MEASURES AND MEASUREMENTS.

CUBIC MEASURES

A solid has three dimensions, length, breadth, and thickness.

A solid that has six equal square sides or *faces* is a cube.

A cube each of whose faces is a *square inch* is a cubic inch. Describe a cubic foot; other cubic units.

Learn the table of cubic measures on page 497.

ONE
CUBIC INCH
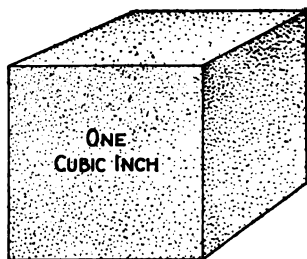
Written Exercises

Reduce to cubic inches:

1. 9 cu. ft.

2. 3 cu. yd.

3. 28 cu. ft.

4. 8 cu. ft. 42 cu. in.

5. 36 cu. ft. 85 cu. in.

Reduce to cubic feet:

6. 35 cu. yd.

7. 8640 cu. in.

8. 9072 cu. in.

9. 33 cu. yd. 17 cu. ft.

10. 48 cu. yd. 25 cu. ft.

Reduce to cubic yards :

11. 81 cu. ft.

12. 162 cu. ft.

13. 432 cu. ft.

14. 1134 cu. ft.

15. 1359 cu. ft.

16. 3348 cu. ft.

17. 46,656 cu. in.

18. 69,984 cu. in.

19. 93,312 cu. in.

20. Change 15⅔ cu. yd. to cubic feet.

21. Change .875 cu. ft. to cubic inches.

22. Reduce 661.5 cu. ft. to cubic yards.

23. Express .0625 cu. yd. in cubic inches.

24. What part of a cubic foot is 864 cu. in.?

25. What part of a cubic yard is 5184 cu. in.?

26. Change 8 cu. yd. 14 cu. ft. 36 cu. in. to cubic inches.

27. Express 216 cu. in. as the decimal part of a cubic foot.

VOLUME OF RECTANGULAR SOLIDS

A solid that has six rectangular faces is a rectangular solid.

A cube is a rectangular solid whose faces are equal squares.

The number of cubic units that a solid contains is its **volume**.

If each cube in this rectangular solid represents some cubic unit, each row represents 4 of those units, each layer 2 × 4 of them, and since there are 3 layers, the rectangular solid contains 3 × 2 × 4 cubic units, that is, its *volume* is 24 cubic units.

If the length, the breadth, and the thickness of a rectangular solid are expressed in *inches*, the volume is found by multiplying the *number* of inches in the length by the *number* of inches in the breadth and that product by the *number* of inches in the thickness, calling the result *cubic inches*. If all the dimensions are in *feet*, the volume is found by finding the product of the *numbers* that measure them, calling the result *cubic feet*. Stated more briefly:

The volume of a rectangular solid is equal to the product of its length, breadth, and thickness, all expressed in like units.

A rectangular solid 4 ft. by 2 ft. by 3 ft. has a volume of 24 cu. ft.

Written Exercises

Find the volume of these rectangular solids:

1. $6'' \times 4'' \times 5''$ 5. $12' \times 9' \times 4'$ 9. 6 yd. × 6 yd. × 14 yd.

2. $8'' \times 7'' \times 6''$ 6. $14' \times 6' \times 5'$ 10. 9 yd. × 8 yd. × 17 yd.

3. $9'' \times 8'' \times 9''$ 7. $18' \times 8' \times 9'$ 11. 8 yd. × 7 yd. × 18 yd.

4. $7'' \times 9'' \times 8''$ 8. $24' \times 7' \times 6'$ 12. 5 yd. × 9 yd. × 25 yd.
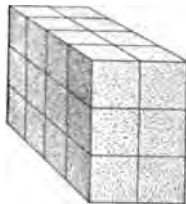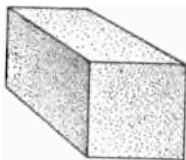
13. Find the volume of a rectangular solid 12 ft. by 8 ft. 6 in. by 4 ft. 4 in.

SOLUTION

Length = 12 ft.; breadth = 8½ ft.; thickness = 4⅓ ft.

Volume = (12 × 8½ × 4⅓) cu. ft., or 442 cu. ft.

Since the dimensions must be expressed in *like units*, 8 ft. 6 in. and 4 ft. 4 in. are first changed to *feet*. Then the *number* of *cubic feet* in the volume is obtained by finding the product of 12, 8½, and 4⅓.

Find the volume of these rectangular solids:

14. $7' \times 6' \times 3' \, 4''$ **17.** $12' \times 6' \times 32''$

15. $8' \times 8' \times 4' \, 3''$ **18.** $18' \times 5' \, 8'' \times 7'$

16. $9' \times 6' \times 5' \, 6''$ **19.** $36' \times 7' \, 2'' \times 6'$

20. The gold mined in a recent year would fill a room 10 ft. square and 9 ft. high. How many cubic feet of gold were mined that year?

21. How many cubic inches of ice are there in a block of ice that is 1 ft. 3 in. long, 10 in. wide, and 10 in. thick?

22. Find the number of cubic feet of stone in a horse block 6 ft. long, 2 ft. wide, and 18 in. thick.

23. One of the largest overhead reservoirs in the world is 320 ft. square and 16 ft. deep. How many cubic feet of water will it hold?

24. The standard size of a bale of cotton is 27 in. by 27 in. by 54 in. How many cubic inches does it contain?

25. Find the number of cubic yards of earth removed in digging a ditch 72 ft. long, 4½ ft. deep, and 2 ft. wide.

26. A granite block exported from this country was 32 ft. long and 3 ft. 6 in. square. Find its volume in cubic feet.

27. A large safe-deposit vault is 109 ft. long, '30 ft. 6 in. wide, and 9 ft. 4 in. high. Find its cubical contents.

LIQUID MEASURES

Oral Exercises

1. Repeat the table of liquid measures.

2. Name some things that are sold by the liquid pint; quart; gallon.

3. Change to pints: 3 qt.; .5 qt.; 8 gi.; 4½ qt.; 24 gi.

4. Change to quarts: 4 gal.; 6 pt.; .5 gal.; 8 pt.; ¾ gal.

5. What part of a gallon is 1 qt.? 2 qt.? 3 qt.? 1 pt.?

Written Exercises

Reduce to the next lower unit:

1. 26 pt.	**5.** 68 qt.	**9.** 104 pt.	**13.** 312 qt.
2. 56 qt.	**6.** 42 pt.	**10.** 132 qt.	**14.** 406 pt.
3. 32 pt.	**7.** 76 qt.	**11.** 214 pt.	**15.** 428 qt.
4. 60 qt.	**8.** 64 pt.	**12.** 228 qt.	**16.** 516 pt.

17–32. In exercises **1–16** reduce to the next higher unit.

Change to the lower of the given units :

33. 19 qt. 1 pt.	**36.** 37 qt. 1 pt.	**39.** 16 gal. 1 qt.
34. 25 pt. 3 gi.	**37.** 31 pt. 2 gi.	**40.** 32 gal. 3 qt.
35. 34 qt. 1 pt.	**38.** 45 qt. 1 pt.	**41.** 68 gal. 2 qt.

Express in the next higher and the given unit:

42. 50 gi.	**45.** 61 pt.	**48.** 115 qt.	**51.** 537 gi.
43. 45 pt.	**46.** 74 qt.	**49.** 314 gi.	**52.** 605 pt.
44. 53 qt.	**47.** 95 gi.	**50.** 421 pt.	**53.** 814 qt.

54. What part of a gallon is 2½ pt. ?

55. Reduce 6 gal. 3 qt. 1 pt. to pints.

56. Change one barrel (31½ gal.) to pints.

57. Reduce 125 pt. to gallons, quarts, and pints.

58. Express 5 pt. as the decimal part of a gallon.

1. Find the cost of 2 gal. of milk at 6 ¢ per quart.

2. How much would I receive for 1 gal. of cream at 16 ¢ a pint?

3. A certain windmill raised 19 qt. of water per minute. How many gallons of water would it raise in 8 minutes?

4. If 1 pt. of wood dye will stain 88½ sq. ft. of hard wood, how many square feet will 1 gal. of dye stain?

5. If the tank of an oil stove holds 9 pt. of oil, how many gallons of oil are required to fill it 8 times?

6. A pipe discharged 22.5 gal. of mineral water per minute. How many pint bottles could be filled from it per minute?

7. One day a milkman sold 164 pint bottles and 130 quart bottles of milk. How many gallons of milk did he sell?

8. In mixing the paint to cover 100 sq. yd. of surface, 3.5 pt. of raw oil were used. How many gallons of such oil would be needed to mix enough paint to cover 1200 sq. yd.?

9. A cubic foot of dry oak will absorb about 1½ qt. of preserving fluid in a week. How many gallons of the fluid will a stick absorb in that time, if it contains 20 cu. ft.?

10. If 5 qt. of water are used in mixing a cubic foot of concrete, how many barrels of water are required for 126 cu. ft. of concrete?

11. The capacity of the largest cask in the world is 649 hhd. Find its capacity in gallons.

12. A motor boat cruised 6256 miles, using .356 qt. of gasoline per mile. How many gallons of gasoline were used on the trip?

13. One year a large sugar-maple tree produced 53.9 gal. of sap so rich that 7 qt. of it produced 1 lb. of sugar. Find the value of the sugar produced from this tree at 14 ¢ per pound.

DRY MEASURES

Oral Exercises

1. Repeat the table of dry measures.   Name some things that are sold by the dry pint ; quart ; peck ; bushel.

2. Change to quarts : 6 pt. ; 8 pk. ; 14 pt. ; .5 pk. ; $\frac{3}{4}$ pk.

3. Change to pecks : 24 qt. ; 7 bu. ; 48 qt. ; .25 bu. ; $2\frac{1}{2}$ bu.

4. What part of a peck is 1 qt. ? 2 qt. ? 4 qt. ? 6 qt. ? 7 qt. ?

Written Exercises

Reduce to the next lower unit :

1. 56 pk.	**5.** 104 qt.	**9.** 156 pk.	**13.** 876 qt.
2. 64 pk.	**6.** 120 qt.	**10.** 208 qt.	**14.** 428 pk.
3. 72 pk.	**7.** 144 qt.	**11.** 228 pk.	**15.** 472 qt.
4. 88 pk.	**8.** 192 qt.	**12.** 256 qt.	**16.** 536 pk.

17–32.   In exercises **1–16** reduce to the next higher unit.

Express in the lower of the given units :

33. 16 qt. 1 pt.	**36.** 34 pk. 4 qt.	**39.** 52 bu. 2 pk.
34. 22 pk. 5 qt.	**37.** 42 bu. 3 pk.	**40.** 65 qt. 1 pt.
35. 28 bu. 2 pk.	**38.** 47 qt. 1 pt.	**41.** 59 pk. 7 qt.

Express in the next higher and the given unit :

42. 35 pt.	**45.** 62 pk.	**48.** 105 qt.	**51.** 309 pt.
43. 57 pk.	**46.** 49 pt.	**49.** 213 pk.	**52.** 364 qt.
44. 98 qt.	**47.** 69 pk.	**50.** 259 qt.	**53.** 454 pk.

54.   Reduce 8.75 bu. to quarts.

55.   What part of a bushel is 4 qt. ?

56.   Change 13 pk. 7 qt. 1 pt. to pints.

57.   Reduce 147 pt. to pecks, quarts, and pints.

58.   Express 6 qt. as the decimal part of a bushel.

Written Exercises

1. Find the cost of a peck of huckleberries at 17 ¢ a quart.

2. A certain bakery uses 3 pk. of apples daily. How many bushels of apples does this bakery use in 30 days?

3. Find the cost of a bushel crate of Lawton blackberries at $10\frac{1}{2}$ ¢ per quart.

4. A certain family eats on the average a peck of potatoes per week. How many bushels of potatoes will they eat in 52 weeks?

5. If a flock of poultry eats 9 qt. of corn a day, how many days will 18 bu. of corn last the flock?

6. If 3 pk. of flaxseed are required to sow an acre of land, how many acres will 12 bu. of seed sow?

7. How many packages of peanuts containing 1 pt. each can be put up from $\frac{3}{4}$ bu. of peanuts?

8. How many bushels of blue-grass seed are required to sow 8 acres, if $2\frac{1}{4}$ pk. are sown per acre?

9. A cranberry picker gathered on the average 75 6-quart measures of cranberries each day. How many bushels of cranberries did he pick per day?

10. If a boy received $2\frac{1}{4}$ ¢ per quart for picking berries, how much would he receive for picking $1\frac{1}{4}$ bu. of berries?

11. One day a huckster sold $1\frac{3}{4}$ bu. of green peas at 25 ¢ per $\frac{1}{2}$ pk. How much did he receive for the peas?

12. Tomatoes were shipped in crates holding 24 qt. each and in baskets holding 20 qt. each. How many more bushels of tomatoes did 16 crates hold than 16 baskets?

13. A dealer buys potatoes at $1.75 per barrel of $2\frac{1}{2}$ bu., and sells them at 30 ¢ a peck. If $\frac{1}{10}$ of the potatoes spoil while in his possession, what profit does he make per barrel?

WEIGHT MEASURES

Oral Exercises

1. Memorize the table of avoirdupois weight.

2. What sort of articles are weighed by avoirdupois weight?

3. From the table on page 498 learn the standard weight of a bushel of wheat; of a bushel of shelled corn; of a bushel of potatoes; of barley; of buckwheat.

NOTE. — The answers in this book that involve the weight of a bushel of various commodities are based on the standard weight in New York state, unless the standard of some other state is specified in the problem.

4. What is the weight of a barrel of flour? of pork? of salt?

5. How many pounds are there in a *long ton?* For what is it used?

Written Exercises

Reduce to ounces :   Reduce to pounds :

1. 7¼ lb.	**6.** .25 lb.	**11.** 16 T.	**16.** .28 T.
2. 34 lb.	**7.** 2 cwt.	**12.** 13½ T.	**17.** 22¾ T.
3. 56 lb.	**8.** 16¼ lb.	**13.** 8¾ cwt.	**18.** 144 oz.
4. 9¾ lb.	**9.** .125 lb.	**14.** 52 oz.	**19.** 192 oz.
5. 89 lb.	**10.** 25⅝ lb.	**15.** 86 oz.	**20.** 288 oz.

Express in the lower of the given units :

21. 15 lb. 9 oz.	**23.** 4 T. 35 lb.	**25.** 8 T. 145 lb.
22. 8 cwt. 8 lb.	**24.** 7 T. 48 lb.	**26.** 9 T. 12 cwt.

Express in pounds and ounces :   Express in tons and pounds :

27. 60 oz.	**29.** 130 oz.	**31.** 2075 lb.	**33.** 4425 lb.
28. 75 oz.	**30.** 248 oz.	**32.** 3164 lb.	**34.** 7128 lb.

Find the weight of :

35. 12 bu. corn	**37.** 28 bu. wheat	**39.** 45 bbl. flour
36. 25 bu. oats	**38.** 36 bu. barley	**40.** 62 bu. potatoes

1. An apple exhibited at a western apple show weighed 34 oz. How many pounds did it weigh?

2. The largest printing press in the world weighs 225,000 lb. Express this weight in tons.

3. A large passenger locomotive with its tender weighs 424,000 lb. Express this weight in tons.

4. A roll of paper weighs .46 of a ton. Find its cost at $2\frac{1}{4}$ ¢ per pound.

5. The statue on the Capitol at Washington weighs 14,985 lb. How many tons does it weigh?

6. A teamster drew 12 bbl. of cement at a load. Find the weight of his load in tons. (1 bbl. cement = 375 lb.)

7. A cubic foot of ice weighs 57.5 lb. Find the weight in tons and pounds of a load of ice containing 60 cu. ft.

8. The yield of a field of broom corn was 120 bales weighing 20.7 tons. Find the average weight of a bale in pounds.

9. In a day, one fishing boat caught 25,000 lb. of fish, another 20,000 lb., and a third 10,000 lb. Find the total value of these catches at $3 per hundredweight.

10. One year the yield from an acre of basket willows was 2.05 tons of sap-peeled rods that sold at 6 ¢ a pound. How much was received for the crop?

11. On a 4-acre cauliflower field 1500 lb. of fertilizer were used per acre. Find its cost at $30.85 a ton.

12. A coal dealer sold coal at $6.75 per short ton. The coal, including expenses, cost $5.35 per long ton. Find his gain per long ton.

TIME MEASURES

Oral Exercises

Unless otherwise specified a "year" means a "common year."

1. Repeat the table of time measures. Name the months of the year and tell the number of days in each.

2. Reduce to days: 2 wk.; 5 wk.; 7 wk.; 9 wk.; 48 hr.

3. Reduce to minutes: 3 hr.; .5 hr.; 120 sec.; 240 sec.

4. Reduce to hours: 120 min.; 180 min.; 2 da.; 300 min.

5. What part of a day is 6 hr.? 12 hr.? 16 hr.? 18 hr.?

Written Exercises

Reduce to the next lower unit:

1. 72 hr.	**5.** 105 da.	**9.** 480 min.	**13.** 288 hr.
2. 91 da.	**6.** 168 hr.	**10.** 660 min.	**14.** 245 da.
3. 96 hr.	**7.** 126 da.	**11.** 780 min.	**15.** 360 hr.
4. 98 da.	**8.** 216 hr.	**12.** 900 min.	**16.** 322 da.

17–32. In exercises **1–16** reduce to the next higher unit.

Express in the lowest of the given units:

33. 11 da. 4 hr.   **36.** 2 yr. 245 da.   **39.** 12 wk. 6 da. 16 hr.

34. 17 wk. 5 da.   **37.** 18 hr. 36 min.   **40.** 31 da. 12 hr. 45 min.

35. 14 da. 9 hr.   **38.** 35 min. 18 sec.   **41.** 18 hr. 35 min. 50 sec.

Express in the next higher and the given unit:

42. 59 da.	**45.** 275 min.	**48.** 124 hr.	**51.** 416 sec.
43. 65 hr.	**46.** 340 sec.	**49.** 145 da.	**52.** 527 min.
44. 88 da.	**47.** 388 min.	**50.** 268 hr.	**53.** 589 sec.

54. Change 5725 min. to days, hours, and minutes.

55. Express 4 hr. 48 min. as the decimal part of a day.

56. Reduce 52 min. 30 sec. to the fractional part of an hour.

1. The year 1911 began on Sunday. On what day of the week did the year 1912 begin? the year 1913?

2. A man paid $8.75 per week for board and room. How much did it cost him per day?

3. If an electric dredge, working continuously, excavates 15,000 cu. yd. of material per day, how much does it excavate per hour?

4. If a match machine produces 3600 boxes of matches per hour, how many boxes of matches does it produce in a second?

5. How many days are there from Feb. 15 to Mar. 15 inclusive in the year 1915? in the year 1916?

6. A machine can count and sort 72,000 coins per hour. How many coins can it count and sort per second?

7. During the deepening of a well, water flowed in at the rate of $12\frac{1}{2}$ gal. per second. At this rate, how much water would flow into the well in an hour?

8. A machine will stick 90,000 pins into paper in one hour. Find its rate per second.

9. A train load of bananas was transported 1344 mi. in 2 da. 15 hr. What was the average rate per hour?

10. The balance wheel of a watch makes about 432,000 oscillations a day. Find the rate of oscillation per second.

11. America's fire loss for five years averaged $30,000 per hour. Find the average loss of property by fire per year.

12. A heavy freight train traveled a distance of 127 mi. in 7 hr. 12 min. Find, to the nearest tenth, its rate per hour.

13. A stamp-canceling machine, working 1 hr. 15 min. 12 sec., canceled 13,912 stamps. Find its capacity per minute.

Oral Exercises

1. Repeat the table of circular measures.
2. Reduce to seconds: 2′; 4′; 6′; 8′; 9′; 10′.
3. Reduce to minutes: 3°; 5°; 120″; 180″; .5°; 360″.
4. What part of a degree is 10′? 15′? 30′? 45′? 50′?
5. What part of a circumference is 90°? 180°? 270°?

Written Exercises

Change to minutes:

1. 22°	5. 660″	9. 20.5°	13. 1140″
2. 25°	6. 780″	10. 17¾°	14. 1380″
3. 64°	7. 840″	11. 41.4°	15. 1680″
4. 85°	8. 960″	12. 65⅛°	16. 2100″

Express in the lowest of the given units:

17. 14′ 25″	20. 15° 24′ 30″	23. 48′ 48″	26. 98° 22′ 52″
18. 22° 30′	21. 28° 16′ 45″	24. 72° 56′	27. 85° 55′ 40″
19. 35′ 45″	22. 40° 18′ 50″	25. 1 cir. 8°	28. 106° 46′ 32″

Express in units of higher denomination:

29. 1350′	32. 3690″	35. 4124′	38. 7830″
30. 2625′	33. 3830′	36. 5463′	39. 6164′
31. 1430″	34. 4945″	37. 5325″	40. 8505″

41. How many degrees are there in ⅜ cir.? in .83⅛ cir.?

42. Through how many degrees does the minute hand of a clock pass in 15 min.? in 45 min.? in 1 hr.? in 1 hr. 30 min.?

43. How many degrees are there in the angle made by the hour and minute hands of a clock at 4 o'clock?

1. Repeat the table of English money.
2. How many centimes are there in a French franc?
3. How many pfennigs are there in a German mark?
4. Reduce to shillings : £2; £4; £5; 48*d*.; 72*d*.
5. Reduce to francs : 200 c.; 500 c.; 150 c.; 325 c.
6. Reduce to pounds : 60*s*.; 80*s*.; 120*s*.; 160*s*.; 180*s*.
7. What part of a mark is 20 pf.? 25 pf.? 50 pf.? 75 pf.?
8. What part of a pound is 5*s*.? 10*s*.? 12*s*.? 15*s*.? 16*s*.?

Written Exercises

Reduce to shillings :   Reduce to sovereigns :

1. £ 35	4. 216*d*.	7. 500*s*.	10. 480*d*.
2. £ 42	5. 324*d*.	8. 680*s*.	11. 720*d*.
3. £ 68	6. 456*d*.	9. 740*s*.	12. 960*d*.

Express :

13. .8 M. as pfennigs.   17. 16*s*. 10*d*. as pence.
14. ¾ fr. as centimes.   18. 330*s*. as pounds and shillings.
15. 19.25*s*. as pence.   19. 1650 pf. as marks and pfennigs.
16. £ 2.75 as shillings.   20. 2780 c. as francs and centimes.

21. In Leeds, England, a carter driving one horse receives 26*s*. per week. Find his wages for a year of 52 weeks.

22. Some German railways sell tea and coffee to employees at 2 pf. a cup. How much does an employee expend for coffee in a year of 52 weeks, if he buys 8 cups a week?

23. The cost of bread in some English cities is a penny per pound. How much does a baker receive for 85 1-lb. loaves of bread, 42 2-lb. loaves, and 64 1½-lb. loaves?

These official equivalents may be learned:

$$1 \text{ pound} = \$4.8665$$
$$1 \text{ franc} = \$ .193$$
$$1 \text{ mark} = \$ .238$$

The monetary unit of Belgium and Switzerland is the same as that of France.

The following units are identical in weight and fineness with the franc: the *peseta* (Spain), the *lira* (Italy), the *drachma* (Greece), and the *bolivar* (Venezuela).

In estimates we think of the pound as $5, the franc as 20 ¢, and the mark as 25 ¢.

Written Exercises

Estimate, then find to the nearest cent, the value in United States money of:

1. £80	**5.** 1200 fr.	**9.** 3180 M.
2. £224	**6.** 1640 M.	**10.** 36.25 lire
3. £40.5	**7.** 122.40 fr.	**11.** 49.80 pesetas
4. £150.75	**8.** 236.50 M.	**12.** 2000 drachmas

13. Reduce £24 8s. 4d. to United States money.

£ s. d.

	24 8 4
12	4
20	8. 33+
	24.4 17⁻

Since $1d. = \frac{1}{12}s.$, $4d. = \frac{1}{3}s. = .33^+s.$ Bringing down the 8s., we have 8.33+s. Since $1s. = £\frac{1}{20}$, $8.33^+s. = £\frac{8.33^+}{20} = £.417^-.$ Bringing down the £24, we have £24.417⁻. Then we multiply $4.8665 by 24.417, obtaining $118.83⁻.

NOTE. — When pounds are expressed to the nearest third decimal place, the error is within ½ of £ .001, equal to about ¼ cent. Hence, *pounds to the nearest thousandth give United States money to the nearest cent.*

Reduce to United States money, to the nearest cent:

14. £8 3s. 4d.	**17.** £12 7s. 7d.	**20.** £100 8s. 1½d.
15. £5 2s. 6d.	**18.** £25 4s. 3d.	**21.** £125 6s. 10d.
16. £7 6s. 9d.	**19.** £66 8s. 5d.	**22.** £440 18s. 9½d.

23. Reduce $225.50 to English money.

$225.50 ÷ $4.8665 = 46.337⁺, the number of pounds.
.337 × 20s. = 6.74s., and .74 × 12d. = 8.88d.
Hence, to the nearest penny, $225.50 = £46 6s. 9d.

NOTE. — To obtain the result correct to the nearest penny, the division need be carried only to the nearest third decimal place.

Reduce to English money, to the nearest penny:

24. $1000	**27.** $48.25	**30.** $77.15	**33.** $122.43
25. $3200	**28.** $62.75	**31.** $84.40	**34.** $760.50
26. $5500	**29.** $90.10	**32.** $50.07	**35.** $325.75

36. Mrs. Hayes purchased 4 Swiss statuettes in Geneva at 2.50 fr. each. Find their cost in United States money.

37. Find the cost in United States money of 30 bouquets purchased in Florence, at ½ lira each.

38. Find the value of a Bank of England note for £1000; of a Bank of Naples note for 1000 lire.

39. A party of 3 traveled by train from Bristol to London, 194 miles, at 2d. each per mile. Find the fare for all in English money ; in United States money, to the nearest cent.

40. A cloth weaver in Germany received 20 pf. per hour. He worked 10 hr. a day and 6 days a week. How much did he earn per week in United States money, to the nearest cent?

41. A man traveling in England exchanged $50 for English money. How many pounds, shillings, and pence did he receive?

42. Great Britain built the New Zealand Government Railway at a cost of £7779 per mile. Find, to the nearest dollar, the cost per mile in United States money.

Oral Exercises

1. How many units are 3 doz. ? 5 doz. ? 6 doz. ?
2. What part of a dozen is 3 ? 4 ? 6 ? 8 ? 9 ? 10 ?
3. Reduce to gross : 48 doz. ; 72 doz. ; 84 doz. ; 96 doz.
4. How many years are 3 score years ? 3 score and 10 years ?

Written Exercises

Reduce :

1. 17 doz. to units
2. 32 gr. to dozens
3. 264 doz. to gross
4. 45 G. gr. to gross
5. 672 units to dozens
6. 768 gr. to great gross
7. $6\frac{1}{2}$ gr. to units
8. 2016 units to gross
9. 9.25 G. gr. to dozens
10. 22 gr. 8 doz. to dozens
11. 15 G. gr. 9 gr. to gross
12. 385 doz. to gross and dozens

13. How many dozen oranges are there in a box containing 150 oranges ?

14. A stationer bought a gross of tablets for $4.68 and sold them for 5¢ each. How much did he gain ?

15. The largest egg hatchery in the world holds 165,000 eggs. Find its capacity in dozens.

16. A merchant bought 2 gross of shoe laces at $1.25 per gross and sold them at 3¢ a pair. Find his profit.

17. In the year 1913 an Indian chief, said to be 6 score and 12 years old, was considered the oldest living human being. In what year was he born ?

18. One year 8028 gross of fountain pens were manufactured in this country. If the average wholesale value of each fountain pen was $.789, what was the value of all ?

Oral Exercises

1. Repeat the table of stationers' measures.

2. Reduce to quires : 4 reams ; 6 reams ; 48 sheets.

3. What part of a quire is 6 sheets? 8 sheets ? 12 sheets ?

4. What part of a ream is 4 quires ? 10 quires ? 15 quires?

NOTE. — Paper is often sold by the 100, 500, and 1000 sheets; also by the pound.

Written Exercises

Reduce :

1. 18 reams to quires	6. 2.5 reams to sheets
2. 24 quires to sheets	7. 86½ bundles to reams
3. 75 bundles to bales	8. 32 reams 14 quires to quires
4. 340 quires to reams	9. 35 quires 18 sheets to sheets
5. 672 sheets to quires	10. 415 quires to reams and quires

11. Miss Tracy bought a 5-quire box of writing paper. How many letters of 1 sheet each can she write on this paper?

12. Find the weight, in pounds, of a bale of paper when a quire weighs 4 oz.

13. How many quires of drawing paper will a class of 36 pupils use in 10 days, if each pupil is supplied with a sheet of paper per day?

14. If 90 sheets of writing paper weigh 1 lb., what is the weight of a ream of such paper?

15. One year 74,510,064 lb. of paper were used in the United States for job printing. Find, to the nearest cent, the value of this quantity of paper at $.084 per pound.

16. What is the cost of the paper and envelopes needed to send out 400 circular letters of 2 sheets each, if the envelopes cost $2.50 per thousand and the paper $1.50 per 500 sheets?

ADDITION OF DENOMINATE NUMBERS

Addition, subtraction, multiplication, and division with denominate numbers are performed just as with other numbers. It is necessary, however, to bear in mind the number of units of any denomination required to make a unit of the next higher denomination, since in denominate numbers it varies, while in integers it is always 10.

Written Exercises

1. Add 14 ft. 11 in. and 9 ft. 6 in.

ft.	in.
14	11
9	6
24	5

As in integers write units of the same denomination in the same column.

For the inches, 6 in. + 11 in. = 17 in., or 1 ft. 5 in.; write 5 under the inches and add 1 ft. to the feet.

For the feet, 1 ft. (carried) + 9 ft. + 14 ft. = 24 ft.; write 24 under the feet.

Hence, the sum is 24 ft. 5 in.

Add :

	gal.	qt.		bu.	pk.		yd.	ft.		lb.	oz.
2.	12	3	3.	14	2	4.	18	2	5.	43	10
	6	2		8	3		12	1		37	14

	ft.	in.		da.	hr.		pk.	qt.		T.	lb.
6.	18	9	7.	25	16	8.	50	.7	9.	5	465
	8	6		16	20		18	2		3	250

	°	′		mi.	rd.		qt.	pt.		hr.	min.
10.	40	45	11.	7	190	12.	18	1	13.	48	30
	24	50		3	245		9	1		12	45

	£	s.		wk.	da.		pt.	gi.		A.	sq. rd.
14.	42	16	15.	50	4	16.	48	2	17.	9	125
	28	19		26	6		39	3		4	108

18–31. Add in exercises 2–9 and 26–31, page 145.

Add :

sq. ft.	sq. in.		bu.	pk.	qt.		hr.	min.	sec.
32. 22	58	**33.** 16	3	5		**34.** 19	48	25
19	69		35	2			22	28	47
8	94		27	3	6		36		38

cu. yd.	cu. ft.		°	′	″		£	s.	d.
35. 15	18	**36.** 25	52	30		**37.** 38	15	11
38	24		18	29	15		26	18	9
27	16			35	45		45		10
44	25		84	48	58		17	14	11

38. Add 18 cu. ft. 360 cu. in.; 16 cu. ft.; 20 cu. ft. 1510 cu. in.

39. Add 24° 14′ 48″ ; 44° 36′ 27″ ; 16° 5′ ; 14° 24′ 4″.

40. Add 48 A. 37.5 sq. rd. ; 72 A. 49.2 sq. rd. ; 95 A. 85 sq. rd. ; 56 A. 122.3 sq. rd. ; 40 A. 140 sq. rd.

41. The sales of ice cream at a store one day were 6 gal. 3 qt. of vanilla cream and 4 gal. 2 qt. 1 pt. of chocolate cream. Find the total amount of cream sold.

42. In making a mixture of chicken feed 3 pk. of corn, 2 pk. 4 qt. of buckwheat, and 2 pk. 2 qt. of wheat were used. How many bushels of grain were used ?

43. At 4.30 P.M. Mrs. Deyo put a 5-pound chicken in the oven to roast. If 15 min. per pound are allowed for roasting chicken, at what time was it ready to serve ?

44. The " Western New York Express " runs from Syracuse to Rochester in 1 hr. 45 min. and from Rochester to Buffalo in 1 hr. 50 min. Find the time required to run from Syracuse to Buffalo.

45. A teamster drew three loads of crushed stone weighing 2 T. 230 lb., 2 T. 260 lb., and 1 T. 1970 lb., respectively. Find the total weight of crushed stone hauled.

SUBTRACTION OF DENOMINATE NUMBERS

Written Exercises

1. From 31 gal. 2 qt. subtract 8 gal. 3 qt.

Since 3 qt. cannot be subtracted from 2 qt., 31 gal. 2 qt. may be expressed as 30 gal. 6 qt. (or thought of as being expressed that way). Thus:

gal.	qt.
31	2
8	3
22	3

For the quarts, 6 qt. − 3 qt. = 3 qt.; write 3 under the quarts.

For the gallons, 30 gal. − 8 gal. = 22 gal.; write 22 under the gallons.

Hence, the whole remainder is 22 gal. 3 qt.

Subtract :

	ft.	in.		gal.	qt.		lb.	oz.		hr.	min.
2.	24	10	**3.**	16	2	**4.**	27	11	**5.**	18	15
	16	8		9	3		14	15		16	30

	pk.	qt.		£	s.		qt.	pt.		yd.	ft.
6.	32	7	**7.**	26	10	**8.**	17		**9.**	34	1
	17	4		17	16		11	1		18	2

10–25. Subtract in exercises **2–17**, page 143.

Subtract :

	sq. ft.	sq. in.		bu.	pk.	qt.		hr.	min.	sec.
26.	38	65	**27.**	16	3	3	**28.**	32	40	55
	12	72		12	1	5		15	45	25

	cu. yd.	cu. ft.		°	′	″		£	s.	d.
29.	42	15	**30.**	65	20	30	**31.**	22	10	6
	18	20		22	30	45		14	15	8

32. How much less than a right angle is 77° 42′ ?

33. How much less than 2 right angles is 154° 28′ 33″ ?

34. How long was it from June 26, 1900 to May 15, 1914?

yr.	mo.	da.
1914	5	15
1900	6	26
13	10	19

The later date is written as the minuend and the earlier date as the subtrahend, the number of the month being used instead of its name.

Subtract as in denominate numbers, considering 30 days as a month and 12 months as a year. The remainder is the difference in time as accurately as it can be expressed in years, months, and days.

Subtract:

yr.	mo.	da.		yr.	mo.	da.		yr.	mo.	da.
35. 1915	4	1	**36.** 1913	8	10	**37.** 1918	2	16
1912	1	15	1909	9	30	1912	6	25

Find the difference in time between to-day and:

38. Mar. 5, 1908   **40.** Oct. 14, 1911   **42.** Feb. 15, 1912

39. July 4, 1916   **41.** Aug. 25, 1915   **43.** Dec. 16, 1925

Subtract:

44. 3 qt. 1 pt. from 6 qt.   **49.** 7s. 9d. from £1 4s.

45. 2 pk. 5 qt. from 9 bu.   **50.** 88 sq. rd. from 2 A.

46. 6 da. 15 hr. from 15 da.   **51.** 875 lb. from 1 T. 250 lb.

47. 576 cu. in. from 3 cu. ft.   **52.** 30 min. 8 sec. from 16 hr.

48. 125 sq. in. from 1 sq. yd.   **53.** 2 yd. 1 ft. 3 in. from 1 rd.

54. A train arrived at a station at 11.47 A.M. and left at 12.02 P.M. How long was the train at the station?

55. John Fish is 6 ft. 1 in. tall and his brother Harvey is 5 ft. 9 in. tall. Find the difference in their heights.

56. A dealer bought 6 gr. of tennis balls and at the end of the season he had 8 doz. and 4 left. How many did he sell?

57. The distance around a room is 63 ft. 6 in. What length of baseboard has it, if the total width of doors is 14 ft. 8 in.?

MULTIPLICATION OF DENOMINATE NUMBERS

Written Exercises

1. Multiply 18 lb. 5 oz. by 9.

lb.	oz.
18	5
	9
164	13

9 times 5 oz. = 45 oz., or 2 lb. 13 oz.
9 times 18 lb. = 162 lb.
162 lb. + 2 lb. (carried) = 164 lb.
Hence, the product is 164 lb. 13 oz.

Multiply:

	ft.	in.		bu.	pk.		hr.	min.		gal.	qt.
2.	7	9	**3.**	5	3	**4.**	3	16	**5.**	5	3
		5			4			6			7

	yr.	mo.		lb.	oz.		£	s.		T.	lb.
6.	8	9	**7.**	8	12	**8.**	7	11	**9.**	3	450
		7			9			8			6

10. 15° 12′ by 6

11. 6s. 5d. by 3

12. £4 8s. by 5

13. 9 ft. 6 in. by 9

14. 3 pk. 3 qt. by 6

15. 7 yr. 8 mo. by 7

16. 8 bu. 3 pk. by 12

17. 5 mo. 21 da. by 2

18. 3 yd. 2 ft. by 18

19. 16 lb. 6 oz. by 9

20. 2 T. 750 lb. by 4

21. 6 rd. 11 ft. by 3

22. 31 gal. 2 qt. by 6

23. 8 wk. 6 da. by 11

24. 2 hr. 12 min. by 10

25. 12 ft. 7 in. by 16

26. 14 rd. 3 yd. by 5½

27. 8° 15′ 20″ by 15

28. £8 7s. 9d. by 4

29. 2 mi. 220 rd. by 3

30. 12 gal. 3 qt. by 21

31. 7 A. 60 sq. rd. by 4

32. 3 bu. 5 pk. 2 qt. by 5

33. 3 yr. 7 mo. 6 da. by 7

34. 8 sq. yd. 6 sq. ft. by 13

35. 14 cu. yd. 9 cu. ft. by 9

36. 4 yd. 2 ft. 8 in. by 15

37. 12 sq. ft. 36 sq. in. by 20

38. 7 cu. ft. 608 cu. in. by 6

39. 3 hr. 20 min. 12 sec. by 24

1. Find the number of feet of picture molding required to go around a square room 15 ft. 9 in. on a side.

2. If 1 bu. 2 pk. of rye are sown per acre, how much seed is required to sow 6 acres?

3. How much time per week does a girl spend practicing music, if she practices 1 hr. 15 min. every day except Sunday?

4. Each day during June, Mrs. Smith used 2 qt. 1 pt. of milk. How many quarts of milk did she use that month?

5. A miner's inch of water is equivalent to a flow of about 11 gal. 2 qt. 1 pt. per minute. Find the flow equivalent to 5 miner's inches.

6. A flock of poultry laid on an average 3 doz. and 5 eggs per day. How many eggs did this flock lay per month?

7. How many feet of barbed wire would be needed to build a fence 5 wires high around a plot 12 rd. 11 ft. square?

8. How many square yards of linoleum are needed to cover the floors of 3 rooms, each 19 sq. yd. 3 sq. ft. in area?

9. At 6 ¢ per foot, find the cost of 7 curtain poles each 4 ft. 6 in. long.

10. If a farmer sows 2 bu. 1 pk. of barley seed per acre, how much seed does he need to sow 13 acres?

11. The weekly wages of a miner in England are £1 8s. 8¼d. Find his wages for 8 weeks.

12. The point of the minute hand of a certain tower clock goes 1 ft. 11 in. per minute. How far does it go per hour?

13. If 41 T. 1333⅓ lb. of coal are burned per hour by the steamship *Mauretania*, how much is burned per day?

14. If each of the 55 shares of a steam gang plow turns a furrow 1 ft. 2 in. wide, what width of land does the plow turn?

DIVISION OF DENOMINATE NUMBERS

Written Exercises

1. Divide 33 mi. 40 rd. by 4.

mi.   rd.

4)33   40

$\overline{\quad 8 \quad 90 \quad}$

33 mi. ÷ 4 = 8 mi. and 1 mi. remainder.
1 mi. 40 rd. = 360 rd.
360 rd. ÷ 4 = 90 rd.
Hence, the quotient is 8 mi. 90 rd.

Divide :

2. 16 ft. 9 in. by 3

3. 26 yr. 6 mo. by 6

4. 21 yd. 2 ft. by 5

5. 82 bu. 2 pk. by 4

6. 22 gal. 2 qt. by 6

7. 84 lb. 14 oz. by 9

8. 53 T. 750 lb. by 7

9. 18 hr. 16 min. by 8

10. 176° 45′ by 15

11. 56 wk. 4 da. by 18

12. 42 mi. 240 rd. by 12

13. 45 min. 40 sec. by 20

14. 120 A. 84 sq. rd. by 16

15. 338 sq. yd. 8 sq. ft. by 25

16. 278 cu. yd. 18 cu. ft. by 12

17. 354 sq. ft. 96 sq. in. by 24

18. How many days will 40 bu. 2 pk. of oats last a horse that eats 1 pk. 4 qt. a day ?

SOLUTION. — Reducing the denominate numbers to a convenient *common denomination*, in this case to pecks, we have

40 bu. 2 pk. = 162 pk., and 1 pk. 4 qt. = 1.5 pk.

Then, dividing 162 pk. by 1.5 pk., the quotient is found to be 108.

Hence, 40 bu. 2 pk. of oats will last the horse 108 days.

19. Divide 94 ft. 6 in. by 6 ft. 9 in.

20. Divide 25 yd. 2 ft. by 3 ft. 6 in.

21. Divide 96 yr. 9 mo. by 4 yr. 6 mo.

22. Divide 79 gal. 3 qt. by 2 gal. 2 qt.

23. Divide 56 hr. 43 min. by 20 min. 30 sec.

24. Divide 25 sq. yd. 6 sq. ft. by 8 sq. ft. 36 sq. in.

Written Exercises

1. How many bushels of oats are there in 6 T. of oats?

2. How many bushels of barley are there in 6 T. of barley?

3. A man spends 1 hr. 40 min. a day going to and from his work. How long does it take him for each of the 4 trips?

4. A farmer drew 285 bu. 2 pk. of potatoes to market in 4 loads. Find the average amount carried per load.

5. The total yield from 5 acres of peanuts was 272 bu. 2 pk. Find the average yield per acre.

6. If 8 cu. yd. of gravel weigh 13 T. 880 lb., what is the weight per cubic yard?

7. A heaped bushel occupies $1\frac{1}{4}$ cu. ft. How many bushels of potatoes are there in a car containing 1000 cu. ft. of them?

8. If 15 bales of hay weighed 1 T. 7 cwt., what was the average weight per bale in pounds?

9. How many sofa pillow covers 18 in. square can be cut from a piece of felt 6 ft. by 4 ft. 6 in.?

10. How many batches of 10 cu. ft. of concrete each had to be mixed to make a wall containing 38 cu. yd. 24 cu. ft.?

11. How many barrels, each holding 2 bu. 3 pk., will it take to hold a crop of 178 bu. 3 pk. of apples?

12. How many strips of 27-inch carpet will be required for a room 15 ft. 9 in. wide, if the strips run lengthwise?

13. One day a motor truck delivered 53 T. 410 lb. of stone in 12 loads. Find the average weight per load.

14. The regulation military pace is 2 ft. 6 in. How many steps will a soldier take in marching a mile?

15. The perimeter of a room is 58 ft. 6 in. How many strips of paper 18 in. wide are needed for the walls?

Drill Exercises

1. Find the area of a rectangle 2′ × 9″.
2. Find ¼ of 16° 32′ 30″; ⅝ of 125 ft. 4 in.
3. What is the volume of a solid 3′ × 4′ × 2′ 6″?
4. Add 2 T. 5 cwt.; 1 T. 12 cwt.; 4 T. 15 cwt.; 3 T. 8 cwt.

5–6. Reduce £1 15s. to United States money, to the nearest cent; $100 to English money, to the nearest penny.

Reduce to the next lower unit:

7. 72 ft.	12. 65 doz.	17. 96s.	22. 186 pk.
8. 84 hr.	13. 125 cwt.	18. 320′	23. 138 hr.
9. 66 pk.	14. 88 qt. (dry)	19. 154 da.	24. 144 sq. ft.
10. 91 da.	15. 78 qt. (liq.)	20. 132 yd.	25. 560 sq. rd.
11. 99 yd.	16. 165 quires	21. 440 rd.	26. 126 cu. ft.

27–46. In exercises **7–26** reduce to the next higher unit.

Reduce:

47. 3.1875 bu. to quarts.
48. 650 pt. to gallons and quarts.
49. ⅝ cu. yd. to cubic feet and cubic inches.
50. ⅞ of a right angle to degrees and minutes.
51. 4 cwt. 80 lb. to the fractional part of a ton.
52. .9 of a day to a compound denominate number.
53. 5344 ft. (height of Mt. Marcy) to miles and feet.

Express in the lower of the given units:

54. 42° 36′	59. £15 18s.	64. 20 A. 127 sq. rd.
55. 18 bu. 3 pk.	60. 68 yd. 2 ft.	65. 62 sq. yd. 8 sq. ft.
56. 32 ft. 7 in.	61. 76 gal. 1 qt.	66. 75 cu. yd. 21 cu. ft.
57. 46 wk. 5 da.	62. 18 hr. 41 min.	67. 16 cu. ft. 780 cu. in.
58. 52 pk. 6 qt.	63. 120 rd. 4 yd.	68. 124 sq. rd. 18 sq. yd.

Express in the next higher and the given unit :

69. 870'   **72.** 368*s.*   **75.** 277 quires   **78.** 780 sq. in.

70. 427 ft.   **73.** 725 pk.   **76.** 386 qt. (dry)   **79.** 463 sq. ft.

71. 362 hr.   **74.** 741 doz.   **77.** 435 qt. (liq.)   **80.** 897 cu. ft.

What decimal part of the next higher unit is :

81. 2 ft. 3 in.   **83.** 7*s.* 6*d.*   **85.** 7 cwt. 50 lb.   **87.** 140 sq. rd.

82. 2 pk. 4 qt.   **84.** 52' 30''   **86.** 43.2 sq. in.   **88.** 648 cu. in.

Add :

	bu.	pk.			•	'			hr.	min.			A.	sq. rd.
89.	65	3	**91.**	75	30		**93.**	15	9		**95.**	75	80
	12	2		22	45			8	20			24	96

	ft.	in.			£	*s.*			mi.	rd.			cu. yd.	cu. ft.
90.	45	3	**92.**	85	16		**94.**	42	120		**96.**	56	12
	16	9		14	4			35	242			13	18

97–104.   Subtract in exercises **89–96.**

Multiply :   Divide :

105. 48 wk. 6 da. by 3   **115.** 64 bu. 2 pk. by 6

106. 32 bu. 3 pk. by 9   **116.** 42 wk. 6 da. by 4

107. 21 ft. 10 in. by 8   **117.** 64 ft. 9 in. by 7

108. 12 mi. 82 rd. by 5   **118.** 94 rd. 4 ft. by 5

109. 38 gal. 3 qt. by 7   **119.** 78 gal. 3 qt. by 9

110. £ 20 18*s.* 9*d.* by 6   **120.** 25 hr. 36 min. by 8

111. 15° 30' 45'' by 10   **121.** 37 mi. 208 rd. by 16

112. 10 hr. 30 min. by 15   **122.** £ 49 12*s.* 6*d.* by 15

113. 38 sq. yd. 8 sq. ft. by 12   **123.** 92 A. 110 sq. rd. by 10

114. 42 cu. yd. 19 cu. ft. by 18   **124.** 88 cu. ft. 576 cu. in. by 18

INDUSTRIAL PROBLEMS

Sewing. **1.** How many pieces of ruffling 6 in. wide can be cut crosswise from 6 yd. of silk?

2. How many yards of cord are required to put around the edge of a pillow, 16 in. by 20 in., if a knot at each corner takes 9 in. and no allowance is made for joining the ends?

3. In making a linen dress, 3 lengths of 37½ in. each were required for the skirt, 22½ in. for the sleeves, and 1¾ yd. for the waist proper. How many yards were required for the dress?

4. A hat frame cost 75 ¢. It was covered with 1½ yd. of velvet at $1.50 per yard and the trimmings for it cost $4.50. If the making cost $1.50, what was the total cost of the hat?

5. A piece of lawn 1 yd. long was only ¼ yd. long after being tucked. How many ¼-inch tucks were made in it?

6. A piece of linen 36 in. wide was tucked for a shirtwaist. If 36 ⅛-inch tucks were made in it, how wide was the piece of linen when tucked?

7. How many pieces of ruffling can be cut from 5¼ yd. of dimity, if the ruffle, having a 1-inch hem and 3 ⅛-inch tucks, is to be 9 in. wide when finished?

Suggestion. — Allow ¼ in. for the first turning, in making hems.

8. How wide must I cut a strip of silk to make a ruffle 5¼ in. wide with a hem of 1⅜ in. on one edge and ⅝ in. on the other? How many such strips can be cut crosswise from 1¼ yd. of silk?

9. A tailor furnished material and findings for a lady's suit as follows: 7 yd. serge at $1.50 a yard; 4 yd. satin at $1 a yard; 1½ yd. canvas at 20 ¢ a yard; wadding, 8 ¢; thread, 20 ¢; weights, 10 ¢; ½ doz. buttons at $1 a dozen. If he charged $15 for making, what was the total cost of the suit?

Cooking. **10.** Mrs. Bowers put a 5-pound roast of beef in the oven at 4.45 P.M. and it was ready to serve at 6.00 P.M. Find the time required per pound to roast the beef.

11. If 1 pt. of berries is required for a pie, how many pies can be made from $\frac{1}{2}$ bu. of berries?

12. Find the cost of the ingredients of a baked custard as follows: 1 pt. milk at 8￠ per quart; 2 eggs at 28￠ per dozen; sugar, $\frac{3}{4}$￠; seasoning, $\frac{1}{12}$￠.

13. If in making currant jelly 1 lb. of sugar is used per pint of currant juice, how much sugar is needed for 1 gal. of juice?

14. Mrs. Chase used 12 oz. of sugar to 1 lb. of oranges for orange marmalade. How many pounds of sugar did she need for 5 lb. of oranges?

15. If in making bread 3 cups of flour are allowed per loaf of bread, how many pounds of flour are required for a batch of 6 loaves? (1 cup of flour weighs 4 oz.)

16. A cooking class made baking powder by mixing 4 oz. of soda, 9 oz. of cream of tartar, and 2 oz. of cornstarch. How' much of each ingredient was needed for 15 lb. of baking powder?

17. Using the following ingredients, a baker made 252 lb. of vanilla-cream biscuit, which he sold at 16￠ per pound:

1 bbl. flour at $5.20 per bbl.	3$\frac{1}{2}$ gal. milk at 6￠ per qt.
20 lb. butter at 26￠ per lb.	2 qt. glycerine at 75￠ per qt.
16 lb. lard at 12$\frac{1}{2}$￠ per lb.	10 oz. soda at 8￠ per lb.
64 lb. powdered sugar at 5$\frac{3}{4}$￠ per lb.	Ammonia, salt, vanilla, 96￠.
2 gal. eggs (10 eggs to a pint) at 24￠ per doz.	

Find the cost; the amount received; the profit.

Lumber. The men listed below were employed by a lumber company to cut down trees, saw them into logs, make roads, haul the logs to the river, and deliver them at the sawmill.

Fill the blanks in exercises **18–26** :

	EMPLOYEES	TIME EMPLOYED	RATE OF WAGES	SEASON'S EARNINGS
18.	1 timekeeper	156 days	$ 1.85 per day	——
19.	1 cook	6 months	—— per month	$ 270.00
20.	1 chore boy	6 months	$30.00 per month	——
21.	34 choppers	156 days	—— per day	$11,934.00
22.	1 scaler	6 months	$50.00 per month	——
23.	23 road makers	72 days	—— per day	$ 2484.00
24.	13 teamsters	156 days	$ 1.90 per day	——
25.	17 log drivers	35 days	—— per day	$ 3123.75
26.	Total cost of this labor			——

Exercises **27–31** are based on a milling season of 184 days and an average daily output of 95,000 ft. of lumber.

27. What was the output for the whole season?

28. The season's expense for labor to operate the sawmill was $14,066.80.   Find the average cost of mill labor per day.

29. Find, to the nearest cent, the average cost of labor per thousand feet of lumber, the average daily pay roll being $76.45.

30. Find the cost of transporting one day's output from the mill to the market at $1.75 per thousand feet.

31. If $\frac{2}{5}$ of the lumber was spruce worth $25.25 per thousand feet, $\frac{1}{10}$ pine worth $45 per thousand feet, and the rest hemlock worth $23 per thousand feet, what was the value of the output for one day?

Oil. 32. In the construction of an oil derrick in Western New York, 9500 ft. of lumber were used at a cost of $25.50 per M. If this was ⅝ of the cost of the derrick, what was its cost?

33. The cost of drilling an oil well 1280 ft. deep was $704. Find the cost per foot.

34. An oil well was equipped with 250 ft. of casing at 45 ¢ a foot, 1280 ft. of tubing at 15½ ¢ a foot, and 1280 ft. of pumping rods at 5½ ¢ a foot. Find the cost.

35. An oil well produced 2352 bbl. of oil in 3 weeks. Find the yield per day.

36. In one month an oil well produced 2880 bbl. of oil. The operator gave ⅛ of it for rent. Find the value of his share at $1.85 per barrel.

37. The distances between the pumping stations on a pipe line from an oil field to a city are 23 mi. 2165 ft., 25 mi. 4858 ft., 25 mi. 3854 ft., 45 mi. 2376 ft., 51 mi. 4963 ft., and 62 mi. 2640 ft. Find the total distance.

Find what decimal part of the crude oil each product is, if 30,800 bbl. of crude oil, when refined, produce:

38. Gasoline, benzine, and naphtha,   3850 bbl.

39. Illuminating oil,   23,100 bbl.

40. Lubricating oil,   924 bbl.

41. Residuum and loss,   2926 bbl.

42. How many tons of refined illuminating oil will a tank freight car hold, if its capacity is 8000 gal.? (1 gal. = 6¼ lb.)

43. Refined oil is often shipped in five-gallon cans, that are packed by two's in wooden cases 20¾ in. long and 10¼ in. wide. How many of these cases can be put in one layer in a steamship's compartment, 41½ ft. long and 31½ ft. wide?
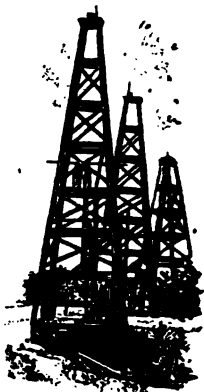
Coal. **44.** The shaft or entrance to an anthracite coal mine was 12 ft. by 18 ft., and extended to a depth of 720 ft. What was the cost of sinking and timbering the shaft at $3.75 per cubic yard?

45. The cost per linear yard for tunneling the underground passage of a coal mine was $3.60 for labor and 90 ¢ for powder. Find the cost of tunneling a passage 567 ft. in length.

46. In a coal mine the coal is carried from the miners to the shaft in cars. If each car weighs 850 lb. and has a capacity of 1 long ton, what is the weight, in tons, of a train of 8 loaded cars?

47. The rails upon which the cars of a certain coal mine run weigh 35 lb. per yard. At $13.20 per long ton, find the cost of the rails for a 2-rail track 480 ft. long.

48. A cage that raises the coal from a mine 840 ft. deep ascends at the rate of 24 ft. per second. How long does it take to reach the surface?

49. It requires on the average $10\frac{3}{4}$ oz. of powder to dislodge 1 ton of coal. Find the cost, at $1.50 per 25-pound keg, of the powder used by a miner and his laborer in a month when they dislodge 132 tons.

50. If $\frac{3}{16}$ of the coal mined is lost by blasting, breaking, and adhering to the slate, what is the loss in one day when 1625 long tons of salable coal are mined?

51. A mine operator sold 925 long tons of chestnut coal at $3.40 per long ton. Find the profit on this sale, his expenses per long ton being $2.08 for mining and 96 ¢ for handling.

52. How much did a retailer gain by selling 850 long tons of chestnut coal at $6.35 per short ton, if the cost per long ton, including freight and handling, was $5.15?

Wheat. 53. A western farmer had two wheat fields, one ¾ mi. by ½ mi., the other ⅔ mi. by ⅝ mi. How many acres of wheat land had he ?

54. How long does it take to plow 300 acres with a gang plow drawn by a traction engine, if it turns 40 acres per day ?

55. If a plow drawn by two horses turns ⅘ of an acre a day, how many such plows are needed to turn 300 acres in 7¼ days ?

56. Find the cost, at $1.20 per bushel, of the seed wheat required for 150 acres, when 5½ pk. are sown per acre.

57. A drill for sowing wheat sowed 24 acres of ground per day. How long did it take to sow a field 216 rd. by 160 rd.?

58. The yield from a 240-acre field of wheat was 36 bu. per acre. How many days of 10 hr. each would it take to harvest the crop with a machine that reaps and thrashes the wheat and puts it into 2-bushel sacks at the rate of 27 sacks per hour ?

59. How many freight cars are required to carry 3600 bu. of wheat, if 18 tons are carried per car ?

60. Find the cost of shipping 2340 bu. of wheat from a Kansas farm to Chicago, the freight rate being 18 ¢ per 100 lb.

61. How much will it cost to store 5760 bu. of wheat in Chicago from Aug. 8 to Sept. 16 inclusive, if the charge is ¾ ¢ per bushel for the first 10 days and ⅝ ¢ for each succeeding 10 days?

62. The cost of transporting wheat from Chicago to New York by water is 4¾ ¢ per bushel and by rail 16 ¢ per 100 lb. How much more does it cost to ship 4800 bu. by rail than by water ?

63. It takes about 4.5 bu. of wheat to make a barrel of flour. How many barrels of flour can be made from 2736 bu. of wheat?

64. If 100 lb. of flour make 135 1-pound loaves of bread, how many loaves can a baker make from 5 bbl. of flour ?

METHODS OF SOLVING PROBLEMS

There is often more than one method of arriving at the answer to a problem, however simple it may be. Suppose that 5000 sheets of paper cost $8. How much will 10,000 sheets cost at the same price?

The answer may be found as follows:

1. **Indirectly,** by finding the cost of 1 sheet, then of 10,000 sheets. Find the answer by this method, **unitary analysis.**

2. **Indirectly,** by finding the cost of 1 thousand sheets, then of 10 thousand sheets. Find the answer by this method.

3. **Directly,** by comparing the cost of 10,000 sheets with that of 5000 sheets. Find the answer by this method, **analysis by comparison.** Which is the best method for this problem?

ORAL ANALYSIS

In the following problems try to discover the best method of solution. Give the answer first, then the steps in the solution, or the **analysis,** in as few words as possible.

One pupil may declare the answer and others give analysis. If more than one analysis is given, the best should be chosen after a comparison.

1. If it requires 7 nails to fasten a shoe on a horse's foot, how many nails are required for 4 shoes?

ANSWER	ANALYSIS
28 nails.	1 shoe requires 7 nails.
	4 shoes require 4 × 7 nails.

2. If there are 8 rows of single desks in a class room and 6 desks in each row, how many children will the room seat?

3. How far is it around a city block each of whose sides is 300 ft. long ?

4. If 1 yd. of ribbon costs 9 ¢, how many yards can be bought for 72 ¢ ?

1 yd. costs 9 ¢; the number of yards that can be bought for 72 ¢ is the same as the number of times 72 ¢ contains 9 ¢.

5. How many yards of lace can be bought for 90 ¢, if 1 yd. costs 10 ¢ ?

6. How many bunches of celery can be bought for 96 ¢, if 1 bunch costs 8 ¢ ?

7. If 1 doz. eggs costs 25 ¢, how many dozen can be bought for $1?

8. If 1 qt. of milk costs 7 ¢, how many quarts can be bought for $1.40 (140 ¢) ?

9. When 1 lb. of coffee costs 30 ¢, how many pounds can be bought for $1.20?

10. Find the cost of 18 oranges at 3 for 10 ¢.

UNITARY ANALYSIS	ANALYSIS BY COMPARISON
3 oranges cost 10 ¢.	3 oranges cost 10 ¢.
1 orange costs ⅓ of 10 ¢.	18 is 6 times 3.
18 oranges cost ¹⁸⁄₃ of 10 ¢.	18 oranges cost 6 × 10 ¢.

Which is the better analysis for this problem?

11. Find the cost of 30 towels at 3 for $1.

12. When 9 lemons cost 18 ¢, how much do 5 lemons cost ?

13. When 12 spools of thread cost 60 ¢, how much do 10 spools cost ?

14. If a street car runs 6 blocks in 5 min., how far does it run in 20 min. ?

15. If 10 bottles of ink cost 50 ¢, how many bottles of ink can I buy for 30 ¢ ?

16. A boy had $5. After buying a hat he had $3 left. How much did he pay for the hat?

ANALYSIS

All his money = $5; one part = $3; other part paid for hat = $5 − $3.

17. Charles bought 8 bananas at 2¢ each and had 14¢ left. How much money had he at first?

18. Clarence had $2. He bought 2 chickens at 75¢ each. How much money had he left?

19. A dealer bought 60 lead pencils at 3¢ each and sold them at 5¢ each. How much did he gain?

Analyze by finding the amounts paid and received; again by finding first the gain on 1 lead pencil. Compare analyses.

20. A grocer bought 80 qt. of berries at 8¢ a quart and sold them at 10¢ a quart. Find his gain.

21. Edwin bought 2 sailboats at 80¢ each and handed the dealer $2. How much change was due him?

22. A chemical engine carries 250 ft. of chemical hose and 750 ft. of engine hose. What part is engine hose?

ANALYSES

1. 750 ft. = 3 × 250 ft.; 750 ft. + 250 ft. = 4 × 250 ft.; 3 of the 4 equal parts of 1000 ft., or ¾ of the whole, is engine hose.

2. 750 ft. + 250 ft. = 1000 ft.; 750 ft. = $\frac{750}{1000}$, or ¾, of 1000 ft.

23. The hook-and-ladder men can erect their ladder in 48 sec., or in —— of a minute.

24. A man had 25 mi. to drive. When he had driven 15 mi., what part of his journey had he completed?

25. Compare 1½ with ¼. When ¼ of a dozen bananas costs 5¢, find the cost of 1½ doz.

ANALYSIS

1½ is 6 times ¼; ¼ doz. bananas cost 5¢; 1½ doz. bananas cost 6 × 5¢.

26. Compare $\frac{1}{4}$ with $1\frac{1}{4}$. When 15 qt. of strawberries cost $\$1\frac{1}{4}$, how many quarts can be bought for $\$\frac{1}{4}$?

27. If my watch gains $\frac{1}{8}$ min. in 3 days, in how many days does it gain $1\frac{1}{8}$ min.?

28. If 24 bunches of celery cost $\$2\frac{2}{5}$, how many bunches can be bought for $\$\frac{4}{5}$?

29. If $2\frac{1}{2}$ lb. of peanut candy cost $50\not{c}$, how much do $6\frac{1}{4}$ lb. cost?

<div align="center">ANALYSIS</div>

$\frac{5}{2}$ lb. cost $50\not{f}$; $\frac{1}{2}$ lb. costs $\frac{1}{5}$ of $50\not{f}$; $\frac{25}{4}$ lb. cost 13 times $\frac{1}{5}$ of $50\not{f}$.

30. If $1\frac{1}{2}$ lb. of steak cost $45\not{c}$, how much do 5 lb. cost?

31. Mr. Jones bought $12\frac{1}{2}$ lb. of lamb for $\$1.50$ and sold me $3\frac{1}{2}$ lb. at cost. How much did I pay for it?

32. If $\frac{2}{5}$ of the value of a store is $\$4000$, how much is the store worth?

33. How many people lost their lives in vessels wrecked on our shores in a year when $\frac{4}{5}$ of them, or 20, perished on the Atlantic coast?

34. If 2 men can do a piece of work in 12 days, how long does it take 4 men to do it?

<div align="center">ANALYSES</div>

1. The amount of work to be done is 2×12 days' work, or 24 days' work. In 1 day 4 men do 4 days' work. To do 24 days' work 4 men will require $\frac{24}{4}$ days, or 6 days.

2. 4 men can do twice as much work as 2 men in the same time, or the same work in half the time. It will take 4 men $\frac{1}{2}$ of 12 days.

35. If 5 men can paint a bridge in 10 days, how long will it take 10 men? How long would it take 2 men?

36. If 40 men can dig a ditch in 5 days, how long will it take 20 men?

37. If 2 men can load a car in $2\frac{1}{2}$ hr., how long will it take 4 men to load the car?

38. If 15 apples cost 25 ¢, how much do 6 apples cost?

39. What is the cost of 20 marbles, if 30 marbles cost 12 ¢?

40. How many sheep can be bought for $12, if 9 sheep cost $36?

41. At a zoölogical garden the animals eat 250 lb. of beef per day. Find the cost at $12 per 100 lb.

42. The daily ration of bread is 100 lb. and of fish 30 lb. Find the amount of each consumed in 30 days.

43. The weekly ration of vegetables is 8 bbl. How many barrels are used per year of 52 weeks?

44. The animals eat 280 bananas per week. How many dozen bananas do they eat per day? •

45. A baby elephant weighs about 200 lb., an adult elephant about 10,000 lb. The adult elephant is how many times as heavy as the baby elephant? how much heavier?

46. If a girl in a factory receives $1¼ for making 4 shirt-waists, how much does she receive for making 16 shirtwaists?

47. If a girl makes 3 shirtwaists in 10½ hr., how long does it take her to make 12 shirtwaists?

48. A farmer sold 6 bu. of oats for 2\frac{1}{10}$. At that rate how much should he receive for 48 bu.?

49. If 12½ yd. of silk cost $15, how much do 25 yd. cost? How much silk can be bought for $60?

50. If 1200 oysters cost $6, how much do 1000 cost?

51. If 100 lb. of meat cost $16, how much do 225 lb. cost? $50 will buy —— lb.

52. If 1000 bricks cost $10, how much do 2500 bricks cost? How many bricks can be bought for $125?

WRITTEN ANALYSIS

Under modern business conditions problems must be solved with the utmost rapidity and with absolute accuracy. An inaccurate result is worthless — it may be disastrous.

No answer may be regarded as accurate unless it has been checked, preferably by solving the problem or parts of it in two different ways. Knowing this, yet keeping in mind the value of time, the business man sets down his work, briefly and neatly, so that he or any other person may review it at a glance with complete understanding.

1. At a manufacturing plant, 9600 men are employed 55 hr. a week. The weekly pay roll is $138,500. Find the average wage per hour to the nearest tenth of a cent.

MENTAL ESTIMATE

10 thousand men working 50 hr. each would do 500 thousand hours' work. 138 thousand dollars for 500 thousand hours' work is about $¼, or 25¢, per hour.

WRITTEN ANALYSIS

1 man does 55 hours' work.
9600 men do 9600 × 55 hours' work.
9600 × 55 hours' work earns $138,500 in wages.
1 hour's work earns $\dfrac{\$138,500}{9600 \times 55}$ in wages.

BUSINESS PROCESS

12	$138500
8	115.417
11	14.427
5	1.312
	.262

After arriving mentally at the result given in the last line of the written analysis, the business man would employ this process instead of the cancellation process, because it is neater, more easily reviewed, and less liable to error.

CHECKS. — 1. The result agrees well with the estimate, consequently no large errors (such as pointing off wrong) have been made.

2. Reviewing the process rapidly, you will find the work to be correct.

In solving each of the following problems:

(1) Read the problem carefully to see what is given and what is asked.

(2) Estimate the result in advance when you can.

(3) Write a brief analysis.

(4) Solve, arranging the computation in a businesslike manner.

(5) Check your result.

NOTE. — These directions should be observed in solving problems wherever practicable *throughout the book.*

2. When 18 bbl. of cabbages can be bought for 22\frac{1}{2}$, how many barrels can be bought for 7\frac{1}{2}$?

3. If 100 lb. of hazelnuts yield 43 lb. of kernels, how many pounds of kernels will 9500 lb. of nuts yield?

4. A merchant bought 96 suits of clothes at $12 each and sold them at $19.50 each. How much did he gain?

5. A dealer bought 125 overcoats at $35 each, and sold them at $60 each. Of this money $425 was lost in bad debts. Find the average gain on each overcoat.

6. Of the pears picked from an orchard, 65 bu. were sold at the home canning factory at 95¢ per bushel. How much did they bring?

7. The late pears were packed in boxes and sold at Christmas time at $1.10 per box. There were 432 boxes. Find the proceeds, deducting $77.50 for freight, etc.

8. When 16 lb. of salt cost $$\frac{1}{10}$, how many pounds are there in a bag that costs 1\frac{2}{3}$?

9. If a 224-pound bag of salt costs 1\frac{2}{3}$, how much does a 56-pound bag cost at the same rate?

10. A locomotive consumes 3000 gal. of water in a run of 100 mi. At this rate, how much water does it consume in going from Albany to Binghamton, a distance of 143 mi.?

11. If a mason's tender carries .7 cu. ft. of mortar per trip and makes 56 trips in a day, how much does he carry per day?

12. If 1 lb. of flour is required for 1.3 lb. of bread, how much bread will a barrel of flour make?

13. A train of 88 coal cars was ⅔ mi. long. How many cars would there have been, if the train had been ¼ mi. long?

14. If .75 lb. of black powder when exploded loosens 1 cu. yd. of rock, how many pounds are needed to loosen 124 cu. yd. of rock?

15. A hoisting engine consumed 40 lb. of coal per hour for 10 hr. a day. Find the cost of 1 day's supply of coal at $2.90 per ton.

16. If a teamster draws 4 loads of gravel per day, each load containing 1.5 cu. yd., how long will it take him to draw the 84 cu. yd. required for a certain piece of work?

17. If a machine makes 350 grape baskets in ⅝ hr., how many baskets does it make in 4¼ hr.?

18. It required 1,836,000 bricks to build an office building. Find the cost of the bricks at $9.25 per M.

19. A stove manufacturer bought 22½ long tons of pig iron at $16.80 per long ton. How much did it cost him? How much per pound did he pay?

20. In this country 5515 tons of old tin were recovered one year. If this was about ⅕ as much as the amount of tin imported, how many tons of tin were imported?

21. A factory is lighted by 50 incandescent electric lamps. If each lamp burns 720 hr. per year at a cost of ⅛⅜ ¢ per hour, how much does it cost per year to light the factory?

22. If it takes 3¾ bu. of seed to sow 4½ acres of land, how many bushels are needed to sow 28⅖ acres?

Find the cost of :

23. 22,000 bricks at $6.50 per M.

24. 18,000 strawberry plants at $2.50 per M.

25. 26,800 lb. of sugar at $4.25 per 100 lb.

26. 44,000 cu. ft. of gas at $.95 per 1000 cu. ft.

27. A carload of wheat, 1080 bu., at $97\frac{3}{8}$ ¢ per bushel.

28. How long does it take a baseball player who can run 100 yd. in 12 sec. to go from home plate to first base (90 ft.)?

29. The average "grounder" during the first hundred yards after it leaves the bat travels at the rate of 60 mi. per hour. How many feet per second does it travel?

30. An orchard yielded $427\frac{1}{2}$ bbl. of apples, averaging $31\frac{3}{8}$ bbl. to 10 trees. Find the number of trees in the orchard.

31. In the settlement of an estate worth $62,500, there were lawsuits that cost the estate $43,750. What part of the estate was left to divide among the heirs ?

32. The senior partner of a firm owned $\frac{4}{5}$ of the business, and sold $\frac{1}{3}$ of his share for $15,120. At that rate how much was the entire business worth?

33. What is the cost of ten pieces of cloth containing $42\frac{1}{2}$, $45\frac{3}{4}$, 46, 44, $47\frac{3}{4}$, 45, $48\frac{1}{2}$, $45\frac{3}{4}$, 47, and $43\frac{3}{4}$ yd., respectively, at $4\frac{3}{4}$ ¢ per yard?

34. In the menagerie Harold saw a test in which a horse pulled a weight of 1875 lb., or $\frac{3}{14}$ of the weight pulled by an elephant. A camel pulled $\frac{11}{15}$ as much as the horse. What was the pulling strength of each animal?

35. A certain group of flour mills turn out 30,000 bbl. of flour per day. If every barrel requires $4\frac{1}{4}$ bu. of wheat, how many carloads of wheat of 900 bu. each and how many carloads of flour of 225 bbl. each are handled in 21 days?

ANALYSIS BY EQUATIONS

1. How many pounds added to 2 lb. will give 4 lb.? This may be stated 2 lb. + ? = 4 lb., or 2 lb. + x = 4 lb.

2. The letter x is a symbol for the **unknown number** of pounds (to be found), the **known numbers** being 2 lb. and 4 lb.

3. The expression,
2 lb. + 2 lb. = 4 lb.,
or   2 lb. + x = 4 lb.,
is an **equation**, because the quantities on the two sides of the sign, = , are equal.

2 lb. + x is the **first member** and 4 lb., the **second member**.

An equation may be compared to a balance scale. The quantities on the two sides are equal, that is, they *balance* each other.

4. Finding the value of the unknown number, x, is **solving the equation.**

5. If there are 6 lb. in the right pan of a scale and 2 lb. in the left pan, how many pounds must be added to the left pan to make the scales balance?

How many pounds might be taken from the right pan to obtain the same result?

6. Find the value of x, if $2 + x = 6$; if $2 = 6 - x$.

The same number may be added to, or subtracted from, both members of an equation without destroying the equality.

7. In $x + 5 = 6$, if we *subtract* 5 from both sides we have,
$$x + 5 - 5 = 6 - 5,$$
or $$x = 1.$$

Compare this with $x = 6 - 5$, in which 5 is **transposed** to the second member with the sign preceding it changed from + to −.

8. In $x - 5 = 6$, if we *add* 5 to both members we have,

$$x - 5 + 5 = 6 + 5,$$
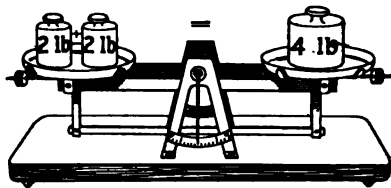
or $\qquad\qquad x = 11.$

Compare this with $x = 6 + 5$, in which 5 is transposed to the second member with the sign preceding it changed from $-$ to $+$.

A number may be transposed from one member of an equation to the other provided the sign $+$ preceding it is changed to $-$ or the sign $-$ to $+$.

9. Just as $2 + 2 + 2 = 3 \times 2$, so $x + x + x = 3 \times x$.

It is written $3x$, however, *without a multiplication sign between the figure and the letter.* What does $4x$ mean? $5x$? $6x$?

10. If $x = 4$, what is the value of $2x$? of $3x$? of $\frac{3}{4}x$?

11. If $4x = 12$, what is the value of $1x$, or of x?

12. If $\frac{1}{4}x = 10$, what is the value of 4 times $\frac{1}{4}x$, or of x?

What must be done to both members of each of the following equations to give an equation whose first member is x?

13. $\frac{1}{3}x = 2$ **14.** $\frac{1}{4}x = 5$ **15.** $3x = 15$ **16.** $5x = 25$

Both members of an equation may be multiplied or divided by the same number without destroying the equality.

Oral Exercises

Solve the following equations :

1. $x + 2 = 5$	**9.** $10 = 2 + x$	**17.** $2x = 8$	**25.** $\frac{1}{2}x = 4$
2. $x - 4 = 6$	**10.** $9 = x + 3$	**18.** $5x = 10$	**26.** $\frac{1}{4}x = 2$
3. $x + 7 = 9$	**11.** $12 = x - 3$	**19.** $6x = 18$	**27.** $\frac{1}{3}x = 3$
4. $x - 3 = 8$	**12.** $14 = x + 7$	**20.** $7x = 21$	**28.** $\frac{1}{5}x = 2$
5. $x + 6 = 9$	**13.** $11 = x - 5$	**21.** $3x = 12$	**29.** $\frac{1}{6}x = 5$
6. $x - 5 = 1$	**14.** $12 = x + 1$	**22.** $8x = 24$	**30.** $\frac{1}{4}x = 9$
7. $x - 4 = 8$	**15.** $13 = 5 + x$	**23.** $9x = 18$	**31.** $\frac{1}{7}x = 4$
8. $x + 3 = 9$	**16.** $15 = 3 + x$	**24.** $5x = 15$	**32.** $\frac{1}{8}x = 6$

Written Exercises

1. What number increased by 5 is equal to 30?

SOLUTION. — Let x = the number.
Then, $x + 5 = 30.$.
Subtract 5 from both members, $x = 25.$

2. If 3 times a number is 18, what is the number?

SOLUTION. — Let x = the number.
Then, $3x = 18.$
Divide both members by 3, $x = 6.$

3. If $\frac{3}{4}$ of a number is 15, what is the number?

SOLUTION. — Let x = the number.
Then, $\frac{3}{4} x = 15.$
Multiply both members by 4, $3x = 60.$
Divide both members by 3, $x = 20.$

4. One third of a number and 10 more equals 20. Find the number.

SOLUTION. — Let x = the number.
Then, $\frac{1}{3} x + 10 = 20.$
Transpose 10, $\frac{1}{3} x = 20 - 10 = 10.$
Multiply both members by 3, $x = 3 \times 10 = 30.$

5. What number increased by 12 equals 36?

6. What number decreased by 14 equals 70?

7. If 4 times a number is 84, what is the number?

8. If $\frac{1}{5}$ of a number is 60, what is the number?

9. What number multiplied by 3 is equal to 78?

10. What number divided by 8 is equal to 62?

11. One half of a number plus 11 equals 37. Find the number.

12. Twelve times a certain number is 15. Find the number.

13. If 20 is added to a certain number and 14 is subtracted from the sum, the result is 19. Find the number.

14. The sum of two numbers is 55 and the larger is 4 times the smaller. What are the numbers?

SOLUTION. — Let	$x =$ the smaller number.
Then,	$4x =$ the larger number,
and	$x + 4x =$ the sum of the numbers.
Therefore,	$x + 4x = 55,$
or	$5x = 55.$
Divide both members by 5,	$x = 11,$ the smaller number.
	$4x = 44,$ the larger number.

15. Separate 116 into two parts, one of which is 3 times the other.

16. The difference between two numbers is 32, and the greater is 3 times the smaller. What are the numbers?

SUGGESTION. $3x - x = 32$; that is, $2x = 32$.

17. The difference between two numbers is 12, and the greater is $\frac{3}{2}$ of the smaller. Find the numbers.

18. What number increased by twice itself equals 30?

19. What number decreased by $\frac{1}{2}$ of itself equals 20?

20. Separate 72 into two parts, one part being $\frac{1}{8}$ of the other.

21. Separate 78 into two parts, one part being $\frac{1}{5}$ of the other.

22. Separate 84 into two parts such that $\frac{1}{2}$ of the larger is equal to the smaller.

23. The sum of two numbers is 43. One number is 7 greater than the other. Find the two numbers.

24. The sum of two numbers is 28. One number is 4 less than the other. Find the two numbers.

25. The difference between two numbers is 36. If one number is $\frac{1}{4}$ of the other, what are the two numbers?

26. What is the number whose double, half, and third added to 33 will give 152?

27. John and Ruth together have $50, and John has 4 times as much as Ruth. How much money has each?

SOLUTION. — Let x = the number of dollars Ruth has,
and $4x$ = the number of dollars John has.
  Then, $x + 4x = 50.$
. $5x = 50.$
                              $x = 10$, the number of dollars Ruth has.
                            $4x = 40$, the number of dollars John has.

28. There are 10 more boys than girls in a school, and the total number of pupils is 100. How many are there of each sex?

SOLUTION. — Let x = the number of girls,
and $x + 10$ = the number of boys.
  Then, $x + x + 10 = 100.$
                            $2x = 90.$
                              $x = 45$, the number of girls.
                         $x + 10 = 55$, the number of boys.

29. Frank and George dug 160 clams. If Frank dug 8 times as many as George, how many clams did each dig?

30. The distance between two cities is 35 mi. by rail. This is $\frac{5}{6}$ of the distance by boat. Find the distance by boat.

31. Two boys bought a boat for $45. One furnished 4 times as much money as the other. How much money did each boy furnish?

32. A steam yacht was sold for $50,000, or $\frac{5}{6}$ of its cost. How much did it cost?

33. A fleet consisted of 525 boats. There were 215 more large boats than small ones. Find the number of small boats.

34. A wagon loaded with coal weighed 4200 lb. The coal weighed 1800 lb. more than the wagon. How much did the wagon weigh? the coal?

35. The width of a room is $\frac{3}{4}$ of its length, and the distance around the room is 70 ft. Find the length and the width.

PERCENTAGE

1. If Richard has 200 words to spell and he misses 5 words out of each *hundred*, how many words does he miss?

2. A certain school has 400 pupils. One stormy day 7 per hundred, or 7 *per cent*, of the pupils were absent. How many were absent?

3. If a man's yearly salary is $1000, and he pays $\frac{12}{100}$, or 12 per cent, of it for house rent, how much does he pay for house rent?

4. Helen's book contains 300 pages. She read .11, or 11 per cent, of it one evening. How many pages did she read that evening?

5. Find .06, or 6 per cent, of $500; 2 per cent of $700; 4 per cent of $600; 8 per cent of $900.

The term **per cent** means per *hundred*, or **hundredths**. The sign for it is %.

5 per cent, 5%, $\frac{5}{100}$, and .05 *all represent the same thing*, namely, 5 *hundredths*.

The general name given to that part of arithmetic that treats of per cents is **percentage**.

The subject of percentage involves no new principle. It is merely an application of fractions and decimals, for per cents are really fractions or decimals whose denominators are 100. In computations it is necessary to express per cents in the fractional or decimal form. The student, then, should be able to use these various forms interchangeably.

Written Exercises

Express as a decimal :

1. 24 %	**3.** 45 %	**5.** 3 %	**7.** 10 %
2. 15 %	**4.** 36 %	**6.** 8 %	**8.** 30 %

Express as a common fraction :

9. 1 %	**12.** 11 %	**15.** 27 %	**18.** 63 %
10. 7 %	**13.** 19 %	**16.** 83 %	**19.** 81 %
11. 9 %	**14.** 17 %	**17.** 51 %	**20.** 77 %

Express as a per cent with the sign % :

21. $\frac{9}{100}$	**24.** .48	**27.** .08	**30.** .60
22. $\frac{13}{100}$	**25.** .75	**28.** .20	**31.** .09
23. $\frac{41}{100}$	**26.** .55	**29.** .02	**32.** .90

Express decimally :

33. 100 % ; 125 % ; 2.3 % ; $33\frac{1}{3}$ % ; $45\frac{3}{4}$ % ; $\frac{1}{2}$ %.

SOLUTIONS

$100\% = 1.00$ $2.3\% = .023$ $45\frac{3}{4}\% = .45\frac{3}{4}$, or .4575
$125\% = 1.25$ $33\frac{1}{3}\% = .33\frac{1}{3}$ $\frac{1}{2}\% = .00\frac{1}{2}$, or .005
It is evident that $\frac{1}{2}$ % means $\frac{1}{2}$ of 1%.   What does $\frac{1}{4}$ % mean ?

34. 6 %	**39.** $6\frac{1}{4}$ %	**44.** 200 %	**49.** $\frac{1}{4}$ %
35. 9 %	**40.** $3\frac{1}{3}$ %	**45.** $12\frac{1}{2}$ %	**50.** .3 %
36. 3.5 %	**41.** 17.4 %	**46.** 175 %	**51.** $\frac{4}{5}$ %
37. 2.2 %	**42.** $15\frac{1}{4}$ %	**47.** $83\frac{1}{3}$ %	**52.** $\frac{5}{8}$ %
38. $8\frac{1}{2}$ %	**43.** $16\frac{3}{8}$ %	**48.** $233\frac{1}{3}$ %	**53.** $\frac{5}{4}$ %

54. Express as a mixed number or as a common fraction in its lowest terms: 125 % ; $12\frac{1}{2}$ % ; 4 % ; $\frac{1}{8}$ %.

SOLUTIONS

$125\% = \frac{125}{100} = 1\frac{1}{4}$ $4\% = \frac{4}{100} = \frac{1}{25}$
$12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{1}{8}$ $\frac{1}{8}\% = \frac{\frac{1}{8}}{100} = \frac{1}{800}$

Express as a common fraction in its lowest terms or as a mixed number:

55. 20 %   **60.** 33⅓ %   **65.** 4 %   **70.** 16⅔ %

56. 40 %   **61.** 37½ %   **66.** 5 %   **71.** 83⅓ %

57. 50 %   **62.** 87½ %   **67.** 8 %   **72.** 133⅓ %

58. 60 %   **63.** 62½ %   **68.** 6¼ %   **73.** 137½ %

59. 75 %   **64.** 66⅔ %   **69.** 8⅓ %   **74.** 266⅔ %

75. Express as a per cent with the sign % : .04½ ; .4½ ; .875 ; 2.75 ; .359 ; .1763.

<div align="center">SOLUTIONS</div>

$.04\frac{1}{2} = 4\frac{1}{2}\%$ $.875 = 87.5\%$, or $87\frac{1}{2}\%$ $.359 = 35.9\%$
$.4\frac{1}{2} = .45 = 45\%$ $2.75 = 2\frac{11}{15} = 275\%$ $.1763 = 17.63\%$

Express as a per cent with the sign % :

76. .90   **81.** .125   **86.** 3   **91.** .00½

77. .8   **82.** 3.75   **87.** 4.5   **92.** .008

78. .03   **83.** .025   **88.** .257   **93.** .4525

79. .3½   **84.** 1.00   **89.** .421   **94.** .1634

80. .03½   **85.** 2.50·   **90.** .043   **95.** .0279

96. Express as a per cent with the sign % : ¾ · ₁/₂₅ ; ⅔ , 1⅕.

<div align="center">SOLUTIONS</div>

$\frac{3}{4} = .75 = 75\%$ $\frac{2}{3} = .66\frac{2}{3} = 66\frac{2}{3}\%$
$\frac{1}{25} = .04 = 4\%$ $1\frac{1}{5} = 1.20 = 120\%$

Find the per cent equivalent of :

97. ½   **101.** 8/20   **105.** ⅛   **109.** ⅛   **113.** 1¾

98. ¼   **102.** 7/10   **106.** ⅞   **110.** ⅔   **114.** 1⅛

99. ⅘   **103.** 3/50   **107.** ⅜   **111.** ¼   **115.** 2⅖

100. 8/10   **104.** 2/25   **108.** ⅝   **112.** ⅚   **116.** 3⅝

When the *per cent* has an easy *fractional equivalent*, work in percentage can be shortened by using the fraction.

This *table of equivalents* should be *memorized*, and applied whenever possible :

$50\% = \frac{1}{2}$	$5\% = \frac{1}{20}$
$25\% = \frac{1}{4}$	$12\frac{1}{2}\% = \frac{1}{8}$
$75\% = \frac{3}{4}$	$37\frac{1}{2}\% = \frac{3}{8}$
$20\% = \frac{1}{5}$	$62\frac{1}{2}\% = \frac{5}{8}$
$40\% = \frac{2}{5}$	$87\frac{1}{2}\% = \frac{7}{8}$
$60\% = \frac{3}{5}$	$6\frac{1}{4}\% = \frac{1}{16}$
$80\% = \frac{4}{5}$	$33\frac{1}{3}\% = \frac{1}{3}$
$10\% = \frac{1}{10}$	$66\frac{2}{3}\% = \frac{2}{3}$
$30\% = \frac{3}{10}$	$16\frac{2}{3}\% = \frac{1}{6}$
$70\% = \frac{7}{10}$	$83\frac{1}{3}\% = \frac{5}{6}$
$90\% = \frac{9}{10}$	$8\frac{1}{3}\% = \frac{1}{12}$

1. What is $\frac{1}{4}$ of 24 ? .25 of 24 ? 25 % of 24 ?

2. What part of 24 is 6 ? How many hundredths of 24 is 6 ? What per cent of 24 is 6 ?

3. 6 is $\frac{1}{4}$ of what number? 6 is .25 of what number? 6 is 25 % of what number?

The three chief types of problems in percentage, illustrated above with the corresponding types in fractions (pages 110–114), are :

1. *Finding a per cent of a number.*

2. *Finding what per cent one number is of another.*

3. *Finding a number when a per cent of it is given.*

The number of which some per cent is found is the **base.**

The *number* of hundredths found is the **rate,** or the **rate per cent.**

The result obtained by finding a per cent of the base is the **percentage.**

Thus, in 25% of 24 = 6, 24 is the *base*, 25 % the *rate*, and 6 the *percentage*.

Finding a per cent of a number.

1. Find $\frac{1}{3}$ of 12; $\frac{3}{4}$ of 16; $\frac{2}{5}$ of 20; $\frac{5}{6}$ of 24; $\frac{7}{8}$ of 32.
How do you find a fractional part of a number?

2. How many are .3 of 20? .12 of 50? .05 of 400?
How do you find a decimal part of a number?

3. Find $\frac{1}{100}$ of 300; 1% of 300; $\frac{7}{100}$ of 500; 7% of 500;
.04 of 200; 4% of 200; .08 of 600; 8% of 600; 12% of 300.

How do you find a per cent of a number?

4. Find 3% of 700; 6% of 400; 50%, or $\frac{1}{2}$, of 20; 25%, or
$\frac{1}{4}$, of 36; 12$\frac{1}{2}$%, or $\frac{1}{8}$, of 40; 20% of 30; 33$\frac{1}{8}$% of 18.

Oral Exercises

Find:

1. 2% of 100	**4.** 3% of 200	**7.** 10% of 800
2. 4% of 300	**5.** 7% of 400	**8.** 11% of 600
3. 6% of 600	**6.** 8% of 700	**9.** 12% of 900

In the following, find the per cents indicated by using the
fractional equivalents given on page 176:

10% of	30% of	20% of	40% of	60% of	50% of
10. 20	**14.** 30	**18.** 25	**22.** 15	**26.** 25	**30.** 14
11. 40	**15.** 50	**19.** 40	**23.** 20	**27.** 30	**31.** 18
12. 80	**16.** 60	**20.** 55	**24.** 30	**28.** 35	**32.** 22
13. 90	**17.** 70	**21.** 75	**25.** 45	**29.** 50	**33.** 30

25% of	75% of	12$\frac{1}{2}$% of	37$\frac{1}{2}$% of	62$\frac{1}{2}$% of	87$\frac{1}{2}$% of
34. 20	**38.** 12	**42.** 32	**46.** 16	**50.** 24	**54.** 32
35. 28	**39.** 16	**43.** 56	**47.** 24	**51.** 48	**55.** 40
36. 32	**40.** 36	**44.** 64	**48.** 40	**52.** 56	**56.** 72
37. 44	**41.** 48	**45.** 96	**49.** 64	**53.** 80	**57** 88

Find the per cents indicated :

5 % of	33⅓ % of	66⅔ % of	16⅔ % of	8⅓ % of	6¼ % of
58. 20	**62.** 21	**66.** 18	**70.** 42	**74.** 24	**78.** 16
59. 60	**63.** 30	**67.** 24	**71.** 54	**75.** 60	**79.** 32
60. 40	**64.** 36	**68.** 27	**72.** 60	**76.** 84	**80.** 64
61. 80	**65.** 45	**69.** 33	**73.** 72	**77.** 96	**81.** 48

Find in the shortest way :

82. 4 % of 600　　**86.** 10 % of 500　　**90.** 15 % of 200

83. 5 % of 400　　**87.** 11 % of 700　　**91.** 12 % of 800

84. 6 % of 700　　**88.** 20 % of 400　　**92.** 16⅔ % of 480

85. 8 % of 900　　**89.** 25 % of 440　　**93.** 33⅓ % of 360

94. How many cows are 70 % of 200 cows ?

95. How many yards are 50 % of 800 yd. ?

96. How many miles are 12 % of 400 mi. ?

97. How many books are 20 % of 120 books ?

98. How many gallons are 25 % of 200 gal. ?

99. How many bushels are 12½ % of 240 bu. ?

100. What is 100 % of 25 ? of 32 ? of 46 ? of any number ? Find 200 % of 25 ; 300 % of 16 ; 500 % of 12.

101. There are 200 children in a school, and 50 % of them are girls. How many girls are there in the school ?

102. In an orchard of 400 trees 3 % were plum trees. How many plum trees were there ?

103. In a football game one team scored 18 points and the other 33⅓ % of that number. What was the score ?

104. Mr. Livingston had $100 and paid 22 % of it for an overcoat. How much did the overcoat cost ?

105. There are 800 books in a library, 12½ % of which are historical. How many historical books are there in the library ?

1. Find 24 % of $47.25.

$47.25
  .24
——————
18900
9450
——————
11.34\emptyset\emptyset$

Since 24% of a number is .24 of it, 24% of $47.25 is .24 of $47.25.
Multiply $47.25 by .24; the result is $11.34.
Hence, 24% of $47.25 = $11.34.

The percentage equals the base multiplied by the rate.

Find :

2. 5 % of $85
3. 7 % of $64
4. 8 % of $73

5. 12 % of 150
6. 15 % of 280
7. 22 % of 350

8. 24 % of $43.50
9. 32 % of $75.25
10. 45 % of $96.80

11. Find 12½ % of $275.20 ; ⅛ % of $275.20.

8)$275.20
$ 34.40

8)$2.7520
$ .344$\emptyset$

Since 12½% of anything = ⅛ of it, the shortest way to find 12½% of $275.20 is to find ⅛ of $275.20, which is $34.40.

Since ⅛% means ⅛ of 1%, we first find 1% of $275.20 by moving the decimal point two places to the left; then find ⅛ of $2.7520, which is $.344.

Find :

12. 50 % of 78
13. 20 % of 85
14. 75 % of 96

15. 16⅔ % of 720
16. 66⅔ % of 474
17. 125 % of 352

18. ½ % of $1410
19. ¾ % of $2560
20. ⅞ % of $3720

Find in the shortest way :

21. 40 % of 65
22. 18 % of 50
23. 25 % of 92
24. 36 % of 75

25. 6 % of $348
26. 9 % of $465
27. ¼ % of $660
28. ⅝ % of $784

29. 33⅓ % of $22.50
30. 62½ % of $31.60
31. 150 % of $42.40
32. 106 % of $36.50

33. How many ounces is 87½ % of a pound?

34. A certain ore yielded 66⅔ % of iron. How many tons of iron were obtained from 48 tons of the ore?

35. Of a bushel of clover seed weighing 60 lb., only 93 % was good seed. How many pounds of good seed were there?

36. How much did a grocer pay for a tub of butter that sold for $ 15, if it cost 88 % of the sum for which it sold?

37. A diver's suit weighs 125 lb. How much does the helmet weigh, if it is 64 % of the entire weight?

38. The wheat necessary to make a barrel of flour that sold at $ 4.75 was purchased for 68 % of the value of the flour. How much was paid for the wheat?

39. Of the 1380 persons on board an Atlantic steamship, the crew was 40 %. How many were in the crew?

40. If a man's salary is $ 1800 per year, and his expenses are 62½ % of that sum, what are his yearly expenses?

41. How many bushels of wheat are there in a schooner's cargo of 27,600 bu. of grain, if 46¾ % of it is wheat?

42. A man willed 37½ % of his fortune of $ 240,000 to a hospital. How much did the hospital receive?

43. One year the value of the imports of the United States was nearly 1760 million dollars. Find, in millions, the approximate value of each import that is represented in the following graphs as a per cent of the total value.
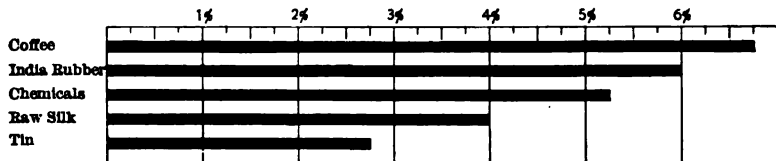
44. The population of a certain town three years ago was 2500. If it has increased 20 %, what is the present population?

SUGGESTION. — The present population = 100 % + 20 %, or 120 %, of 2500.

45. A canning factory using 6600 bu. of tomatoes a season increased its capacity 16⅔ %. How many bushels did it then use?

46. Formerly there were 400 fishermen in a certain town. If the number has decreased 5 %, how many fishermen are there now?

SUGGESTION. — The number there now = 100 % — 5 %, or 95 %, of 400.

47. Chickens lose 20 % of their weight in being dressed for market. A butcher bought 40 lb. of live chickens. How much did they weigh when dressed?

48. One week Frank made $3.20 by selling papers. How much did he make the next week, if his earnings decreased 25 %?

49. A farmer took 125 tons of sugar beets to the factory where the dirt, called *tare*, was removed. If 8 % of the weight was tare, how many tons of clean beets had he?

50. A mason's helper working for $1.75 per day had his pay increased 14⅔ %. How much did he then receive per day?

51. A man, who had a pile of wood containing 234 cords, sold 16⅔ % of it. How many cords did he have left?

52. Mr. Charles invested $6540 in business, and a year later increased his capital 16⅔ %. How much was his capital then?

53. In a certain village there are 1528 people, all but 12½ % of whom are native born. How many are native born?

54. Two years ago a lawyer's income was $4868, but last year it increased 12½ %. What was his income last year?

55. The bark from a cork tree, 48 lb. in all, was reduced 33⅓ % in weight by boiling. Its weight after being boiled was reduced 25 % by scraping. Find its final weight.

Finding what per cent one number is of another.

1. What part of 8 is 4 ?　of 8 is 6 ?　of 9 is 3 ?　of 9 is 6 ? How do you find what part one number is of another ?

2. What part of 100 is 31 ?　What per cent of 100 is 31 ? What per cent of 100 is 5 ?　20 ?　25 ?　50 ?　75 ?　$33\frac{1}{3}$?

3. What part of 10 is 5 ?　How many *hundredths* of 10 is 5 ?　What per cent of 10 is 5 ?　What per cent of 12 is 3 ?

4. What per cent of a number is $\frac{1}{4}$ of it ?　What per cent of 9 is 3 ?　6 ?

Observe that the per cent one number is of another is simply their fractional relation expressed as *hundredths.*

<center>Oral Exercises</center>

What per cent of

1. 6 is 3 ?	**6.** 12 is 3 ?	**11.** 10 is 6 ?	**16.** 12 is 10 ?
2. 8 is 2 ?	**7.** 20 is 4 ?	**12.** 12 is 8 ?	**17.** 20 is 12 ?
3. 6 is 2 ?	**8.** 12 is 9 ?	**13.** 10 is 8 ?	**18.** 24 is 15 ?
4. 8 is 8 ?	**9.** 16 is 2 ?	**14.** 24 is 9 ?	**19.** 44 is 22 ?
5. 6 is 4 ?	**10.** 24 is 4 ?	**15.** 20 is 6 ?	**20.** 36 is 24 ?

21. A quart is what per cent of 1 gal. ?

22. What per cent of a pound are 12 oz. ?

23. What per cent of an hour are 20 min. ?

24. Anna read 10 pages of a book containing 50 pages. What per cent of the book did she read ?

25. Mr. Carter had $ 24 in his pocketbook and spent $ 6 for a ton of coal.　What per cent of his money did he spend ?

26. Mrs. Hicks used 25 yd. of thread from a 200-yard spool. What per cent of the spool of thread did she use ?

27. From an Alabama farm worked on shares 36 bales of cotton were secured, of which 12 went to the owner of the land. What per cent of the cotton did the landowner receive ?

1. What per cent of 125 is 6.25 ?

$$\begin{array}{r} .05 \\ 125\overline{)6.25} \\ \underline{6\ 25} \end{array}$$

Since 6.25 is $\frac{6.25}{125}$ of 125 and $\frac{6.25}{125} = .05$, 6.25 is 5 *hundredths*, or 5% of 125.

The rate equals the percentage divided by the base.

2. What per cent of $8.64 is $3.24 ?

SOLUTION. $3.24 ÷ $8.64 = .375, or .37½ ; that is, $3.24 is 37½% of $8.64.

What per cent of the first number is the second ?

3. 50, 2	**7.** 72, 36	**11.** 160, 60	**15.** $750, $150
4. 60, 3	**8.** 70, 42	**12.** 180, 45	**16.** $290, $5.80
5. 50, 3	**9.** 96, 64	**13.** 210, 35	**17.** $120, $7.50
6. 75, 9	**10.** 90, 75	**14.** 225, 75	**18.** $105, $8.75

What per cent of

19. 900 ft. are 450 ft. ?   **22.** 888 mi. are 148 mi. ?

20. 640 in. are 160 in. ?   **23.** 760 pk. are 570 pk. ?

21. 870 rd. are 580 rd. ?   **24.** 944 hr. are 354 hr. ?

25. A man who owned a farm of 330 acres rented 220 acres of it. What per cent of the farm did he rent ?

26. Of 450 crates of oranges from Florida, 9 crates were spoiled on the way. What per cent of the fruit was spoiled ?

27. What per cent of the work of painting a house did a painter do in 6 days, if he did all the work in 25 days ?

28. What per cent of the 800 paper mills of this country does New York contain, if it contains 180 paper mills ?

29. If the income from an acre of strawberries was $495 and the cost of production was $198, what per cent of the income was the cost ?

30. In a certain school there are 750 pupils, of whom 270 are in the high school department.   What per cent of the pupils of the school are in the high school ?

31. Of the 7500 new books printed in America one year, 475 were for children.   What per cent were children's books ?

32. A man who owed $20,000 could pay only $4900.   What per cent of his debts could he pay ?

33. A 20-dollar gold coin was worn so that it had lost 10¢ in value.   What per cent of its weight had been worn away ?

34. In crossing the Atlantic a vessel used 8570 tons of the 4760 tons of coal on board.   What per cent of the supply was used ?

35. One year a National League baseball team won 103 games of the 151 games played.   What per cent of the games did the team win ?

Give baseball per cents, or " averages," to the nearest tenth of 1%.

36. A player who led his team in batting came to the bat 490 times during a season and made 171 safe hits.   What was his batting average ?

This diagram, or graph, 4 in. long, represents the value of all the boots and shoes made in the United States in a certain year.

Mass.   N.Y.   N.H.   O.   Pa. Me. Ill. Other States

Find, from the graph, the per cent of the total value each state produced, the length of its part being:

37. Ohio,   .27 in.   **41.** Pennsylvania,   .21 in.

38. Maine,   .19 in.   **42.** Massachusetts,   1.79 in.

39. New York, .39 in.   **43.** New Hampshire,   .37 in.

40. Illinois,   .16 in.   **44.** Other states,   .62 in.

45. A grocer mixed 12 lb. of Java coffee with 36 lb. of Mocha. What per cent of the mixture was Java ?

SUGGESTION. — What per cent of 12 + 36 is 12?

46. One season a basket-ball team won 20 games and lost 12. What per cent of the games played were won ?

47. Mr. Joy has traveled 120 mi. of a 300-mile journey. What per cent of his journey has he still to go ?

SUGGESTION. — What per cent of 300 is (300–120) ?

48. An electric railway has 24 mi. of road. If all but 9 mi. of it are within city limits, what per cent of it is in the city?

49. During a certain month 210 girls were employed in a factory, and the next month 280. Find the per cent of increase.

SUGGESTION. — The increase = 70 girls. What per cent of 210 is 70?

50. A man burned 3000 ft. of gas in his house one month and 2400 ft. the next. What was the per cent of decrease ?

51. Flour that sold for $4.85 a barrel was advanced in price to $5.82. Find the per cent of advance.

52. A 15-pound ham weighed but 11 lb. when roasted. Find the per cent of decrease in weight.

53. The length of a dry dock was increased from 600 ft. to 650 ft. Find the per cent of increase.

54. A farmer raised 250 bu. of oats, and sold all but 65 bu. What per cent of his crop did he sell ?

55. A ton of squash stored in a dry room weighed only 1488 lb. after four months. Find the per cent of decrease in weight.

56. Mr. Lang bought a lot for $837 and built a house on it at a cost of $4743. What per cent of his total investment was the cost of the house ?

Finding a number when a per cent of it is given.

1. If $\frac{3}{4}$ of a number is 9, what is the number?

2. If $\frac{7}{100}$ of a number is 21, what is $\frac{1}{100}$ of it? $\frac{100}{100}$ of it? What is the number?

3. If 7 % of a number is 21, what is 1 % of it? 100 % of it? What is the number?

4. If 12 % of a number is 48, what is 1 % of it? What is the number? How does this result compare with $48 \div .12$?

5. If $37\frac{1}{2}$ %, or $\frac{3}{8}$, of a number is 18, what is the number?

Oral Exercises

Find the number of which

1. 3 is 1 %	**6.** 80 is 10 %	**11.** 12 is $12\frac{1}{2}$ %
2. 6 is 3 %	**7.** 25 is 50 %	**12.** 20 is $33\frac{1}{3}$ %
3. 8 is 2 %	**8.** 24 is 40 %	**13.** 35 is $62\frac{1}{2}$ %
4. 20 is 5 %	**9.** 72 is 12 %	**14.** 50 is $66\frac{2}{3}$ %
5. 90 is 9 %	**10.** 36 is 75 %	**15.** 60 is $83\frac{1}{3}$ %

16. If 50 % of a man's working day is 4 hr., how many hours per day does he work?

17. A baseball team lost 8 games, or 25 % of the total number of games played. How many games were played?

18. If a fast freight goes 20 % of its journey from New Orleans to Louisville in 10 hr., how long does the trip take?

19. If it takes 25 % of a man's salary to pay his rent, which is $15 a month, what is his monthly salary?

20. How many acres are there in a farm of which $66\frac{2}{3}$ %, or 40 acres, are tillable?

21. A fruit dealer sold $37\frac{1}{2}$ % of a box of grapefruits. How many grapefruits did the box contain at first, if he sold 24?

1. Find the number of which 84 is 24 % ; 126 is 87½ %.

SOLUTIONS

a

24 % = .24 = ₁₂₄₀₀.
₁₂₄₀₀ of the number = 84.
₁₀₀ of the number = 84 ÷ 24 = 3.5.
The number = 100 × 3.5 = 350.
Or, the number = 84 ÷ .24 = 350.

a

87½ % of a number = ⅞ of it.
⅞ of the number = 126.
⅛ of the number = 126 ÷ 7 = 18.
The number = 8 × 18 = 144.
Or, the number = 126 ÷ ⅞ = 144.

●*b*

Let x = the number.
Then, .24 x = 84.
Divide by .24, x = 84 ÷ .24 = 350.
Hence, the number is 350.

●*b*

Let x = the number.
Then, ⅞ x = 126.
Divide by ⅞, x = 126 ÷ ⅞ = 144.
Hence, the number is 144.

The base equals the percentage divided by the rate.

Find the number of which

2.	30 is 4 %	**7**	60 is 16 %	**12.**	$140 is 12¼ %
3.	22 is 5 %	**8.**	52 is 25 %	**13.**	$220 is 16⅔ %
4.	50 is 8 %	**9.**	74 is 40 %	**14.**	$3.50 is 66⅔ %
5.	75 is 6 %	**10.**	720 is 32 %	**15.**	$6.25 is 125 %
6.	98 is 7 %	**11.**	960 is 75%	**16.**	$7.35 is 105 %

17. What is the total weight of a bale of Florida sponges, if the covering weighs 2 lb., or 4 % of the total weight ?

18. In a certain town 240 people, or 30 % of the inhabitants, are engaged in manufacturing. What is the population ?

19. If the 400 books on travel in a library are 5 % of all the books, how many volumes does the library contain ?

20. If $275 will pay 55 % of a debt, how much is the debt ?

21. A farmer sold 315 bu. of wheat, which was 35 % of his crop. How many bushels of wheat did he raise ?

* Students who have covered the work of pages 168–172 may prefer to do the exercises of this case by the models shown in *b*.

22. How much steel was used in constructing a building, if 12 % of it, or 1200 tons, was used in the dome?

23. What is the combined area of the parks in a certain city, if the largest contains 319 acres, or 58 % of the whole area?

24. A manufacturer received 500 lb. of yarn, which was 8 % of his order. How much yarn had he ordered?

25. In drying, some wheat decreased in weight 2 %, or 775 lb. Find its weight before drying.

26. In six hours 6000 tons of coal were loaded on a vessel. If this was 80 % of her load, how many tons did she carry?

27. How many tons of sugar cane are required to produce 8510 lb. of sugar, if 9 % of the cane is converted into sugar?

28. One year the income of a certain publication from advertisements alone was $5463, or 37½ % of its total income. Find the total income.

29. A man spent $1239 one year. If this was 84 % of his salary, how much was his salary?

30. How many feet of barbed wire are there on a spool, if 12 ft. is ⅘ % of it?

31. If coal yields 65.9 % ot its weight in coke, how many pounds of coal will yield 5931 lb. of coke?

32. When 6875 tons of ice had been put into an ice house on Lake Erie, 62½ % of its capacity was occupied. Find the capacity of the ice house in tons.

33. If 66⅔ %, or 8640 lb., of a crop of seed cotton was seed, how many pounds of seed cotton were raised?

34. Some corn in the ear lost 28.5 %, or 2422½ lb., in weight while it was stored in a corn crib through the winter. Find its weight when put into the crib.

35. What number increased by 32 % of itself equals 165 ?

Suggestion.   165 is 100 % + 32 %, or 132 %, of what number? .

36. What number increased by 75 %, or ¾, of itself equals 672 ?

37. A clerk's salary was increased 18 %. He then received $1652 a year. What was his salary before the increase ?

Suggestion. — After the increase the clerk's salary was 118 % of what it was before, that is, $1652 is 118 % of his salary before the increase.

38. The cost of living for a family one year was $442, or 4 % more than for the year before. Find the cost for the year before.

39. What number decreased by 15 % of itself equals 255 ?

Suggestion.   255 is 100 % − 15 %, or 85 %, of what number?

40. What number decreased by 16⅔ %, or ⅙, of itself equals 485 ?

41. Some flour was damaged 25 % in transportation. If 600 bbl. arrived in good condition, how many were shipped ?

Suggestion.   600 is 100 % − 25 %, or 75%, or ¾, of what number?

42. A man who had property worth $15,600 found that this was 60 % less than he owed. How much did he owe ?

43. A typist can transcribe her notes at the rate of 50 words per minute, which is 25 % faster than she could do it six months ago. What was her rate then ?

44. If cloth shrinks 5½ % of its length in being sponged, what was the original length of a sponged piece 37.8 yd. long ?

45. Coal sold at $5.50 per ton one winter, or 8⅓ % less than the price the winter before. Find the price the winter before.

46. I paid $1250 for my automobile, or 60 % more than for my horse and carriage. How much did my horse and carriage cost ?

1. Given the base 245, and the rate 8%; find the percentage.

2. If the percentage is $5.50 and the base $22, find the rate.

3. Find the base, if the percentage is $75 and the rate 4%.

Find the value of x in each exercise in the table :

Base	Rate	Percentage	Base	Rate	Percentage
4. $45	6%	x	**12.** 54 mi.	x	18 mi.
5. 150	x	12	**13.** 260	105%	x
6. x	25%	34	**14.** x	47%	23½
7. 75 qt.	12%	x	**15.** $\frac{1}{16}$	x	$\frac{1}{16}$
8. .25	x	.04	**16.** x	½%	$3.15
9. x	12½%	$1.60	**17.** $28.80	37½%	x
10. 680	35%	x	**18.** 6 gal.	x	4 qt.
11. 88 ft.	x	110 ft.	**19.** x	6¼%	45 lb.

20. If 18.8% of the weight of potatoes is starch, how many pounds of starch are there in 60 lb. of potatoes?

21. A sixth grade class in history had 60 lessons during the year, 10 of them about Washington and 9 about Lincoln. What per cent of the time was devoted to each?

22. How many sweet potato plants does it take to yield a barrel of sweet potatoes, if 25 plants yield 20% of a barrel?

23. Mr. Thompson's flock of 240 sheep was decreased 12½% by an epidemic. How many sheep had he after the epidemic?

24. A part of the St. Lawrence River was dredged from a depth of 10 ft. to a depth of 30 ft. What per cent was the depth increased?

25. Walter lost 5% in weight one year and then weighed 114 lb. How much did he weigh at the beginning of the year?

Find the per cent of a 175-acre farm devoted to each crop, there being :

26. 14 acres of rye   **29.** 10.5 acres of corn

27. 42 acres of hay   **30.** 5.25 acres of barley

28. 28 acres of oats   **31.** 8.75 acres of potatoes

32. What is the width of a street, if 60 % of the width, or 48 ft., lies between the curbstones ?

33. A sample of coal was found to be 73.19 % pure carbon. How much carbon is there in a short ton of such coal?

34. The total yield of some date palms was 128 bunches of dates. What was the average yield per tree, if it was $6\frac{1}{4}$ % of the total yield ?

35. An orange tree yielded 50 % more oranges the second time it bore than the first time. If the second crop amounted to 420 oranges, how many oranges were there in the first crop ?

36. If 15 lb. of cream contain $3\frac{3}{4}$ lb. of butter fat, what is the per cent of butter fat in the cream ?

37. If a codfish trawl line contains 750 hooks, and average catch fills 80 % of them, how many fish are taken average each time ?

38. Mr. Day paid $208 for a horse and $112 for a harness. What per cent of the total was the cost of

39. It cost Mr. James $196.50 to raise and cherries. If he ·paid 30 % of this for picking the did the picking cost ?

40. If 28 shirts is $\frac{2}{5}$ % of the weekly output of a how many shirts are made there each week ?

41. A vessel that has 3255 mi. to go covers 1· tance in one day. Find the distance traveled in

42. Mr. Ford had 76 % of his money invested the rest, $7680, in the bank. How much mone

Loss
7 %
8 %

Loss
$10
$30
$60

Rate or Loss
$16\frac{2}{3}$ %
$12\frac{1}{2}$ %
$37\frac{1}{2}$ %

43. A steer weighing 1350 lb. yielded 783 lb. of dressed beef. What per cent of his weight was beef?

44. How many bushels of wheat are necessary to make 225 bbl. of flour, if wheat yields 75 % of its weight in flour?

45. During transportation a carload of lambs weighing 25,000 lb. lost $7\frac{1}{2}$ % in weight. Find the loss in weight.

46. If 22.9 % of the cost of operating a mine was for labor costing $6434.90, what was the cost of operating the mine?

47. The output of a starch factory is 6.48 tons of starch per day. How many bushels of potatoes are used, if they yield 18 % of their weight in starch?

48. Lake Erie has an area of 9960 sq. mi. If it were larger by 24 sq. mi., its area would be 32 % of that of Lake Superior. How large is Lake Superior?

49. An automobile truck with its load weighed 18,700 lb. the load was 64 % of the total weight, how many pounds did
² load weigh?
pouna.,

21. A gas meter that ran 2.5 % too fast registered the pas-
year, 10 ^,600 cu. ft. of gas in a year. What was the actual
What per c ^ gas that passed through the meter?

22. How cre of land produced 18,756 lb. of sugar beets.
barrel of swe ounds of sugar did these beets contain, if a test

23. Mr. Tl to be $16\frac{2}{3}$ % sugar?
by an epidemi opulation of a city decreased $24\frac{1}{2}$ % one year, and

24. A part ³0, what was the population before the decrease?
depth of 10 ft manufacture of 432 lb. of plate glass, 151.2 lb. of
depth increase b. of limestone, 43.2 lb. of soda, 36.72 lb. of

25. Walter .nd 54 lb. of other materials were used. What
114 lb.   How nwhole was each substance?

PROFIT AND LOSS

1. A watch that cost $80 was sold at a *profit* of 25 % of the cost. What was the gain? the selling price?

2. A bicycle that cost $48 was sold at a *loss* of 12½ % of the cost. What was the loss? the selling price?

3. An article that cost $5 was sold for $5¼. How much was gained? What per cent of the cost was gain?

4. If a man gains (or loses) $30 on some goods and his gain (or loss) is 33⅓ % of the cost, what is the cost?

Problems in *profit and loss* are *percentage* problems in which :

The *base* is the cost or sum invested.

The *rate* is the per cent of gain or loss.

The *percentage* is the profit or loss.

Oral Exercises

Find the gain:

	Cost	Rate of Gain
1.	$30	10 %
2.	$60	50 %
3.	$40	20 %

Find the loss:

	Cost	Rate of Loss
4.	$200	5 %
5.	$500	7 %
6.	$900	8 %

Find the rate of gain:

	Cost	Gain
7.	$12	$3
8.	$24	$8
9.	$54	$9

Find the rate of loss:

	Cost	Loss
10.	$80	$10
11.	$60	$30
12.	$90	$60

Find the cost :

	Gain	Rate of Gain
13.	$20	40 %
14.	$27	30 %
15.	$50	8⅓ %

Find the cost :

	Loss	Rate of Loss
16.	$70	16⅔ %
17.	$80	12½ %
18.	$30	37½ %

Written Exercises

Find the gain or loss and the selling price :

	Cost	Rate of Gain			Cost	Rate of Loss
1.	$150	10%		8.	$280	5%
2.	$340	15%		9.	$465	4%
3.	$235	20%		10.	$808	7%
4.	$376	25%		11.	$425	3%
5.	$430	60%		12.	$550	8%
6.	$525	40%		13.	$682	6¼%
7.	$624	50%		14.	$576	8⅓%

Find the rate of gain or loss :

	Cost	Selling Price			Cost	Selling Price
15.	$1.20	$1.35		22.	$4.02	$3.35
16.	$2.80	$3.50		23.	$5.70	$3.99
17.	$3.15	$4.20		24.	$6.75	$4.05
18.	$7.50	$8.85		25.	$8.72	$5.45
19.	$5.50	$7.70		26.	$7.60	$5.70
20.	$6.20	$8.06		27.	$8.80	$7.70
21.	$6.40	$9.60		28.	$9.50	$7.60

Find the cost and the gain or loss :

	Selling Price	Rate of Gain			Selling Price	Rate of Loss
29.	$2.70	12½%		36.	$2.10	12½%

HINT. $2.70 is 112½% of the cost.   HINT. $2.10 is 87½% of the cost.

30.	$5.40	50%		37.	$2.75	8⅓%
31.	$3.30	10%		38.	$5.84	33⅓%
32.	$4.05	25%		39.	$4.20	12½%
33.	$6.89	30%		40.	$3.75	16⅔%
34.	$7.50	20%		41.	$4.50	37½%
35.	$6.16	40%		42.	$5.82	62½%

Written Exercises

1. When a hat that cost $3.50 was sold at an advance of 50 %, what was the gain ?

2. Coffee bought for 28¢ a pound was sold for 35¢ a pound. What was the gain per cent ?

3. A farm was sold at a loss of 20 % of the cost. If the loss on an acre was $12, how much did the farm cost per acre ?

4. A merchant sold a rug for $48, thus gaining $8. What was the gain per cent ?

5. A horse bought for $250 was sold for $150. What was the loss per cent ?

6. Find the selling price of cloth bought at 75¢ a yard and sold at a profit of 20 %.

7. Some goods that cost $375 were damaged by fire and sold at a loss of 12 %. How much was lost ?

8. Some goods were sold at a profit of $30. If 12 % was gained, what was the cost of the goods ?

9. A carriage was bought for $120 and sold for $100. Find the loss per cent.

10. Find the gain on a stove that cost $36 and was sold for 15 % more than it cost.

11. By selling coal at $6 a ton a gain of 20 % was made. How much did the coal cost?

12. A man bought a house for $4000 and later sold it at a gain of 15 %. What was the selling price ?

13. I bought a phonograph for $25 and sold it at a loss of 34 %. How much did I lose ?

14. What per cent is gained by selling tea at 55 ¢ per pound, if it cost 40 ¢ ?

15. A furniture dealer sold a parlor suit at a gain of 16⅔ %. If he sold it for $210, how much did he pay for it ?

16. A stationer bought blank notebooks at $1.08 per dozen and sold them at 15 ¢ each. What per cent did he gain ?

17. A dry goods merchant sold flannel for 36 ¢ per yard at a gain of 12½ %. How much did the flannel cost him ?

18. When raw silk costs $3.84 per pound, for how much per pound must it be sold to make a profit of 16⅔ % ?

19. Grain that cost 66 ¢ per bushel was damaged so that it was sold for 41¼ ¢ per bushel. What was the per cent of loss ?

20. The gain on a sale of machinery was $157.53, which was 30 % of the cost. Find the cost.

21. A haberdasher sold shirts at $1.50 each, thereby gaining 20 %. How much did he pay for them per dozen ?

22. A quantity of leather that cost $1648 was sold at a gain of 12½ %. For how much was it sold?

23. A man who paid $8250 for a yacht sold it at a loss of 14 %. What was the selling price ?

24. Find the cost per dozen pairs of infants' woolen hose that retail for 25 ¢ a pair at a gain of 60 %.

25. A lumber merchant sold 4800 ft. of lumber for $103.68, gaining 8 %. How much did it cost him per 1000 ft. ?

26. Apples are purchased for 84 ¢ per bushel and sold by the quart at a gain of 33⅓ %. Find the selling price per quart.

27. Find the gain per cent, to the nearest tenth, on neckties bought at $1.32 per dozen and sold at 25 ¢ each.

MARKING GOODS

Merchants mark their goods to show the *cost price* and the *selling price*, usually writing the first above a horizontal line and the second below.

To prevent customers from reading the mark and discovering the cost, some private mark is often adopted, the most usual device being some word or words of ten different letters to represent the ten Arabic numerals. This is called a **key**.

Thus, if the key is *white sambo*, the corresponding letters and figures are

$$w - h - i - t - e \quad s - a - m - b - o$$
$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 0$$

To avoid repeating a letter and thus giving a clue to the key, an extra letter, called a **repeater**, is used.

Thus, when the cost of an article is $1.75 and the selling price, $2.99, if the key is *white sambo* with k for the repeater, the mark would be $\frac{w\,a\,e}{h\,b\,k}$.

Sometimes the cost is given in one key and the selling price in another, but often the selling price is expressed without the use of a private mark.

In exercises, use key *white sambo*, repeater k, unless otherwise specified.

Written Exercises

Fill in the selling price at $33\frac{1}{3}\%$ above cost :

1. $\dfrac{h\,t}{\cdots}$ 3. $\dfrac{t\,i\,h}{\cdots}$ 5. $\dfrac{w\,t\,k}{\cdots}$ 7. $\dfrac{b\,s}{\cdots}$ 9. $\dfrac{h\,m\,k\,s}{\cdots}$

2. $\dfrac{s\,b}{\cdots}$ 4. $\dfrac{s\,b\,o}{\cdots}$ 6. $\dfrac{a\,m\,b}{\cdots}$ 8. $\dfrac{w\,i\,h}{\cdots}$ 10. $\dfrac{t\,s\,i\,e}{\cdots}$

Interpret each mark and find the per cent of gain :

11. $\dfrac{w\,h\,e}{w\,e\,o}$ 13. $\dfrac{t\,m}{e\,t}$ 15. $\dfrac{s\,t}{m\,k}$ 17. $\dfrac{h\,k\,e}{h\,m\,k}$ 19. $\dfrac{i\,a\,e\,o}{t\,i\,h\,e}$

12. $\dfrac{h\,t\,o}{i\,h\,o}$ 14. $\dfrac{a\,e}{w\,h\,o}$ 16. $\dfrac{i\,s}{t\,h}$ 18. $\dfrac{i\,t\,o}{t\,h\,e}$ 20. $\dfrac{e\,t\,a\,e}{s\,a\,m\,b}$

Mark articles to sell at $37\frac{1}{2}\%$ above the following costs:

21. 24 ¢   **23.** 88 ¢   **25.** $2.80   **27.** $4.48   **29.** $176

22. 40 ¢   **24.** 96 ¢   **26.** $7.20   **28.** $6.80   **30.** $224

Fill in the cost mark, the selling price being 20% above cost:

31. $\dfrac{\cdots}{t\,m}$ **33.** $\dfrac{\cdots}{i\,o\,k}$ **35.** $\dfrac{\cdots}{m\,t}$ **37.** $\dfrac{\cdots}{i\,t\,h}$ **39.** $\dfrac{\cdots}{i\,s}$

32. $\dfrac{\cdots}{s\,k}$ **34.** $\dfrac{\cdots}{a\,e\,o}$ **36.** $\dfrac{\cdots}{b\,s}$ **38.** $\dfrac{\cdots}{t\,b\,m}$ **40.** $\dfrac{\cdots}{s\,m\,t\,s}$

41. How shall a book dealer mark a book that cost $1.20 so that he may give a reduction of 16% from the marked price and still make a profit of 40%?

SOLUTION. — The selling price = 140% of $1.20, or $1.68.

Since the selling price is 16% less than the marked price, $1.68 is 84% of the marked price. The marked price = $1.68 ÷ .84, or $2, and the complete mark should be $\dfrac{w\,h\,o}{h\,o\,k}$.

Mark articles that cost as follows so that there will be a profit of 25% after making the indicated reduction:

42. 32 ¢, 20% **45.** 96 ¢, 25% **48.** $3.40, 15%

43. 72 ¢, 10% **46.** 64 ¢, $33\frac{1}{3}\%$ **49.** $6.72, $12\frac{1}{2}\%$

44. 84 ¢, 30% **47.** 92 ¢, $37\frac{1}{2}\%$ **50.** $8.40, $16\frac{2}{3}\%$

Find the cost of one article when billed by the dozen as follows, and give the complete mark, profit 40%:

Use the key *birthplace,* repeater *n,* for the cost, and *white sambo,* repeater *k,* for the selling price.

51. Hats, $21   **57.** Shirts, $18

52. Caps, $15.60   **58.** Gloves, $16.80

53. Scarfs, $24   **59.** Shoes, $30

54. Neckties, $7.20   **60.** Rubbers, $8.40

55. Raincoats, $54   **61.** Sweaters, $53.40

56. Overalls, $6.60   **62.** Hunting coats, $42

COMMERCIAL DISCOUNT

Any deduction made from a price, debt, or amount is a discount.

To meet the varying conditions of the market, discounts are often given from the prices published in catalogues and price lists, the *list prices* usually being higher than the market prices.

A discount from a list price is a **trade discount**.

Sometimes, when goods are sold to be paid for at a future time, a discount is allowed for immediate payment.

A discount allowed for immediate payment is a **cash discount**.

A discount made for payment within a definite time is a **time discount**.

The general name **commercial discount** refers to either *trade*, *cash*, or *time discount*.

Discounts are usually reckoned as some per cent of the price or amount, and are often spoken of as so many " per cent off."

The per cent of discount is the **rate of discount**.

If an article listed or marked $1.50 is sold at "10 % off," the discount is 10 % of $1.50, or $.15; and the selling price is $1.50 − $.15, or $1.35.

Sometimes discounts are expressed in fractional form, thus: "10 % off" is " $\frac{1}{10}$ off."

The price, or cost, after the discount has been taken off is the **net price**, or **net cost**.

Oral Exercises

Find the discount when the list price and the rate of discount are :

1. 20 ¢, 5 %	**6.** $12, 50 %	**11.** $18, 33$\frac{1}{8}$ %
2. 50 ¢, 6 %	**7.** $25, 20 %	**12.** $64, 12$\frac{1}{2}$ %
3. 25 ¢, 8 %	**8.** $36, 25 %	**13.** $48, 16$\frac{1}{8}$ %
4. 80 ¢, 10 %	**9.** $50, 30 %	**14.** $72, 37$\frac{1}{2}$ %
5. 50 ¢, 12 %	**10.** $45, 40 %	**15.** $80, 62$\frac{1}{2}$ %

Written Exercises

Find the discount on articles listed as follows:

1. $15, 2% off	**9.** $2.40, 15% off	**17.** $20.00, $6\frac{1}{4}$% off
2. $18, 4% off	**10.** $3.50, 20% off	**18.** $18.60, $8\frac{1}{3}$% off
3. $24, 3% off	**11.** $6.25, 12% off	**19.** $44.56, $12\frac{1}{2}$% off
4. $45, 5% off	**12.** $5.72, 25% off	**20.** $28.05, $33\frac{1}{3}$% off
5. $38, 7% off	**13.** $7.35, 20% off	**21.** $50.40, $16\frac{2}{3}$% off
6. $78, 6% off	**14.** $9.60, 30% off	**22.** $70.24, $37\frac{1}{2}$% off
7. $89, 8% off	**15.** $8.56, 25% off	**23.** $94.32, $12\frac{1}{2}$% off
8. $66, 9% off	**16.** $9.75, 40% off	**24.** $85.50, $33\frac{1}{3}$% off

Find the total discount and the total net cost:

	ARTICLES	LIST PRICE	RATE OF DISCOUNT
25.	12 tents	$28.00 each	3 %
26.	36 hatchets	.65 each	40 %
27.	20 shotguns	32.00 each	$37\frac{1}{2}$%
28.	15 rowboats	35.00 each	10 %
29.	30 hammocks	7.25 each	20 %
30.	18 gun cases	3.00 each	$16\frac{2}{3}$%
31.	24 revolvers	7.50 each	$33\frac{1}{3}$%
32.	48 shell bags	.50 each	10 %
33.	36 fishing rods	1.60 each	$12\frac{1}{2}$%
34.	24 hunting coats	2.25 each	$16\frac{2}{3}$%
35.	50 wool blankets	6.75 each	$33\frac{1}{3}$%
36.	30 hunting knives	1.75 each	20 %
37.	48 pairs hip boots	5.25 a pair	$12\frac{1}{2}$%
38.	60 pairs storm boots	4.75 a pair	2 %

Written Exercises

1. Find the discount on a lamp listed at $9.50, ½ off.

2. The list price of an electric chafing dish outfit was $26 with a discount of 50 %. Find the net price.

3. What is the discount on a mandolin that is listed at $25, if it is sold at 5 % off for cash?

4. Find the net price of a box of soap costing $4 less a discount of 12½ %.

5. The list price of a wagon is $85, from which 10 % discount is offered. Find the net price.

6. At a sale some tapestry curtains were marked down 20 % from the regular price, $5.25 per pair. Find the selling price.

7. The regular price of a refrigerator was $24.75. If it was marked down 33⅓ % for a sale, what was the selling price?

8. How much does a lady pay for 4 yd. of linen, if the price, $.50 per yard, is marked down 8 %?

9. Find the net price of a desk listed at $ 82.50, discount 20 %.

10. There was a discount of 50 % on a set of Shakespeare's works, the list price of which was $41.50. Find the discount.

11. The list price of a set of Dickens was $22.50. At a discount of 33⅓ %, for how much was the set sold?

12. A discount of 40 % was allowed on a set of books by Victor Hugo, listed at $16.25. Find the net price.

13. A merchant sold gocarts at 30 % off. What was the selling price of one marked $18.50?

14. Mr. Burton bought a suit of clothes at 25 % off. If the suit was marked $27, how much did he pay for it?

15. Find the list price of a truck sold at $1.80, discount 60%.

SUGGESTION. $1.80 is 100% − 60%, or 40%, of the list price.

16. What was the list price of a pair of canvas mittens sold for 12¢, if there was a discount of 20%?

17. How much must a dealer pay for 9 cameras listed at $25 each, if there is a discount of 15%?

18. Find the cash cost of 220 ft. of garden hose at $7\frac{1}{2}$¢ per foot, discount for cash 10%.

19. Find the net cost of 3 cases of crayons, each holding 100 gross, at a discount of 20%, list price 10¢ per gross.

20. The net cost of a wheelbarrow was $6.90. If a trade discount of 40% was given, what was the list price?

21. Silk mufflers listed at $2.25 each are sold to a retailer at a discount of $16\frac{2}{3}$%. If the retailer buys 3 doz., how much discount does he receive?

22. The net cost of a mantel upon which a cash discount of 3% was given was $27.16. Find the regular price.

23. A merchant bought 25 lb. of Japan tea for $13.50. If the list price was 60¢ per pound, what was the rate of discount?

24. A manufacturer purchased 3600 yd. of gingham at $6\frac{3}{4}$¢ per yard, and received 3% discount by paying within ten days. What was the net cost of the goods?

25. Mr. Munn bought 15 vises for a Manual Training School, listed at $9.50 each. If the net cost was $85.50, what was the rate of discount?

26. A merchant in Minneapolis bought at $62\frac{1}{2}$% discount a carload of damaged wheat weighing 42,000 lb. If the original price was $.72 per bushel, what was the total net cost?

DISCOUNT SERIES

Catalogues that contain price lists are often expensive. To avoid reprinting them with each change in the market, dealers change discounts instead of list prices. If the market price rises, a smaller discount is given; if the price falls, the discount is increased, usually by allowing a second discount.

If an order is large, an additional discount may be given.

It often happens, then, that several successive discounts are allowed. The first is a discount from the list price, the second from the remainder, the third from the second remainder, etc. Two or more successive discounts form a **discount series**.

Written Exercises

1. What is the net price of an article listed at \$17.40, if the discounts are 25 % and 20 % ?

List price	\$17.40
Less 25 %	4.35
Remainder	13.05
Less 20 %	2.61
Net price	\$10.44

The first discount is 25 % of \$17.40, or \$4.35.
Subtracting, we find the remainder to be \$13.05.
The second discount is 20 % of \$13.05, or \$2.61.
Subtracting, we find the net price to be \$10.44.

NOTE. — Find the discounts in the order (20 %, 25 %) and thus show that the result is the same in whatever order they are taken.

Find the net price of articles listed and discounted as follows :

2. \$40 ; 15 %, 5 %

3. \$60 ; 25 %, 8 %

4. \$25 ; 20 %, 10 %

5. \$70 ; 30 %, 20 %

6. \$35 ; 12 %, 10 %

7. \$56 ; 50 %, 25 %

8. \$95 ; 40 %, 30 %

9. \$89 ; 60 %, 20 %

10. \$3.20 ; $12\frac{1}{2}$ %, 10 %

11. \$5.70 ; 40 %, $16\frac{3}{4}$ %

12. \$8.40 ; $33\frac{1}{3}$ %, $12\frac{1}{2}$ %

13. \$400 ; 50 %, 10 %, 5 %

14. \$625 ; 20 %, 20 %, 3 %

15. \$800 ; $37\frac{1}{2}$ %, 15 %, 4 %

16. \$750 ; $33\frac{1}{3}$ %, 10 %, 2 %

17. \$920 ; $62\frac{1}{2}$ %, 20 %, 5 %

Written Exercises

Find the total discount and the total net cost:

ARTICLES	LIST PRICE	RATES OF DISCOUNT
1. 3 doz. food choppers	$24.00 per doz.	30 %, 10 %
2. 4½ doz. meat cleavers	9.00 per doz.	33⅓%, 5 %
3. 6 doz. butcher knives	6.50 per doz.	40 %, 10 %
4. 5½ doz. coffee mills	8.00 per doz.	35 %, 20 %
5. 15 lawn mowers	10.00 each	60 %, 10 %, 5%

6. Find the net cost to a retailer of 2½ doz. hammocks the list price of which is $16 per dozen; discounts 20 %, 12½ %.

7. Ford & Bond sold me 4½ doz. thermometers at $15 per dozen; discounts 20 %, 7½ %. Find the total discount.

8. When the list price of sash cord is 30¢ per pound, with discounts of 15 % and 5%, how much do 200 lb. cost?

9. How much must be paid for 6 sets of harness, the list price being $15.20 each, if there are discounts of 25 % and 5 %?

10. In paying for 2 doz. beaver traps, listed at $16.50 per dozen, a sporting goods dealer received discounts of 60 % and 10 %. How much did he pay for the traps?

11. Ryder & Co. bought 720 lb. of galvanized sheet iron at discounts of 70 % and 2½ %. If the list price was 15¢ per pound, what was the net cost?

12. How much did I pay for 96,000 shipping tags listed at 25¢ per thousand; discounts 25 %, 5 %?

13. A dealer bought 12 6-pound sledges, listed at 30¢ per pound, at discounts of 75 % and 10 %. Find the net cost.

14. Mr. Fuller bought 150 ft. of steel fence at $1.10 per foot, and 2 gates at $3.50 each. If he received a discount of 15 %, and 5 % for cash, what was the net cost?

15. What single discount is equal to the series 15 %, 10 % ?

<div align="center">SOLUTION</div>

List price,	100	% of the list price
Less 15%,	15	% of the list price
Remainder,	85	% of the list price
Less 10%,	8.5	% of the list price
Net price,	76.5	% of the list price

The single discount 100%–76.5%, or 23.5%, equals the series 15%, 10%.

Find the single discount equal to the series :

16. 10 %, 5 %	**21.** 20 %, 12 %	**26.** 40 %, 10 %, 5 %
17. 15 %, 6 %	**22.** 30 %, 10 %	**27.** 50 %, 20 %, 4 %
18. 25 %, 2 %	**23.** 16 %, 12½ %	**28.** 25 %, 33⅓ %, 2 %
19. 40 %, 4 %	**24.** 55 %, 33⅓ %	**29.** 60 %, 12½ %, 6%
20. 50 %, 8 %	**25.** 40 %, 16⅔ %	**30.** 20 %, 37½ %, 3 %

31. How much less is the net cost of 280 lb. of Manila rope listed at 12½ ¢ per pound when the discount is 25 % than when it is the series 20 %, 5 % ?

32. Which is better, discounts of 40 %, 20 % or 50 %, 10 % ?

33. Which is better, discounts of 25 %, 10 % or 20 %, 15 % ?

34. Which is better, discounts of 10 %, 16⅔ % or 25 %, 2 % ?

35. Find the list price, the net price being $10.44 and the discounts, 25 %, 20 %.

<div align="center">SOLUTION</div>

The series 25%, 20% = a single discount of 40%.
Since $10.44 is 60% of the list price, the list price = $10.44 ÷ .60, or $17.40.

Find the list price from the given net price and discounts :

36. $63; 30 %, 10 %	**39.** $360; 20 %, 10 %
37. $96; 40 %, 20 %	**40.** $18.90; 60 %, 12½ %
38. $75; 33⅓ %, 25 %	**41.** $97.20; 40 %, 16⅔ %

42. A music dealer received $270 cash for a piano after giving the discounts, 40 %, 10 %. What was his asking price ?

BILLS SUBJECT TO DISCOUNT

NOTE. — If necessary at this point, the pupil may refresh his memory in regard to bills by reading pages 384–386.

When there are discounts that are the same for all items of a bill, it is customary to find the amount of the bill and then the discount on that amount.

The amount of a bill before it is discounted is the **gross amount**; the gross amount less all discounts is the **net amount**.

Written Exercises

Find the net amount, given the gross amount and discount :

1. $28; 4%   5. $45; 20%, 5%   9. $360; 12½%, 2%
2. $45; 2%   6. $72; 25%, 6%   10. $498; 16⅔%, 4%
3. $76; 10%   7. $60; 15%, 3%   11. $560; 37½%, 15%
4. $94; 25%   8. $90; 30%, 8%   12. $744; 33⅓%, 12½%

13. Find the discount at 50% on a bill of $125 for furniture.

14. From a bill of $170 for work benches a discount of 10% was allowed. Find the net amount of the bill.

15. Shafer & Son received discounts of 40% and 10% on a bill of $350 for silver plate. Find the total discount.

16. A bill for school supplies amounted to $475.50. Find the cash payment, the discount being 33⅓%, and 5% for cash.

17. What is the difference between a discount of 35% on a bill of $320, and two successive discounts of 20% and 15%?

18. From a bill of $840 for window glass, 65% and 20% discounts were allowed. What was the net amount of the bill?

19. Find the total discount on a bill of goods amounting to $1280, with discounts of 12½% and 30%.

20. What is the net amount of a bill of $496 for tinware, if the discounts are 75% and 10%, and 5% for cash?

The terms upon which goods are sold, that is, the conditions as to discounts and time of payment, are printed or written on the bill.

The following illustrates a form of bill with discounts :

Boston, Mass. *April 7, 1911*

Mr. Ralph Boyd, 72 Tremont St.

Bought of **A.G. CROOKS & CO.,** **284 FEDERAL ST.**

WHOLESALE GROCERS

Terms: *Trade 5%, 2% cash.*

10 pails ginger,	250 lb 23 d	57	50		
15 boxes cloves,	150 lb 15 d	22	50		
20 „ cinnamon,	200 lb 27 d	54			
		134			
Discounts 5%, 2%		9	25	124	75
Received payment.					
A. G. Crooks & Co					

Since the trade discount is the same for all items, the series 5 %, 2 % is taken from the gross amount. When the items of a bill have different trade discounts, each discount is deducted from the item to which it applies. If goods were sold to be paid for in 90 days with the privilege of paying in 10 days and getting a discount of 2 %, the bill would have on it " Terms : net 90 days, 2 % in 10 days," or, more briefly, " Terms : *n*/90, 2/10."

Written Exercises

Supply necessary data, make out bills, and find net amounts, assuming that the buyer takes advantage of the time discounts:

1. 30 yd. tailors' canvas at 15¢ a yard. Terms : 6/30.

2. 50 lb. Japan tea at 60 ¢ a pound. Discounts, 10%, 5%.

3. 5 clocks at $20 each. Discounts, 50 %, 10 %, 5%.

4. 75 rolls paper at 40¢ each. Terms : 15%, 2/30.

5. 1 set historical maps, $21; 1 encyclopedia, $18 ; 1 dictionary, $12 ; 3 book holders at $4.80 each. Terms : 33⅓ %, 5 %.

6. 25 women's watches @ $24 ; 20 men's watches @ $16; 12 men's watches @ $20.   Terms : 50 %, 10 %, 6 %.

NOTE. — The symbol @, meaning *at*, is followed by the *price of a unit.*
In the extension of an item or in finding a discount give the. result to the nearest cent.   (See note, page 98.)

7. 8 grindstones @ $5.50 ; 15 oilstones @ 60¢ ; 1¼ doz. scythes @ $8.   Discounts, 33⅓ %, 10 %.

8. 72 sets irons @ $1.50 ; 36 teakettles @ 65¢ ; 60 coffee pots @ 45¢ ; 8 doz. egg beaters @ 85¢.   Terms : 40 %, 15 %.

9. 15 sets drafting instruments @ $10 ; 30 amber triangles @ 50¢ ; 60 bottles India ink @ 25¢ ; 75 erasers @ 10¢ ; 9 doz. thumb tacks @ 10¢.   Discount, 20 %.

10. 10 electric bells @ 90¢ ; 72 dry batteries @ 25¢ ; 9 electric stoves @ $8.40 ; 24 electric irons @ $8.50 ; 12 percolators @ $17.50 ; 15 electric cookers @ $10.   50 % off.

11. 3 doz. hammers @ $6 ; 2½ doz. screwdrivers @ $7 ; 1¾ doz. saws @ $16 ; 15 bench planes @ $1.90.   25 % and 10 % off.

12. 30 linen tablecloths @ $3.50 ; 40 doz. napkins @ $3 ; 60 doz. handkerchiefs @ $2.25.   Discounts, 15 %, 5 %.

13. 25 doz. rolls paper towels @ $6 ; 50 gal. liquid soap @ $1.20 ; 30 soap fountains @ $2.60.   Terms : 33⅓ %, 5 %.

14. 9 chiffoniers @ $24 ; 12 tables @ $30 ; 9 rocking chairs @ $18 ; 18 desk chairs @ $11 ; 6 china cabinets @ $26 ; 6 brass beds @ $28 ; 15 stands @ $4.   Discounts, 50 %, 2 %.

15. 6 doz. wrenches @ $10 ; 6 doz. wrenches @ $14 ; 500 machine bolts at $3.36 per hundred ; 150 carriage bolts at $3.04 per hundred.   Discounts, 70 %, 10 %.

16. 4500 1-pound paper bags at $1.60 per M ; 2000 5-pound paper bags at $3.25 per M ; 1500 10-pound paper bags at $5.10 per M.   Terms : 60 %, 10 %, 5 %.

Miscellaneous Exercises

1. A bill of goods amounting to $600 was discounted 3 % for cash. How much was the discount ?

2. Find the discount on 3 doz. drawing boards listed at $1 each with a discount of 25 %.

3. What single discount is equal to the series $33\frac{1}{3}$ %, 10 % ?

4. A plumber's bill for supplies amounted to $375, discounts 40 %, and 4 % for cash. What was the net amount of the bill ?

5. Supply necessary data, make out the bill, and find the net amount: 18 trowels @ $1.50; 24 picks @ $1.95; 30 shovels @ $1.25; 36 hoes @ 75¢; 9 wheelbarrows @ $7. Terms: 40%.

6. A hardware merchant ordered 250 lb. of horseshoe nails at $.22 per pound. If he received a discount of 40 %, and 5 % for cash, what was the net cost ?

7. A jeweler bought a clock listed at $33.32, discounts 50 %, 10 %, 5 %, and sold it at the list price. Find his gain.

8. How much less does a dealer pay for 360 yd. of Brussels carpet listed at $1.25 per yard, if he makes his payment in ten days with 4 % off instead of in thirty days with 3 % off ?

9. A grocer bought sugar at $5 per hundredweight with a cash discount of 1 %. If he sold it at 6¢ a pound, what was his gain per barrel of 365 lb. ?

10. What is the difference on a bill of $440 between a direct discount of 35 % and successive discounts of 20 % and 15 % ?

11. A retail druggist gets a discount of 40 % from the wholesale list price and sells at the list price. What per 'cent of profit does he make ?

12. A merchant retailed toilet soap at 3 cakes for 25¢. If he bought the soap at $7 per gross with a discount of 25 %, and 5 % for cash, what was his gain per gross ?

COMMISSION AND BROKERAGE

A person who buys or sells goods or transacts business for another is an **agent**, a **commission merchant**, or a **broker**.

The person for whom business is done is the **principal**.

In general, a commission merchant actually receives the goods, while a broker simply arranges for their sale or purchase, the goods being shipped directly from the seller to the purchaser.

The compensation of an agent is usually reckoned as some per cent of the *value* involved, and is the **commission** or **brokerage**.

Thus, a seller's commission is some per cent of the amount of sales; a buyer's commission is some per cent of the cost; a collector's commission is some per cent of the money collected.

Sometimes commission is computed at a certain price per unit, as grain per bushel, pork per barrel, cotton per bale.

The sum left after the commission and other expenses have been paid is the **net proceeds**.

Oral Exercises

Find the commission :

Amount of Sales	Rate of Commission		Amount of Sales	Rate of Commission
1. $ 300	2 %	10.	$ 1200	4 %
2. $ 500	4 %	11.	$ 1400	2 %
3. $ 800	3 %	12.	$ 4000	5 %
4. $ 600	5 %	13.	$ 2500	3 %
5. $ 400	8 %	14.	$ 8000	7 %
6. $ 900	6 %	15.	$ 7000	6 %
7. $ 150	2 %	16.	$ 9000	8 %
8. $ 250	4 %	17.	$ 5000	9 %
9. $ 160	5 %	18.	$ 5600	10 %

Written Exercises

Find the commission :

	SALES	RATE OF COMMISSION		PURCHASES	RATE OF COMMISSION
1.	$325	2 %	11.	$1750	$\frac{1}{2}$ %
2.	$450	5 %	12.	$5260	$\frac{1}{4}$ %
3.	$740	4 %	13.	$4720	$\frac{1}{8}$ %
4.	$390	6 %	14.	$6080	$1\frac{1}{2}$ %
5.	$880	3 %	15.	$3760	$1\frac{1}{8}$ %
6.	$565	8 %	16.	$2956	$2\frac{1}{2}$ %
7.	$985	2 %	17.	$7432	$2\frac{1}{4}$ %
8.	$674	7 %	18.	$6736	$2\frac{3}{4}$ %
9.	$798	8 %	19.	$5068	$3\frac{1}{4}$ %
10.	$839	10 %	20.	$9176	$6\frac{1}{4}$ %

Find the net proceeds :   Find the total (gross) cost :

	SALES	RATE OF COMMISSION		PURCHASES	RATE OF COMMISSION
21.	$1400	3 %	33.	$1650	$\frac{1}{2}$ %
22.	$2500	5 %	34.	$2800	$\frac{1}{8}$ %
23.	$6800	4 %	35.	$3400	$\frac{1}{4}$ %
24.	$7300	2 %	36.	$1950	$1\frac{1}{8}$ %
25.	$5800	7 %	37.	$5380	$2\frac{1}{2}$ %
26.	$49.50	6 %	38.	$2360	$2\frac{1}{4}$ %
27.	$37.40	5 %	39.	$8240	$3\frac{1}{4}$ %
28.	$48.50	8 %	40.	$4752	$2\frac{3}{4}$ %
29.	$89.50	2 %	41.	$6750	$3\frac{1}{2}$ %
30.	$65.50	6 %	42.	$6146	$4\frac{1}{2}$ %
31.	$56.50	4 %	43.	$9564	$3\frac{3}{4}$ %
32.	$90.50	8 %	44.	$7692	$6\frac{1}{4}$ %

Written Exercises

Note. — In finding cost or commission give results to the nearest cent.

1. At 2 % find the commission on sales amounting to $348.

2. A book agent's sales one month amounted to $275. How much did he make that month, if his commission was 40 % ?

3. The value of the maps sold by an agent one summer was $635. Find his commission at 50 %.

4. A commission merchant sold 100 bbl. of flour at $6.75 a barrel. Find his commission at 3 %.

5. At 2 % commission, how much will an agent receive for selling property to the value of $825.50?

6. How much was the builder's commission at 10 % on a house that cost $4500? Find the architect's commission at 5 %.

7. An agent received $64.25 for collecting rents. How much did he collect, if his commission was 5 % ?

8. How much commission was received for selling 50 bbl. of apples at $1.80 per barrel, if the rate of commission was 7 % ?

9. An agent received 20 % commission for selling pictures. How much did he earn by selling 50 pictures at $2.85 each?

10. A commission merchant bought 560 bu. of potatoes at 60¢ a bushel. Find his commission at 5 %.

11. Find the commission, at 2 %, on 400 tons of coal sold for $5.25 a ton.

12. A lawyer received $18.10 for collecting a debt of $362. What was his rate of commission?

13. An agent collected $745 for his principal and charged 7 % commission. Find the commission and the net proceeds.

14. At 1½ %, how much was a real estate agent's fee for selling some city lots valued at $2425?

Find the commission and the net proceeds:

	PRODUCE	QUANTITY	PRICE	RATE OF COMMISSION
15.	Onions	250 bu.	85 ¢	6 %
16.	Cabbages	500 bbl.	$1.25	5 %
17.	Apples	360 bbl.	$1.75	10 %
18.	Peaches	120 baskets	75 ¢	8 %
19.	Blackberries	960 qt.	12 ¢	$7\frac{1}{2}$ %
20.	Butter	2880 lb.	$28\frac{3}{4}$ ¢	5 %
21.	Cheese	1280 lb.	$15\frac{1}{4}$ ¢	5 %
22.	Eggs	2250 doz.	24 ¢	5 %
23.	Chickens (live)	1760 lb.	16 ¢	10 %
24.	Chickens (dressed)	2450 lb.	20 ¢	5 %

25. A lawyer collected 80 % of a debt of $2500 and charged 10 % commission. How much did the creditor receive?

26. What is the weekly income of a clerk who receives $8 a week and a commission of 4 % of his sales, if his sales average $250 a week?

27. A commission merchant purchased rice for me to the amount of $2840.50. Find the commission at $3\frac{1}{2}$ %.

28. How much brokerage did a broker receive on 20,000 bu. of wheat, if he charged $\frac{1}{8}$ ¢ per bushel?

29. A commission merchant bought 8000 bu. of corn at 56 ¢ per bushel. Find his commission at $2\frac{1}{2}$ %.

30. If a real estate agent charged $2\frac{1}{4}$ a month for renting a house for $62\frac{1}{2}$ a month, what was his rate of commission?

31. How much did a broker receive for purchasing 91,000 lb. of coffee for an importer at 9 ¢ per pound, brokerage $\frac{1}{2}$ % ?

32. How much business must a commission merchant do to earn $125, if his average commission is $2\frac{1}{2}$ % ?

33. An agent sold 2000 bu. of corn at 72¢ a bushel. He paid $134.40 freight and $22.50 storage. Find the net proceeds, or amount remitted to the principal, after the agent has deducted a commission of 2 %.

SOLUTION

Amount of sales, 2000 × 72 ¢ =	$1440.00
Commission, 2 % of $1440 = $ 28.80	
Freight 134.40	
Storage 22.50	185.70
Net proceeds remitted	$1254.30

Find the commission and the net proceeds :

ARTICLE	QUANTITY	PRICE	EXPENSES	RATE OF COMMISSION
34. Oats	3000 bu.	39¢	$ 54.50	2 %
35. Wheat	4000 bu.	83½¢	125.00	½ ¢ per bu.
36. Pork	100 bbl.	$28.50	56.00	3 %
37. Eggs	1500 doz.	26¢	5.00	5 %
38. Books	225	$1.75	10.50	40 %
39. Cotton	4750 lb.	10¼¢	45.75	2¼ %
40. Silk	1000 yd.	$1.25	25.00	7 %

41. My agent sold goods for me to the amount of $4620. If he paid $85 for cartage and other expenses and charged 3 % commission, what amount did he remit to me ?

42. An agent sold 4 sewing machines at $35 each, receiving a commission of 40 %. The expense of delivering them was $2.25 each. Find the commission and the net proceeds.

43. What were the net proceeds of a sale of 240 bbl. of flour at $6.50 per barrel, if the agent charged a commission of 2½ % and paid $20.50 freight charges ?

44. Find the commission and the amount remitted on a sale of 225 bales of cotton, of 500 lb. each, at 11¢ per pound, commission 3 %, freight and other expenses, $125.

Miscellaneous Exercises

1. What were the net proceeds of a sale of 150 doz. bunches of celery at 36 ¢ a dozen, if the rate of commission was 5 % ?

2. A collector succeeded in collecting 75 % of a debt of $4000 and charged 15 % commission. How much did the creditor receive ?

3. Find the net proceeds from the sale of 250 bags of coffee, averaging 132 lb. each, at 14½ ¢ per pound, commission at 2 %.

4. What was the architect's commission on an office building that cost $ 24,000, if he received 1¾ % for drawing the plans, and 3¼ % for superintending the construction ?

5. Find the net proceeds from the sale of 1840 lb. of dressed poultry at 15 ¢ per pound, commission 5 %.

6. If a lumber agent sold 80,000 ft. of lumber at $ 24 per M and received a commission of 4¼ %, how much was his commission ?

7. The rent of a house was $ 37½ per month, and the agent retained out of that sum a commission of $1¼ per month. What was the rate of commission ?

8. My agent sold a city house for me for $ 5500, commission 2 % ; also some country property for $ 2800, commission 3 %. Find my net proceeds from the two sales.

9. Find the auctioneer's commission on a sale of rugs, the amount of the sale being $ 97,546.50, and the rate of commission 2 %.

10. A real estate agent was paid $120 for collecting rents to the amount of $2400. What was his rate of commission ?

11. How many 5-dollar books must a book agent sell to earn $240, if his commission is 40 % ?

12. If a salesman received $3750 one year as commission on sales amounting to $50,000, what was his rate of commission?

13. What was the income last year of a commercial traveler whose sales amounted to $105,620, if his commission was 2¾%?

14. An agent bought a block of 14 houses, each house worth $3200, on a commission of 1½%. Find his commission.

15. A commercial traveler receives a salary of $25 a week and 2% commission on his sales. If his sales amount to $75,000 in a year, what is his income for the year?

16. Find the rate of commission, when the net proceeds from a sale of $5000 are $4875.

17. One month an agent secured 346 subscriptions to a dollar magazine, earning $86.50. Find his rate of commission.

18. The commission at 3½% for selling a quantity of oats was $189. How many bushels were sold, if the price received was 40 ¢ per bushel?

19. A Cincinnati packer sent 150 bbl. of mess pork to a commission merchant, who sold it at $24.60 a barrel, paying $77 freight charges, 10 ¢ a barrel cartage, and 4 ¢ a barrel storage. Find the commission at 2¾% and the net proceeds.

20. A commercial traveler who sold goods on a commission had an income one year of $3000 by selling goods to the amount of $120,000. What was his rate of commission?

21. The net proceeds of a sale of produce were $2451. If the commission was $129, what was the rate of commission?

22. A salesman was offered his choice of a salary of $2500 a year, or $1500 a year with a commission of 2% on his sales, or 6% commission on all sales. If his sales amounted to $60,000, how much better than the first offer was the second? the third?

REVIEW

Express as a per cent, explaining each process:

1. $\frac{1}{4}$ 3. $\frac{1}{20}$ 5. $.5\frac{1}{2}$ 7. .875

2. $\frac{1}{8}$ 4. $2\frac{3}{5}$ 6. $.02\frac{1}{4}$ 8. 4.25

What is the meaning of *per cent?* What name is given to that part of arithmetic that treats of per cents?

9. Find 25 % of 40; 16 % of 20; $37\frac{1}{2}$ % of 96; 75 % of 276.

To what is the product of the base and the rate equal?

Define base; rate; percentage.

10. What per cent of 96 is 48? of 60 min. is 40 min.?

How do you find what per cent one number is of another? Given the base and the percentage, how is the rate found?

11. Find the number of which 20 is 25 %; 42 is $87\frac{1}{2}$ %.

If the percentage and the rate are given, how is the base found?

12. What number increased by 25 % of itself equals 30?

13. What number decreased by $37\frac{1}{2}$ % of itself equals 45?

14. Find the gain when the cost is $25 and the rate of gain 20 %. Upon what is the per cent of gain or loss reckoned?

15. If the selling price is $3.75 and the rate of loss is $6\frac{1}{4}$ %, what is the cost? Explain.

Using the key *corn basket*, repeater *h*, mark articles to sell at $33\frac{1}{3}$ % above the following costs:

16. 27 ¢   17. *s b*   18. *e t*   19. $1.50   20. *o h b*

Explain why a *key* is used in marking goods.

21. Find the discount and the net price when the list price is $50 and the rate of discount is 25 %. Explain.

Define discount; net price; rate of discount.

Name three kinds of commercial discount. Define each.

22. When the net price is $1.50 and the discount ⅛ off, what is the list price? Explain.

23. Find the net price of an article listed at $6, discounts 33⅓ % and 20 %. What are two or more successive discounts called?

24. What single discount is equivalent to the series, 20 %, 10 %, 5 %? Explain the process.

25. Define gross amount of a bill; net amount.

26. Find the commission and the net proceeds when the amount of sales is $2475 and the rate of commission is 3 %. Upon what is the commission or brokerage usually reckoned?

Define commission; net proceeds.

Written Exercises

Express as a decimal: Express as a per cent (%):

1. 6 %	**4.** 37½ %	**7.** ⅓	**10.** ⅙
2. 8 %	**5.** 16⅔ %	**8.** ¾	**11.** ⅞
3. 9 %	**6.** 140 %	**9.** ⅝	**12.** $\frac{3}{10}$

Express as a common fraction in its lowest terms or as a mixed number:

13. 5 %	**15.** 25 %	**17.** 60 %	**19.** 37½ %	**21.** 112½ %
14. 4 %	**16.** 20 %	**18.** 80 %	**20.** 16⅔ %	**22.** 133⅓ %

Find the value of x in each of the following:

	BASE	RATE	PERCENTAGE		BASE	RATE	PERCENTAGE
23.	80	6 %	x	**28.**	x	8 %	52
24.	90	x	4½	**29.**	840	75 %	x
25.	x	25 %	21	**30.**	x	35 %	343
26.	72	x	9	**31.**	720	x	180
27.	x	20 %	18	**32.**	x	33⅓ %	322

Fill the blank spaces in the following :

	Cost	Rate of Gain	Gain	Selling Price
33.	$4.40	——	$1.10	——
34.	$5.80	——	——	$6.96
35.	——	40 %	——	$10.85
36.	——	30 %	$2.01	——

	Cost	Rate of Loss	Loss	Selling Price
37.	$3.50	10 %	——	——
38.	——	——	30 ¢	$4.70
39.	——	12½ %	——	$4.20
40.	$6.80	——	34 ¢	——

Find the commission and net proceeds :

	Amount of Sales	Rate of Commission		Amount of Sales	Rate of Commission
41.	$250	2 %	45.	$1150	3 %
42.	$375	4 %	46.	$1280	4 %
43.	$480	6 %	47.	$2650	7 %
44.	$525	8 %	48.	$3875	9 %

Find the discount on an article listed as follows :

49. $16, 5 % off   53. $4.40, 15 % off   57. $18, 30 %, 15 % off

50. $22, 7 % off   54. $6.75, 20 % off   58. $34, 25 %, 10 % off

51. $35, 9 % off   55. $8.60, 35 % off   59. $42, 33⅓ %, 5 % off

52. $48, 8 % off   56. $9.80, 45 % off   60. $68, 40 %, 12½ % off

Find the single discount equal to the series :

61. 25 %, 4 %   63. 50 %, 10 %   65. 70 %, 10 %, 2 %

62. 30 %, 6 %   64. 40 %, 15 %   66. 75 %, 20 %, 12½ %

Note. — Students who have covered the work of pages 168–172 may, if they wish, solve some of the following problems by use of the equation and x.

67. What was the loss per cent on a farm bought for $3200 and sold for $2400?

68. If beef loses 20% of its weight by roasting, what is the weight of 14 lb. of roast beef before being roasted?

69. Codfish bought for $12.50 per hundredweight was sold in 10-pound boxes at $1.50 per box. Find the gain per cent.

70. A merchant bought 30 bbl. of flour at $5.50 per barrel, discount 1% for cash. Find the net cost of the flour.

71. A house was sold for $4050 at a gain of 12½%. How much did the house cost?

72. One week an agent sold 9 vacuum cleaners at $11.50 each, commission 33⅓%. Find his income for the week.

73. A grocer paid $19.20 for a tub of butter weighing 60 lb. What was his selling price per pound, if he gained 12½%?

74. A man whose income is $1500 a year pays 20% of it for rent, 25% of it for household expenses, and 30% of it for all other expenses. How much does he save in a year?

75. A real estate agent sold a farm for $6750, charging 4% commission. Find the net proceeds from the sale.

76. A crop of potatoes was sold for $743.75. What was the expense of raising them, if it was 48% of the selling price?

77. If 32% of the people in a certain city are voters, and there are 40,176 voters, what is the population of the city?

78. An agent bought for his principal 325 cwt. of sugar at $4.25 per hundredweight. Find his commission at 4%.

79. If 150 lb. of "lead," used in a medium grade of lead pencils, consists of 88¼ lb. of graphite and the rest clay, what is the per cent of each substance in the mixture?

80. A dealer was obliged to sell at 92 % of the cost a lot of pineapples for which he had paid $115.50.   Find his loss.

81. A merchant's expenses were $50 per day.   If his expenses were 25 % of his income, how much was his income per day?

82. My agent sold goods for me to the amount of $2430. If he paid $44 for cartage and other expenses and charged 2 % commission, how much were the net proceeds?

83. The circulation of a publication has increased 26 % in three years, and is at present 177,030.   What was the circulation three years ago?

84. What is the difference, on a bill of $425, between a discount of 20 % and successive discounts of 10 % and 10 %?

85. An agent collected debts to the amount of $1250.   He paid his principal $1175 of it.   Find his rate of commission.

86. The Hudson River ice crop in a poor season was only 1,440,000 tons, or 68 % less than the crop of the previous season. How many tons of ice were harvested the previous season?

87. A merchant had a stock of hats that cost $2.50 each. When he had sold 75 % of them, the cost value of the rest was $360.   How many of these hats did he have in stock at first?

88. A man bought a ring for $35, or $12\frac{1}{2}$ % less than its value.   If he sold it for $12\frac{1}{2}$ % more than its value, what was his gain?

89. A manufacturer sold 50 pieces of velveteen at auction at $48 per piece.   On one half of them he gained 20 % and on the rest he lost 20 %.   How much did he lose by the sale?

90. A dealer gained 20 % on flour that he sold for $6 a barrel.   The market advanced, and he sold the rest of his stock for $6.50 a barrel.   What was his per cent of gain after the advance?
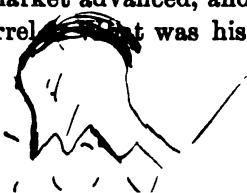
91. A dealer bought goods at 20 % less than their market value, and sold them at 20 % above the market value. What per cent did he gain ?

SUGGESTION. 80 % of the market value represents the cost; 120 % of the market value, the selling price; and 40 % of the market value, the gain. What part of 80 % is 40 %? what per cent?

92. What per cent is gained by buying clothing at 10 % below market value and selling it at 12¼ % above ?

93. At what price must goods that cost $ 1.80 per yard be marked so that the seller may deduct 10 % from the marked price and still gain 20 % ?

94. A dry goods merchant sold tablecloths at 20 % less than the marked price, yet gained 16⅔ %. At what price were they marked, if they cost him $ 3.60 apiece ?

95. A manufacturer marked his goods down 12½ % from the list price on account of a fall in the market value, and allowed an additional discount of 4 % for cash. What was the entire discount, expressed in per cent ?

96. Supply necessary data, make out the bill, and find the net amount : 8 doz. erasers @ $ 1.70 ; 2 100-gr. cases crayons @ $ 16 ; 6 boxes colored crayons @ $ 1.20 ; 1½ doz. blackboard pointers @ $ 1. Discounts, 33⅓ %, 5 %.

97. A merchant's complete stock of goods cost $ 50,000. He sold it at an average advance of 15 % above cost and lost 2¼ % of his sales in bad debts. Find his gain.

98. Maxwell & Thurston, commission merchants, buy for Stafford & Sons, grocers, 625 lb. of butter @ 28 ¢, 345 lb. of butter @ 30 ¢, 3000 eggs at $ 2.25 per hundred, 126½ lb. of cheese @ 14½ ¢, and 20 bbl. of sweet potatoes @ $ 2.75. The commission is 2 %, and the cartage, $ 3.50. What is the indebtedness of the grocers to the commission merchants ?

INTEREST

Money paid for the use of money is **interest**.

The money for the use of which interest is paid is the **principal**.

The sum of the principal and the interest is the **amount**.

Interest is reckoned as a certain *per cent* of the principal, and the **rate of interest** is the per cent paid for the use of the principal for *one year*.

In ordinary interest calculations, a month is regarded as 30 days, and a year as 12 months, or 360 days.

Oral Exercises

Find the interest for 1 year on :

1. $100 at 8%	**6.** $40 at 4%	**11.** $150 at 8%
2. $400 at 8%	**7.** $70 at 5%	**12.** $900 at 7%
3. $800 at 6%	**8.** $50 at 8%	**13.** $1000 at 6%
4. $800 at 4%	**9.** $80 at 2%	**14.** $2500 at 2%
5. $600 at 5%	**10.** $90 at 6%	**15.** $4000 at 5%

Find the interest at 5% on :

16. $100 for 1 yr.; for 2 yr.	**23.** $400 for 1 yr.; for 5 yr.
17. $400 for 1 yr.; for 4 yr.	**24.** $600 for 1 yr.; for 8 yr.
18. $200 for 1 yr.; for 5 yr.	**25.** $500 for 1 yr.; for 4 yr.
19. $800 for 1 yr.; for 4 yr.	**26.** $800 for 1 yr.; for 8 yr.
20. $500 for 1 yr.; for 8 yr.	**27.** $700 for 1 yr.; for 2 yr.
21. $600 for 1 yr.; for 2 yr.	**28.** $900 for 1 yr.; for 2 yr.
22. $800 for 1 yr.; for 2 yr.	**29.** $1000 for 1 yr.; for 2 yr.

1. What is the interest on $75.50 for 3 yr. at 6 %?

SOLUTION

Principal,	$75.50
Rate,	.06
Interest for 1 year,	4.53\cancel{00}$
	3
Interest for 3 years,	$13.59

The interest equals the principal multiplied by the rate multiplied by the number expressing the time in years.

Find the interest on :

NOTE. — *Estimate* results and compare with answers to written work.

2. $500 for 3 yr. at 6 %

3. $400 for 4 yr. at 4 %

4. $600 for 2 yr. at 3 %

5. $900 for 3 yr. at 5 %

6. $250 for 5 yr. at 2 %

7. $325 for 2 yr. at 4 %

8. $460 for 2 yr. at 8 %

9. $615 for 4 yr. at 6 %

10. $575 for 3 yr. at 5 %

11. $835 for 1 yr. at 7 %

12. $318 for 2 yr. at 6 %

13. $740 for 5 yr. at 4 %

14. $635 for 2 yr. at 7 %

15. $775 for 3 yr. at 5 %

16. $936 for 4 yr. at 6 %

17. $705 for 2 yr. at 8 %

18. $896 for 3 yr. at 4 %

19. $1200 for 1 yr. at $2\frac{1}{2}$ %

20. $1600 for 2 yr. at $3\frac{1}{2}$ %

21. $2400 for 5 yr. at $2\frac{1}{4}$ %

22. $5240 for 3 yr. at $5\frac{1}{2}$ %

23. $3180 for 4 yr. at $4\frac{1}{2}$ %

24. $4760 for 2 yr. at $3\frac{1}{2}$ %

25. $5648 for 5 yr. at $2\frac{1}{4}$ %.

26. $38.00 for 4 yr. at 3 %

27. $36.50 for 1 yr. at 2 %

28. $27.40 for 2 yr. at 5 %

29. $43.75 for 5 yr. at 4 %

30. $57.50 for 3 yr. at 6 %

31. $64.25 for 4 yr. at 8 %

32. $83.50 for 2 yr. at 4 %

33. $75.50 for 5 yr. at 2 %

34. $98.50 for 2 yr. at 6 %

35. $70.80 for 3 yr. at 5 %

Finding interest for years and months.   ·

<div align="center">Oral Exercises</div>

Find the interest for 6 months ($\frac{1}{2}$ year) on :

1. \$100 at 4 %	**3.** \$800 at 3 %	**5.** \$3000 at 2 %
2. \$400 at 7 %	**4.** \$700 at 6 %	**6.** \$5000 at 4 %

Find the interest for 4 months ($\frac{1}{8}$ year) on :

7. \$200 at 3 %	**9.** \$50 at 3 %	**11.** \$240 at 2 %
8. \$400 at 6 %	**10.** \$90 at 4 %	**12.** \$120 at 5 %

Find the interest for 3 months ($\frac{1}{4}$ year) on :

13. \$300 at 4 %	**15.** \$110 at 8 %	**17.** \$2000 at 5 %
14. \$600 at 2 %	**16.** \$250 at 4 %	**18.** \$2400 at 2 %

Find the interest for 2 months ($\frac{1}{6}$ year) on :

19. \$500 at 6 %	**21.** \$60 at 5 %	**23.** \$1200 at 2 %
20. \$400 at 3 %	**22.** \$80 at 3 %	**24.** \$4000 at 9 %

<div align="center">·</div>

<div align="center">Written Exercises</div>

1. Find the amount of \$284.60 for 2 yr. 8 mo. at 5 %.

<div align="center">Solution</div>

Principal,	\$284.60	
Rate,	.05	
Interest for 1 year,	\$14.2300	
	2⅔	(2 yr. 8 mo. = 2⅔ yr.)
Interest for 2 years,	\$28.46	
Interest for ⅔ of a year,	9.487	(to the nearest mill)
Interest for 2⅔ years,	\$37.95	(to the nearest cent)
Principal,	284.60	
Amount,	\$322.55	

Note. — In computing interest, it is sufficiently accurate to express inter-
mediate results to the nearest mill and the final result to the nearest cent.

Find the interest: Find the amount:

	PRINCIPAL	TIME	RATE		PRINCIPAL	TIME	RATE
2.	$130	1½ yr.	5%	28.	$1200	1 yr. 9 mo.	6%
3.	$175	1⅓ yr.	4%	29.	$1900	2 yr. 6 mo.	3%
4.	$250	2½ yr.	2%	30.	$3500	2 yr. 4 mo.	2%
5.	$460	3¼ yr.	6%	31.	$2700	1 yr. 3 mo.	6%
6.	$350	2¾ yr.	3%	32.	$4300	4 yr. 4 mo.	5%
7.	$620	4⅛ yr.	4%	33.	$3580	3 yr. 9 mo.	4%
8.	$450	3¾ yr.	5%	34.	$5025	4 yr. 3 mo.	3%
9.	$375	2¼ yr.	6%	35.	$3750	2 yr. 9 mo.	2%
10.	$520	4⅜ yr.	3%	36.	$4680	2 yr. 3 mo.	2½%
11.	$275	3⅖ yr.	2%	37.	$6125	3 yr. 6 mo.	3½%
12.	$525	1¾ yr.	8%	38.	$8075	1 yr. 8 mo.	2¼%
13.	$895	2¾ yr.	6%	39.	$5915	3 yr. 4 mo.	4½%
14.	$618	4¾ yr.	5%	40.	$7162	4 yr. 6 mo.	5½%
15.	$370	6 mo.	2%	41.	$43.50	5 yr. 3 mo.	8%
16.	$465	3 mo.	4%	42.	$25.75	2 yr. 4 mo.	2%
17.	$190	2 mo.	5%	43.	$37.60	3 yr. 8 mo.	4%
18.	$572	4 mo.	6%	44.	$53.12	4 yr. 4 mo.	5%
19.	$325	8 mo.	8%	45.	$67.25	2 yr. 8 mo.	6%
20.	$450	9 mo.	3%	46.	$45.45	3 yr. 2 mo.	3%
21.	$650	6 mo.	5%	47.	$72.25	4 yr. 8 mo.	5%
22.	$925	2 mo.	3%	48.	$38.75	2 yr. 9 mo.	2%
23.	$775	4 mo.	4%	49.	$56.35	3 yr. 3 mo.	4%
24.	$496	3 mo.	2%	50.	$85.40	1 yr. 4 mo.	3%
25.	$842	8 mo.	8%	51.	$63.55	4 yr. 6 mo.	6%
26.	$638	9 mo.	6%	52.	$91.15	3 yr. 8 mo.	5%
27.	$574	10 mo.	7%	53.	$75.50	2 yr. 2 mo.	4%

Finding interest for years, months, and days.

1. Find the interest on $ 650 for 2 yr. 4 mo. 15 da. at 6 %.

SOLUTION

Since 15 da. is $\frac{1}{2}$ mo., 4 mo. 15 da. $= 4\frac{1}{2}$ mo.; since $4\frac{1}{2}$ mo. is $\frac{3}{8}$ yr., 2 yr. 4 mo. 15 da. $= 2\frac{3}{8}$ yr.

Then, the interest $= 2\frac{3}{8} \times 6 \%$ of $ 650

$$= \frac{19}{8} \times \frac{\cancel{6}^{3}}{\cancel{100}_{\cancel{50}}} \times \$ \cancel{650}^{13} = \frac{\$ 741}{8} = \$ 92.62\frac{1}{2}, \text{ or } \$ 92.63.$$

The rate and time are expressed as common fractions, and the process is *shortened* by cancellation. Inaccuracies arising from inexact intermediate results are thus avoided.

Find the interest, at the given rate, for the time indicated, on:

2–3. $ 500, 4 %, 1 yr. 5 mo. 10 da. ; 1 yr. 3 mo. 6 da.

4–5. $ 150, 5 %, 2 yr. 2 mo. 12 da. ; 4 yr. 4 mo. 15 da.

6–7. $ 350, 2 %, 4 yr. 7 mo. 15 da. ; 3 yr. 6 mo. 20 da.

8–9. $ 425, 6 %, 3 yr. 4 mo. 24 da. ; 4 yr. 1 mo. 18 da.

10–11. $ 275, 3 %, 2 yr. 9 mo. 10 da. ; 3 yr. 5 mo. 12 da.

12–13. $ 560, 6 %, 3 yr. 1 mo. 15 da. ; 5 yr. 9 mo. 18 da.

14–15. $ 680, 8 %, 5 yr. 2 mo. 20 da. ; 4 yr. 1 mo. 24 da.

16–17. $ 750, 5 %, 1 yr. 3 mo. 18 da. ; 2 yr. 10 mo. 15 da.

Find the amount of :

18–19. $ 300, 5 %, 1 yr. 1 mo. 6 da. ; 1 yr. 7 mo. 6 da.

20–21. $ 250, 6 %, 3 yr. 1 mo. 10 da. ; 2 yr. 4 mo. 24 da.

22–23. $ 460, 4 %, 2 yr. 8 mo. 24 da. ; 3 yr. 6 mo. 12 da.

24–25. $ 575, 2 %, 4 yr. 5 mo. 18 da. ; 5 yr. 8 mo. 12 da.

26–27. $ 340, 3 %, 5 yr. 7 mo. 15 da. ; 4 yr. 10 mo. 20 da.

28–29. $ 775, 6 %, 2 yr. 10 mo. 12 da. ; 3 yr. 10 mo. 24 da.

Determine the time between dates as shown on page 146, and find the interest on :

30. $300, at 4 %, from May 7, 1914 to June 22, 1915.

31. $500, at 6 %, from Aug. 5, 1913 to Oct. 17, 1916.

32. $450, at 3 %, from Apr. 9, 1912 to Aug. 24, 1914.

33. $625, at 5 %, from Jan. 29, 1914 to Mar. 5, 1915.

34. $540, at 2 %, from June 3, 1915 to Apr. 18, 1917.

35. $775, at 4 %, from Sept. 20, 1913 to Aug. 8, 1916.

36. $1500, at 6 %, from Mar. 8, 1913 to June 26, 1916.

37. $3250, at 5 %, from Jan. 4, 1914 to May 19, 1915.

38. $4725, at 4 %, from Apr. 26, 1915 to Sept. 20, 1917.

39. $25.00, at 3 %, from July 8, 1913 to Feb. 23, 1915.

40. $42.25, at 4 %, from May 8, 1914 to Nov. 28, 1916.

41. $53.50, at 6 %, from Oct. 14, 1912 to Jan. 20, 1915.

Find the amount of :

42. $600, at 2 %, from Feb. 4, 1913 to Sept. 10, 1916.

43. $850, at 3 %, from Nov. 5, 1914 to June 20, 1916.

44. $725, at 6 %, from Mar. 22, 1912 to July 7, 1914.

45. $940, at 5 %, from Aug. 12, 1914 to Apr. 24, 1917.

46. $34.50, at 4 %, from June 8, 1913 to Oct. 6, 1915.

47. $65.80, at 5 %, from Sept. 1, 1912 to July 25, 1916.

48. $74.75, at 8 %, from July 8, 1915 to Oct. 14, 1918.

49. $93.25, at 8 %, from Oct. 25, 1913 to Apr. 13, 1917.

50. $235.50, at 2 %, from Jan. 6, 1914 to Nov. 21, 1918.

51. $510.56, at 6 %, from May 5, 1912 to Oct. 17, 1915.

52. $845.75, at 8 %, from June 9, 1913 to Mar. 19, 1917.

53. $650.25, at 5 %, from Dec. 28, 1915 to Feb. 4, 1918.

PROMISSORY NOTES

Edward Payne buys of John Barton silk costing $5000, and promises to pay for it in three months, with interest at 6%. He then gives Mr. Barton a *note* as follows:

$5000.   *Albany, A. Y., Oct. 9, 1914.*

----Three months----after date----I----promise to pay to the order of John Barton.---------------------------------------

Five thousand and $\frac{00}{100}$~~~~~~~~~~~~~~~~Dollars.

    Value received, with interest at 6%.

                         Edward Payne.

A written promise made by one person to pay to another a definite sum of money at a specified time is a **promissory note**, or simply a **note**.

The *essentials* of a note are as follows:

1. It must be signed by the person who promises to pay. This person is the **maker**, or **drawer**.

2. It must designate by name, or otherwise, the person to whom the money is to be paid. This person is the **payee**.

3. The *sum* to be paid must be definite. It is the **face** of the note.

The face of a note is usually written both in words and in figures.

4. The *time* of payment must be definite.

It may be payable *on demand*; at a *specified time* after date, which must appear on the note; on or before a *specified date*; etc. When no time of payment is mentioned, the note is payable on demand.

A promise to pay " when trade will permit " is not binding.

NOTES.— 1. Unless the *place* of payment is stated, the residence or place of business of the maker is understood, and this must appear on the note.

2. When a note reads "with interest," but does not give the rate of interest, the legal rate in the state where the note was made may be collected.

The legal rate of interest in New York state is 6 %.

3. If a note does not contain the words "with interest," no interest may be collected for the time it has to run; but if the note is not paid when due, it then begins to draw interest at the legal rate.

4. The words "value received" are usually included in a note, though they are not essential.

The note on page 229 is a **time note** because it is payable *three months after date*. A note, like the following, which is payable *on demand*, is a **demand note**.

$300.⁵⁰   *Buffalo, N. Y., Sept. 1, 1914.*

On demand   I   promise to pay to _____

Mary Glover_____or order,

Three hundred and $\frac{50}{100}$ ~~~~~~~~~~~~~~~*Dollars.*

Value received.   *James Ward.*

The words "or order" mean that James Ward will pay $300.50 to any one to whom Mary Glover may order it paid.

A note payable "to bearer" is payable to any one who presents it.

A note that is or may be made payable to a person other than the payee may be sold and is said to be **negotiable**.

If a note is payable "to bearer," the payee may sell it, or *negotiate* it, by delivering it to the buyer, and any buyer may negotiate it in the same way.

If a note is payable "to order," a simple delivery is not sufficient to negotiate it, for it is payable to the payee until he orders it paid to another.

If a note is payable to the payee only, no words "to order" or "to bearer" being inserted, it is **non-negotiable**.

The payee of a note may order it paid to another person by writing his name on the back of it together with any instructions or conditions he deems best. This is **indorsing** the note.

The person who indorses a note is an **indorser**; the person who lawfully owns a note is the **holder**. The payee of a note is the first holder; any purchaser of it becomes the holder.

The principal forms of indorsement are illustrated below:

1. Suppose that John Barton, the payee of the note on page 229, sells the note to Oliver Evans and indorses it **in blank**.

This indorsement is really an agreement between Mr. Barton and all future holders of the note to this effect: "For value received, I, John Barton, transfer to the *bearer* all my right and title in this note. I guarantee that it will be paid when due, if not by the maker, then by myself."

INDORSEMENT IN BLANK

John Barton.

2. If Mr. Barton indorses in full when he sells the note to Mr. Evans, no one except Mr. Evans can negotiate it, for the indorsement makes the note payable to his order.

This form of indorsement also guarantees payment. It is useful in sending a note through the mail, for if lost or stolen, it is of no value to the person obtaining it.

INDORSEMENT IN FULL

Pay to the order of Oliver Evans.
John Barton.

3. Suppose that Mr. Barton wishes to avoid responsibility for the payment of the note. He will write "without recourse" over his name.

- This is a **qualified** indorsement *in blank*.

A qualified indorsement may be made either *in blank* or *in full*.

QUALIFIED INDORSEMENT

Without recourse.
John Barton.

When a note becomes payable it is said to **mature.**

In a few states a note matures three days after the time specified therein. Most states including New York, however, have abolished these **days of grace.**

If a note falls due on Sunday, a Saturday half-holiday, or a legal holiday, it is usually payable on the next *succeeding* business day as in New York state; but in some states it must be paid on the preceding business day.

When the time of a note is given in *days*, the note matures on the expiration of the exact number of days named.

Thus, a note dated May 15, due 60 days after date, is due July 14.

When the time of a note is given in *months*, the note is due on the same date of the month of maturity as the date given on the note, if the month of maturity has such a date.

Thus, a note dated May 15 and due in 2 mo. is due July 15; but a note dated Dec. 29, 30, or 31 and due in 2 mo. is due the last day of February.

If, when a note is unpaid at its maturity, the holder fails to protest it, that is, to notify the indorsers in a manner prescribed by law that it is unpaid, they are released from responsibility regarding its payment.

Oral Exercises

Examine the note on page 229 and answer these questions :

1. Who is the maker? the payee?

2. Where is the note payable?

3. What is the face of the note? the date?

4. How long does the note have to run?

5. What is the date of maturity of the note?

6. Is the note negotiable or non-negotiable? interest-bearing or non-interest-bearing?

7. Answer the same questions in regard to the note on p. 230.

Answer questions 1–6 in regard to each of these notes :

8. *$500.*   *Ithaca, N. Y., Dec. 16, 1914.*

Ninety days after date, I promise to pay to F. W. Thayer, or bearer, Five hundred Dollars, with interest.   Value received.

                                        James Marshall.

9. *$2000.*   *Buffalo, N. Y., Nov. 20, 1914.*

Three months after date, for value received, I promise to pay to Monroe Havens, Two thousand Dollars.

                                        Homer Smith.

1. Write a negotiable note for $1000 with yourself as maker, and J. J. Barber as payee, due in 1 yr. with interest at 6%. Find the amount due when the note matures.

2. Write a non-negotiable note for $200 payable to yourself by John H. Fassett, without interest.

Write notes from the following data, supplying the year, and find the amount due on each at maturity:

DATE	FACE	TIME	PAYEE	MAKER	RATE
8. Jan. 5	$150	6 mo.	A. B. Bent	J. C. Bolt	5%
4. May 2	$500	4 mo.	E. D. Gage	Eva Hunt	6%
5. Aug. 3	$200	1 yr.	Benn Hall	Ray Ford	4%
6. Oct. 7	$600	2 mo.	C. L. Ames	E. A. Day	5%
7. Feb. 6	$800	2 yr.	O. C. Burt	F. R. Earle	6%
8. Nov. 18	$950	60 da.	Roy Jones	M. Avery	7%
9. Apr. 24	$480	30 da.	N. Smith	Ira Butts	3%
10. Dec. 15	$398	90 da.	A. R. Salls	S. Brown	8%
11. Mar. 30	$765	3 mo.	F. Lucas	Earl Bell	6%

12. Supplying data, write a negotiable note with two indorsements, the first *in full*, the second *in blank.*

13. Write a note with two indorsements in full, the second being *without recourse.*

14. Write two negotiable notes differing in form, for $795.36, payable by yourself, in 3 mo., to John Fenton, with interest.

15. Properly indorse the notes in exercise **14** to transfer one to the bearer and the other to Richard Gray or order.

16. Suppose that you have sold Scott & Lyons goods for $1000, terms half cash and half payable in 60 days. Draw the note, with their signature, making it negotiable. Indorse it to J. West so that you will not be responsible for its payment.

Find the date of maturity and the amount due at maturity :

	DATE	FACE	TIME	RATE
17.	Aug. 2, 1913	$700	6 mo.	6%
18.	Sept. 12, 1914	$400	90 da.	5%
19.	May 6, 1912	$550	3 mo.	4%
20.	July 14, 1913	$780	4 mo.	4½%
21.	June 16, 1915	$950	60 da.	5½%
22.	Feb. 27, 1914	$125.50	5 mo.	6%
23.	Mar. 11, 1913	$150.80	30 da.	5½%
24.	Oct. 7, 1915	$960.75	2 yr.	5%
25.	Apr. 23, 1914	$645.25	120 da.	3½%

26. On Aug. 5, 1914, Dugan & Co. bought hay of R. Day to the value of $800. They paid half cash and gave a note for the rest payable to R. Day, or bearer, in 30 days. Write the note.

27. Loren Niles borrowed $150 of Royal Gordon, July 10, 1914, and gave his note payable to Royal Gordon, or order, in 90 days with interest at 5%. How much was due Mr. Gordon when the note matured? Write the note.

28. On Aug. 18, 1914, W. Rogers gave F. C. Hayes a note for $2500 payable 6 mo. from date with interest at 5%, but the note was not paid until May 18, 1915. Find the amount paid.

29. Sept. 2, 1914, in payment of a debt to Martin Cole, Charles Hutt gave two negotiable notes, one for $1500 payable in 1 yr. with interest at 6%, the other for $1250 payable in 60 days with interest at 4%. Write the notes. Find the amount of each at maturity.

30. Joseph Somers gave Henry Finch a note for $350, dated Aug. 1, 1914, payable on demand. Payment was demanded Sept. 14, but was not made until Dec. 1. Write the note. How long should it bear interest? Find the amount due when the note was paid, the legal rate of interest being 6%.

PROBLEMS IN INTEREST

Finding the rate.

1. What is the interest on $100 for 2 yr. at 1%?

2. Since $100 yields $2 interest in 2 yr. at 1%, at what rate will it yield $4 interest in the same time? $6 interest?

3. At what rate will $300 yield $27 interest in 3 yr.?

Written Exercises

1. At what rate must $800 be invested to yield an interest of $176 in 5 yr. 6 mo.?

SOLUTIONS

(a) The interest on $800 at 1% for 5½ years is 5½ × 1% of $800, or $44. Since a rate of 1% for the given time yields $44, it will require a rate of as many per cent to yield $176 as $44 is contained times in $176, or 4%.

(b) Let x = the rate. Then, the interest on $800 for 5½ years is $5\frac{1}{2} \times \frac{x}{100}$ of $800, or 44 x dollars. Since this interest is 176 dollars, 44 x = 176. Therefore, $x = 4$, and the rate is 4%.

The rate equals the given interest divided by the interest for the given time at 1%.

Find the rate at which the given principal (P) will yield the given interest (I) in the given time (T):

	P	I	T		P	I	T
2.	$500	$50	2 yr.	8.	$600	$126	5 yr. 3 mo.
3.	$300	$60	4 yr.	9.	$900	$180	3 yr. 4 mo.
4.	$400	$72	3 yr.	10.	$2600	$234	2 yr. 3 mo.
5.	$700	$70	2½ yr.	11.	$4000	$900	4 yr. 6 mo.
6.	$650	$130	5 yr.	12.	$4500	$585	2 yr. 2 mo.
7.	$2000	$210	3½ yr.	13.	$2250	$765	5 yr. 8 mo.

14. At what rate will $600 amount to $720 in 4 yr.?

15. At what rate will $3200 amount to $3616 in 3 yr. 3 mo.?

Finding the time.

1. What interest will $100 earn in 1 yr. at 6%?

In what time will $100 at 6% earn $6? $12? $18?

2. What interest will $200 earn in 1 yr. at 6%?

In what time will $200 at 6% earn $12? $24? $18? $30?

Written Exercises

1. In what time will $750 at 6% yield $250 interest?

Solutions

(a) The interest for 1 yr. at 6% is 6% of $750, or $45.

Hence, the time required is $\frac{250}{45}$ yr., or $5\frac{5}{9}$ yr., or 5 yr. 6 mo. 20 da.

(b) Let x = the number of years. Then, $\frac{6x}{100} \times 750 = 250$.

That is, $45x = 250$.

Therefore, $x = 5\frac{5}{9}$, and the time is $5\frac{5}{9}$ yr., or 5 yr. 6 mo. 20 da.

Hence, $750 at 6% will yield $250 in 5 yr. 6 mo. 20 da.

The time (in years) equals the given interest divided by the interest at the given rate for 1 year.

Find the time in which the given principal (P) will yield the given interest (I) at the given rate (R):

P	I	R		P	I	R		P	I	R
2. $50	$4	4%		**5.** $600	$72	3%		**8.** $2600	$234	6%
3. $40	$8	5%		**6.** $750	$60	6%		**9.** $4000	$350	5%
4. $80	$6	6%		**7.** $960	$84	5%		**10.** $6500	$325	8%

11. In what time will any principal double itself at 4%?

Solution.—To double itself, a principal must gain 100% of itself in interest; $100\% + 4\% = 25$. Hence, the time is 25 years.

In how many years will any principal double itself at

12. 5%?   **13.** 2%?   **14.** 6%?   **15.** 8%?   **16.** $4\frac{1}{2}$%?

17. How long will it take $600 to amount to $1000 at 6%?

18. How long will it take $1800 to amount to $2000 at 4%?

Finding the principal.

1. What principal at 6 % in 1 yr. will earn $6? $12? $18?

2. What principal at 5 % in 2 yr. will earn $100? $200?

1. What principal at 5 % in 2 yr. will yield $500 interest?

SOLUTIONS

(*a*) Since $1 at 5% in 2 yr. yields $.10 interest, it will take as many dollars of principal to yield $500 interest as $.10 is contained times in $500, which is 5000. Hence, the principal is $5000.

(*b*) Let x = number of dollars in the principal. Then, $2 \times .05 x = 500$.
That is, $.10 x = 500.$
Therefore, $x = 500 \div .10 = 5000.$
Hence, the principal required is $5000.

The principal equals the given interest divided by the interest on $1 for the given time at the given rate.

What principal will yield an annual income of

2. $125 at 5 %?   **4.** $750 at 6 %?   **6.** $1000 at 2 %?

3. $500 at 4 %?   **5.** $900 at 4½ %?   **7.** $3000 at 7½ %?

Find the principal that will yield the given interest (I) in the given time (T) at the given rate (R):

	I	T	R		I	T	R
8.	$24	2 yr.	6 %	**13.**	$100	60 da.	5 %
9.	$90	3 yr.	3 %	**14.**	$120	1½ yr.	4 %
10.	$18	5 yr.	6 %	**15.**	$390	3¼ yr.	4 %
11.	$100	4 yr.	5 %	**16.**	$500	2 yr. 6 mo.	4 %
12.	$256	2 yr.	8 %	**17.**	$480	3 yr. 4 mo.	6 %

18. What principal will amount to $504 in 2 yr. at 6 %?

SUGGESTION. — Since in 2 yr. at 6% $1 amounts to $1.12 and the required principal to $504, the required principal = ($504 ÷ $1.12) times $1.

19. What principal will amount to $828 in 3¾ yr. at 4 %?

Miscellaneous Exercises

Estimate, then find the interest on :

1. $400 for 2 yr. at 5 %
2. $570 for 3 yr. at 3%
3. $480 for 4 yr. at 6 %
4. $648 for 6 mo. at 7%
5. $720 for 4 mo. at 4 %
6. $840 for 8 mo. at 3 %
7. $975 for 9 mo. at 6 %

8. $660 for 15 dà. at 3 %
9. $812 for 20 da. at 6%
10. $900 for 12 da. at 5 %
11. $1100 for 1½ yr. at 6 %
12. $1200 for 2¼ yr. at 4 %
13. $1179 for 3⅓ yr. at 5 %
14. $1480 for 2¾ yr. at 5½ %

15. $16.20 for 2 yr. 6 mo. at 5 %
16. $63.50 for 1 yr. 8 mo. at 4 %
17. $72.80 for 3 yr. 9 mo. at 6 %
18. $84.52 for 2 yr. 2 mo. at 5 %
19. $76.65 for 1 yr. 4 mo. at 4 %
20. $88.20 for 2 yr. 3 mo. at 6 %

21–23. $450 for 1 yr. 6 mo. 20 da. at 6 % ; at 4 % ; at 5 %
24–26. $576 for 2 yr. 8 mo. 12 da. at 3 % ; at 7 % ; at 4 %
27–29. $660 for 3 yr. 5 mo. 18 da. at 5 % ; at 6 % ; at 8 %
30–32. $747 for 1 yr. 9 mo. 24 da. at 7 % ; at 4 % ; at 6 %
33–35. $936 for 4 yr. 3 mo. 15 da. at 4 % ; at 8 % ; at 5 %

Find the date of maturity and the amount due at maturity :

	DATE	FACE	TIME	RATE
36.	Apr. 8, 1914	$600	2 yr.	5 %
37.	May 14, 1913	$540	6 mo.	4 %
38.	Aug. 10, 1914	$690	4 mo.	6 %
39.	July 29, 1914	$450.80	9 mo.	3½ %
40.	Oct. 15, 1913	$620.60	15 da.	4½ %
41.	Dec. 24, 1915	$840.75	20 da.	5½ %

Fill the blank spaces in the following:

	PRINCIPAL	RATE	TIME	INTEREST	AMOUNT
42.	$320	6%	2 yr.	——	——
43.	$480	——	1 yr. 6 mo.	$36	——
44.	——	7%	2 yr. 3 mo.	$88.20	——
45.	$675	4%	——	——	$765
46.	$720	——	1 yr. 9 mo.	——	$757.80
47.	$840	6%	——	$170.10	——
48.	——	5%	2 yr. 6 mo.	$116.25	——
49.	$1200	——	1 yr. 2 mo.	——	$1312
50.	$1280	4%	——	$166.40	——
51.	——	6%	2 yr. 5 mo.	——	$1603
52.	$1460	——	4 yr. 7 mo.	$200.75	——
53.	$1650	5%	——	——	$1963.50
54.	——	7%	2 yr. 8 mo.	$230.72	——
55.	$1860	——	3 yr. 4 mo.	——	$2077
56.	$2450	4%	——	$465.50	——

57. The annual net income of a farm that cost $10,500 was $525. What rate of interest did the farmer's investment earn?

58. A man borrowed money at 7½%, and retained it until the interest equaled the principal. How long did he have it?

59. I deposited $3200 in a bank and 6 mo. later was credited with $56 interest. What rate of interest was allowed?

60. A man borrowed $3000 and gave a written promise to pay the lender $3050 in 4 mo. How much of the $3050 was interest? What rate of interest was paid?

61. A man bequeathed to a college enough money so that the interest on it at 3½% would provide for 10 annual scholarships of $350 each. Find the amount of the gift or endowment.

62. At what rate will $2000 amount to $2400 in 5 yr.?

63. What principal will yield $1000 interest in 60 da. at 5%?

64. A man lived on the income from $50,000, ¼ of it being invested at 7 % and the rest at 5 %. Find his annual income.

65. If $475 is put at interest at 6 % May 15, 1914, when will it amount to $489.25?

66. James Wright borrowed $1550 on April 1. On Oct. 1 following he paid the amount, which was $1596.50. What rate of interest did he pay?

67. A resident of Colorado Springs borrowed money in the East at 5 % and loaned it on good security at home at 8 %. Find his annual gain on an investment of $75,000.

68. A house that cost $5400 rented for $576 per year. Taxes and repairs cost the owner $157.50 a year. Find the net annual income and the rate of interest realized on the investment.

69. D. Clark bought a farm of O. Engell for $3750. He paid $1250 cash and gave a note for the rest payable in 1 yr. 6 mo. with interest at 4 %. Find the amount of the note at maturity.

70. A coal dealer purchased his coal at $3.20 per ton cash on April 1, and sold it 6 mo. later. If money was worth 5 %, what was the real cost of the coal to him at the time of sale?

71. The profits earned by a power plant in a year, after deducting all expenses and allowing for depreciation of the property, were $8187.50. The plant cost $25,000. What rate of interest was earned on the investment?

72. On Oct. 15, Albert Wilcox borrowed $500 of William Harris. He gave his note payable to William Harris, or order, in 9 mo. with interest at 5 %. Mr. Harris sold the note to Henry Hines. Draw the note and indorse it in full. Find the amount due at maturity.

SIX PER CENT METHOD

Much of the money lent in the money market is lent for terms of 90 days, 60 days, 30 days, or less. For such terms and a rate of 6 % the most convenient unit of time is 2 months, or **60 days,** ⅙ of a year.

The interest for 60 *days at* 6 % *is* 1 % *of the principal, found by moving the decimal point two places toward the left.*

By adding or subtracting parts or multiples of the interest for 60 days, the interest for any term may be found.

The following processes illustrate the method of finding the interest at 6 % on $5000, for the terms mentioned, from the interest for 60 days :

80 days	10 days	6 days	5 days
2)$50.00	6)$50.00	10)$50.00	12)$50.00
$25.00	$ 8.33	$ 5.00	$ 4.17

90 days	98 days	11 days	125 days
2)$50.00	2)$50.00	6)$50.00	$50.00
25.00	10) 25.00	10) 8.333	12) 50.00
$75.00	2.50	.833	4.17
	$77.50	$ 9.17	$104.17

This method is sometimes known as the **banker's method.**

Oral Exercises

Find the interest at 6 % on :

1. $100 for 60 da.   7. $600 for 30 da.   13. $6000 for 5 da.
2. $400 for 30 da.   8. $500 for 60 da.   14. $4000 for 6 da.
3. $200 for 90 da.   9. $800 for 90 da.   15. $2000 for 15 da.
4. $300 for 60 da.   10. $900 for 20 da.   16. $3000 for 10 da.
5. $800 for 80 da.   11. $400 for 15 da.   17. $5000 for 12 da.
6. $600 for 90 da.   12. $300 for 30 da.   18. $9000 for 20 da.

Written Exercises

Estimate, then find the interest at 6 % on :

1. $3000 for 40 da.	**10.** $8000 for 3 da.	**19.** $700 for 33 da.
2. $6000 for 70 da.	**11.** $9000 for 8 da.	**20.** $660 for 22 da.
3. $9000 for 24 da.	**12.** $6000 for 7 da.	**21.** $450 for 72 da.
4. $4000 for 36 da.	**13.** $5000 for 9 da.	**22.** $720 for 65 da.
5. $8000 for 45 da.	**14.** $7200 for 90 da.	**23.** $650 for 48 da.
6. $5000 for 42 da.	**15.** $2500 for 13 da.	**24.** $840 for 96 da.
7. $6000 for 80 da.	**16.** $4200 for 25 da.	**25.** $360 for 85 da.
8. $3000 for 34 da.	**17.** $3500 for 16 da.	**26.** $850 for 120 da.
9. $2000 for 72 da.	**18.** $8300 for 80 da.	**27.** $390 for 100 da.

From the interest on any principal at 6 % the interest at other rates may be found by adding or subtracting certain parts of the interest at 6 %, as here illustrated :

Adding	$\frac{1}{6}$ of itself for the interest at 7 %.
Subtracting	$\frac{1}{6}$ of itself for the interest at 5 %.
Adding	$\frac{1}{3}$ of itself for the interest at 8 %.
Subtracting	$\frac{1}{3}$ of itself for the interest at 4 %.
Adding	$\frac{1}{4}$ of itself for the interest at $7\frac{1}{2}$ %.
Subtracting	$\frac{1}{4}$ of itself for the interest at $4\frac{1}{2}$ %.
Dividing by	2 for the interest at 3 %.

Find the interest on :

28. $250 for 60 da. at 7 %	**36.** $840 for 12 da. at $7\frac{1}{2}$ %
29. $500 for 30 da. at 5 %	**37.** $450 for 15 da. at $4\frac{1}{2}$ %
30. $400 for 90 da. at 8 %	**38.** $750 for 10 da. at 4 %
31. $500 for 60 da. at 4 %	**39.** $800 for 45 da. at 3 %
32. $160 for 63 da. at 3 %	**40.** $900 for 25 da. at 5 %
33. $775 for 33 da. at 4 %	**41.** $650 for 150 da. at 8 %
34. $640 for 18 da. at 8 %	**42.** $500 for 120 da. at $4\frac{1}{2}$ %
35. $850 for 70 da. at 5 %	**43.** $800 for 125 da. at $7\frac{1}{2}$ %

For convenience, bankers often use the following *table of days intervening between dates:*

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
Mar.	306	337	365	31	61	92	122	153	184	214	245	275
Apr.	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	333	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

The number of days from any day of one month to the *same* day of another month is found by starting at the name of the first (in the left-hand column) and following across to the column headed with the name of the second. (Should Feb. 29th of a leap year intervene between dates, add 1 day.)

Suppose it is required to find the number of days from June 5 to Nov. 10. From the table we find that it is 153 days from June 5 to Nov. 5; adding 5 days, we find the required time to be 158 days.

Written Exercises

Find the interest at 6 % for the terms mentioned :

1. $5000, Mar. 6 to May 5
2. $3000, June 2 to July 5
3. $5000, July 5 to Nov. 2
4. $4500, Aug. 8 to Oct. 7
5. $7225, Sept. 1 to Dec. 3
6. $8400, Nov. 30 to Feb. 1
7. $6300, Dec. 14 to June 12
8. $9475, July 17 to Mar. 14

9. $1600, July 25 to Nov. 22
10. $22.50, June 30 to Aug. 2
11. $35.75, Sept. 26 to Nov. 15
12. $15,000, Aug. 21 to Sept. 4
13. $50,000, Dec. 22 to Jan. 7
14. $35,000, Sept. 7 to Apr. 4
15. $125,000, May 4 to July 1
16. $500,000, Nov. 6 to Feb. 2

The following modification of the six per cent method is often used, the work being further shortened by **cancellation**.

When the rate is 6 %, the interest for 60 days is 1 % of the principal, and for 6 days the interest is $\frac{1}{10}$ % of the principal, or .001 of the principal.

The interest at 6 % for any number of days is .001 of the principal multiplied by the number of days divided by 6.

The interest at any other rate is found by taking such a part of the interest at 6 % as the given rate is of 6 %.

Written Exercises

1. Find the interest on $6228 for 93 days at 6 % ; also at 4 %.

SOLUTIONS

$$\$1.038 \atop \$\cancel{6.228} \times \dfrac{93}{\cancel{6}} =$$

$$\$1.038 \times 93 = \$96.53$$

$$\$1.038 \atop \$\cancel{6.228} \times \dfrac{\overset{31}{\cancel{93}}}{\cancel{6}} \times \dfrac{\overset{2}{\cancel{4}}}{\underset{2}{\cancel{6}}} =$$

$$\$1.038 \times 31 \times 2 = \$64.36$$

Find, using cancellation, the interest on the following for the given time and rate :

2. $3000, 12 days, 6 %

3. $5000, 8 days, 4 %

4. $7500, 15 days, 3 %

5. $4365, 60 days, 5 %

6. $8000, 10 days, 4 %

7. $7200, 90 days, 4 %

8. $3420, 42 days, 6 %

9. $6250, 36 days, 4½ %

10. $7200, 93 days, 3½ %

11. $5820, 80 days, 5½ %

12. $2400, July 6 to Oct. 4, 5 %

13. $6300, June 4 to Aug. 3, 4 %

14. $7800, Nov. 10 to Feb. 10, 3 %

15. $8100, Dec. 12 to Feb. 1, 6 %

16. $9600, May 16 to Aug. 14, 5 %

17. $2700, Mar. 1 to June 2, 7 %

18. $75.60, Apr. 1 to June 30, 7½ %

19. $43.20, Nov. 4 to Jan. 3, 4½ %

20. $85.50, Aug. 7 to Dec. 11, 5 %

21. $64.80, Mar. 10 to June 8, 4 %

Another form of the *six per cent method* often used, especially when the time is not expressed in days, makes the interest on $1 for the given time the basis of computation.

Thus, the interest on $1 for 1 year at 6% is $.06; for 1 month, $.005; for 6 days, $.001; and for 1 day, $ 000⅙.

Written Exercises

1. Find the interest on $240.50 for 2 yr. 4 mo. 14 da. at 6%.

SOLUTION

The interest on $1 for 2 yr.   = $.12
The interest on $1 for 4 mo.   = .02
The interest on $1 for 14 da.   = .002⅓
The interest on $1 for 2 yr. 4 mo. 14 da. = $.142⅓
The interest on $240.50 = 240.50 × $.142⅓, or $34.28.

Find the interest at 6% on:

2. $42.50 for 7 yr. 4 mo. 20 da.

3. $39.75 for 5 yr. 3 mo. 15 da.

4. $425.10 for 6 yr. 5 mo. 10 da.

5. $365.42 for 8 yr. 10 mo. 24 da.

6. $875.50 for 4 yr. 11 mo. 18 da.

7. $920.65 for 4 yr. 10 mo. 17 da.

8. $1500 from Mar. 14, 1913 to Apr. 3, 1914.

9. $4800 from June 24, 1914 to Dec. 15, 1915.

10. $3200 from Dec. 18, 1912 to Feb. 24, 1914.

11. $64.45 from Feb. 28, 1914 to Oct. 31, 1915.

12. $75,000 from Aug. 16, 1913 to Feb. 12, 1917.

13. $56,000 from Feb. 24, 1914 to Dec. 17, 1916.

14. $375.60 from May 22, 1910 to Mar. 16, 1915.

15. $840.07 from Dec. 10, 1909 to Nov. 20, 1914.

16. $225,000 from Oct. 4, 1913 to Feb. 11, 1916.

17. $500,000 from Aug. 1, 1913 to Jan. 31, 1914.

From the interest at 6%, find the interest at the given rate:

18. $300 for 1 yr. 5 mo. at 5%.

19. $250 for 3 yr. 4 mo. at 8%.

20. $625 for 2 yr. 9 mo. at 4%.

21. $5000 for 3 yr. 5 mo. 10 da. at 5%.

22. $4000 for 2 yr. 7 mo. 24 da. at 8%.

23. $5000 for 4 yr. 9 mo. 11 da. at 8%.

24. $5000 for 4 mo. 2 da. at 3% ; at 3½ %.

25. $7500 for 2 mo. 5 da. at 5% ; at 5½ %.

26. $3685 from Feb. 17, 1913 to July 9, 1914 at 5%.

27. $2440 from June 11, 1912 to Sept. 8, 1913 at 4%.

28. $75.50 from Nov. 22, 1912 to May 10, 1913 at 7%.

29. $825.25 from Sept. 8, 1913 to Apr. 3, 1914 at 7½ %.

30. $25,000 from May 16, 1913 to Aug. 14, 1915 at 3%.

Using any convenient method, find the interest at 6%, 4%, 4½ %, 3%, and 5% on :

31. $5000 for 2 da.

32. $5000 for 5 da.

33. $5000 for 10 da.

34. $5000 for 30 da.

35. $5000 for 60 da.

36. $5000 for 90 da.

37. $6440 for 63 da.

38. $3200 for 75 da.

39. $5000 for 120 da.

40. $5000 for 125 da.

41. $133.20 for 21 da.

42. $786.32 for 27 da.

43. $2500 for 60 da.

44. $13,200 for 30 da.

45. $432.60 for 11 da.

46. $635.80 for 57 da.

47. $752.19 for 93 da.

48. $563.22 for 3 mo. 18 da.

49. $287.10 for 5 mo. 12 da.

50. $1000 for 1 yr. 2 mo. 9 da.

51. $1200 for 2 yr. 7 mo. 3 da.

52. $16,000, Feb. 1 to Feb. 28.

53. $45,000, Jan. 10 to Jan. 20.

54. $500,000, June 7 to July 1.

Those who have much interest to compute expedite their work by the use of **interest tables,** illustrated by the following incomplete table, based on a year of 360 days and a rate of 6 %:

Years	$1000	$2000	$3000	$4000	$5000	$6000	$7000	$8000	$9000
1	60	120	180	240	300	360	420	480	540
2	120	240	360	480	600	720	840	960	1080
3	180	360	540	720	900	1080	1260	1440	1620
...	...	...	...	...	...	...	...	...	...
Months	**$1000**	**$2000**	**$3000**	**$4000**	**$5000**	**$6000**	**$7000**	**$8000**	**$9000**
1	5	10	15	20	25	30	35	40	45
2	10	20	30	40	50	60	70	80	90
3	15	30	45	60	75	90	105	120	135
...	...	...	...	...	...	...	...	...	...
Days	**$1000**	**$2000**	**$3000**	**$4000**	**$5000**	**$6000**	**$7000**	**$8000**	**$9000**
1	.167	.333	.50	.667	.833	1.00	1.167	1.333	1.50
2	.333	.667	1.00	1.333	1.667	2.00	2.333	2.667	3.00
3	.500	1.000	1.50	2.000	2.500	3.00	3.500	4.000	4.50
4	.667	1.333	2.00	2.667	3.333	4.00	4.667	5.333	6.00
5	.833	1.667	2.50	3.333	4.167	5.00	5.833	6.667	7.50
6	1.000	2.000	3.00	4.000	5.000	6.00	7.000	8.000	9.00
7	1.167	2.333	3.50	4.667	5.833	7.00	8.167	9.333	10.50
8	1.333	2.667	4.00	5.333	6.667	8.00	9.333	10.667	12.00
...	...	...	...	...	...	...	...	...	...

Written Exercises

1. Find, by the interest table, the interest on $2300 for 2 yr. 2 mo. 7 da. at 6 %.

SOLUTION. — Since the interest on any number of *hundred* dollars is .1 of that on the *same* number of *thousand* dollars, we have,

$$\begin{array}{cccc} & \text{2 yr.} & \text{2 mo.} & \text{7 da.} \\ \text{Int. on } \$2000 = & \$240.00 + & \$20.00 + & \$2.333 \\ \text{Int. on} \quad 300 = & 36.00 + & 3.00 + & .350 \\ \text{Int. on } \$2300 = & \$276.00 + & \$23.00 + & \$2.683 = \$301.68 \end{array}$$

Find, by the interest table, the interest at 6 % on:

2. $2600, 1 yr. 2 mo.   **6.** $500, 2 yr. 1 mo. 5 da.

3. $8300, 3 mo. 4 da.   **7.** $800, 1 yr. 3 mo. 2 da.

4. $6700, 2 yr. 3 da.   **8.** $400, 3 yr. 2 mo. 8 da.

5. $4900, 3 yr. 1 mo.   **9.** $700, 2 yr. 3 mo. 6 da.

PARTIAL PAYMENTS

It frequently happens that the maker of a note cannot pay the whole amount at one time, but instead he makes **partial payments**.

A record of the partial payments is written, or indorsed, on the back of the note with the dates when they were made, as in the margin.

Received on the
Within Note:
Aug. 7, 1913, $200
May 5, 1914, $500

UNITED STATES RULE

Written Exercises

1. A note for $1000, dated Jan. 1, 1913, interest 6%, was indorsed as follows: Mar. 22, 1913, $125; June 1, 1914, $50; Aug. 4, 1914, $75; Nov. 1, 1914, $150. Find the amount due Jan. 1, 1915.

SOLUTION

Principal, Jan. 1, 1913		$1000.00
Int. to Mar. 22, 1913, — 2 mo. 21 da.		13.50
Amount		1013.50
First payment		125.00
New principal, Mar. 22, 1913		$888.50
Int. to June 1, 1914, — 1 yr. 2 mo. 9 da.		63.53
Second payment (which is less than int. due)	$50.00	
Int. from June 1 to Aug. 4, — 2 mo. 3 da. on $888.50		9.33
Amount		961.36
Third payment (to be added to second)	$75.00	125.00
New principal, Aug. 4, 1914		$836.36
Int. to Nov. 1, 1914, — 2 mo. 27 da.		12.13
Amount		848.49
Fourth payment		150.00
New principal, Nov. 1, 1914		$698.49
Int. to Jan. 1, 1915, — 2 mo.		6.98
Amount due Jan. 1, 1915		$705.47

United States Rule. — *Find the amount of the principal to a time when a payment, or the sum of two or more payments, equals or exceeds the interest due, and from the amount subtract such payment or payments.*

With the remainder as a new principal proceed as before.

In other words, partial payments are applied first to pay the interest due, and then, if anything is left, to reduce the principal.

Most of the states, including New York, have adopted the United States rule for computing the indebtedness when partial payments have been made.

2. A note for $80, dated Oct. 1, 1913, had written across the back of it "Received on the within note, Jan. 1, 1914, $47." How much was due July 1, 1914, with interest at 4%?

3. Find the amount due Oct. 1, 1914, on a note for $100, dated May 1, 1914, interest 6%. Indorsement: Aug. 1, 1914, $50.

4. Find the amount due July 15, 1914, on a note for $150, dated Oct. 15, 1913, with interest at 5%. Indorsement: April 15, 1914, $125.

5. A note for $1000, bearing 5% interest, and dated Jan. 2, 1914, had these payments indorsed on it: April 2, 1914, $200; July 2, 1914, $300. How much was due Oct. 2, 1914?

6. A note for $400, dated April 1, 1913, had indorsed on it the following payments: Sept. 1, 1913, $130; Dec. 1, 1913, $100. How much was due April 1, 1914, with interest at 4%?

7. A note for $1600, bearing 6% interest, dated Nov. 1, 1912, had these indorsements: Feb. 1, 1913, $600; Oct. 1, 1913, $200; Nov. 1, 1913, $400. How much was due May 1, 1914?

8. A note for $1250, given Mar. 1, 1911, had these indorsements: July 1, 1911, $25; Dec. 1, 1912, $200; Oct. 11, 1913, $600. Find the amount due April 4, 1914, interest at 6%.

9. A note for $4000, bearing 5% interest, dated Aug. 15, 1912, had these indorsements: Jan. 2, 1913, $40; May 12, 1913, $500. How much was due June 1, 1914?

MERCANTILE RULE

Business men often settle notes and accounts on which partial payments have been made, and which do not run longer than one year by the following rule:

Mercantile Rule. — *Find the amount of the principal at the time of settlement.*

Find the amount of each payment from the time it was made until the time of settlement.

From the amount of the principal subtract the sum of the amounts of the payments.

The following exercises are to be solved by this rule.

Written Exercises

1. Find the amount due Oct. 15, 1914, on a note for $200, dated June 15, 1914, interest 6%. Indorsement: Aug. 12, 1914, $125.

2. Find the amount due May 12, 1914, on a note for $600, dated Nov. 24, 1913, interest 4%. Indorsement: Feb. 18, 1914, $360.

3. A note for $1200 given May 18, 1913, had the following indorsements: Oct. 12, 1913, $300; Mar. 6, 1914, $50. How much was due May 18, 1914, with interest at 5%?

4. A note for $850, dated June 2, 1913, had these indorsements: Oct. 2, 1913, $100; Feb. 2, 1914, $150; Apr. 2, 1914, $75. How much was due May 2, 1914, interest at 6%?

5. A note for $175, dated June 24, 1913, had these indorsements: July 24, 1913, $75; Aug. 18, 1913, $25. How much was due Dec. 24, 1913, interest at 4%?

6. A note for $1600, dated Jan. 2, 1914, had the following indorsements: Mar. 2, 1914, $300; July 1, 1914, $25; Oct. 17, 1914, $80. How much was due Jan. 2, 1915, interest at 6%?

ACCURATE INTEREST

Interest computed by taking the exact number of days between dates and reckoning 365 days for a year is **accurate**, or **exact, interest**.

Accurate interest does not differ from ordinary interest for whole years, but for terms less than a year it is $\frac{360}{365}$ of ordinary interest, reckoned for the actual number of days.

Accurate interest is used by the United States government, by some banks, and to some extent in other business.

Written Exercises

1. Find the accurate interest on $400 for 45 da. at 5%.

SOLUTION. — Accurate interest, $\dfrac{\overset{9}{\cancel{45}}}{\underset{73}{\cancel{365}}} \times \dfrac{5}{\cancel{100}} \times \$\overset{4}{\cancel{400}} = \$\dfrac{180}{73} = \$2.47.$

Find the accurate interest on:

2. $200 for 90 da. at 6% **6.** $5000 for 120 da. at 3%

3. $500 for 33 da. at 6% **7.** $6400 for 111 da. at 4%

4. $750 for 60 da. at 5% **8.** $7200 for 214 da. at 5%

5. $480 for 45 da. at 4% **9.** $8100 for 179 da. at 4½%

Using the table for days on page 243, find the accurate interest at 6% on:

10. $150, Mar. 2 to July 5 **14.** $5000, Feb. 1 to Oct. 1

11. $440, Aug. 1 to Dec. 1 **15.** $5000, May 1 to Nov. 1

12. $765, May 1 to Sept. 1 **16.** $4640, June 1 to Nov. 4

13. $480, June 1 to July 9 **17.** $3875, July 6 to Dec. 19

18. How much less is the accurate interest on $10,000 from June 15 to Sept. 15 at 3% than the ordinary interest?

19. The United States pays accurate interest. How much interest is saved thereby by the United States on $2,500,000, borrowed from Mar. 1 to July 1 at 4½%?

COMPOUND INTEREST

Interest on the principal and its unpaid interest, combined at regular intervals, is **compound interest.**

Interest may be compounded with the principal annually, semiannually, or quarterly, etc.

Compound interest cannot usually be enforced by law, even though it is specified in the contract. Nevertheless the subject is of importance, since it is at the basis of computations concerning investments, for there is nothing to hinder a man from drawing his interest and investing it.

Written Exercises

1. Find the amount of $400 for 2 yr. 4 mo. at 6%, interest compounded annually; also find the compound interest.

SOLUTION

Principal	$400
Interest for 1st yr. at 6%	24
Principal beginning 2d yr.	$424
Interest for 2d yr. at 6%	25.44
Principal beginning 3d yr.	$449.44
Interest for 4 mo. at 6%	8.99
Amount for 2 yr. 4 mo. at 6%	$458.43

The compound interest = $458.43 − $400, or $58.43.

NOTE. — Unless otherwise specified, interest is understood to be compounded *annually*. If compounded semiannually, the rate must be considered one half the annual rate mentioned; if quarterly, one fourth, etc.

The amount is found for the greatest number of *entire* periods, as years, half years, etc., and the simple interest upon this for the rest of the time is added. Intermediate results here should not be limited to the nearest mill.

Find the interest and the amount at 4% compound interest:

2. $200, 3 yr.   5. $300, 1½ yr.   8. $425, 3 yr. 8 mo.

3. $400, 2 yr.   6. $550, 3¼ yr.   9. $850, 2 yr. 6 mo.

4. $700, 4 yr.   7. $650, 2⅓ yr.   10. $975, 4 yr. 3 mo.

11-19. Do exercises 2-10 with interest compounded semiannually.

20. Find the amount of $1000 for 10 yr. at 4% compound interest; at 4%, compounded semiannually.

SOLUTIONS

1. By the table on the next page the amount of $1 at 4% compound interest for 10 yr. is $1.480244. Hence, the amount of $1000 for the same time at the same rate is 1000 × $1.480244, or $1480.24.

2. If the rate is 4% compounded semiannually, there are 20 interest periods and the rate for each is 2%. Therefore, the amount of $1000 for 10 yr. at 4%, compounded semiannually, is the same as the amount of $1000 for 20 yr. at 2%, compounded annually. By use of the table this amount is found to be $1485.95.

Find, by the table, the amount at compound interest of:

21. $400 for 2 yr. at 5%   **25.** $1000 for 8 yr. at 7%

22. $600 for 3 yr. at 4%   **26.** $3000 for 6 yr. at 6%

23. $900 for 5 yr. at 2%   **27.** $5000 for 10 yr. at $3\frac{1}{2}$%

24. $800 for 4 yr. at 6%   **28.** $3250 for 12 yr. at $4\frac{1}{2}$%

29. $1500 for 3 yr. 6 mo. at 6%

30. $4000 for 8 yr. 3 mo. at 4%

31. $6000 for 6 yr. 8 mo. at $2\frac{1}{2}$%

32. $7000 for 14 yr. 9 mo. at 3%

33. $5000 for 8 yr. 2 mo. 6 da. at 6%

34. $8250 for 10 yr. 4 mo. 12 da. at 7%

35. $75.50 for 15 yr. 6 mo. 3 da. at 4%

36. $86.48 for 18 yr. 10 mo. 15 da. at 6%

37. $8000 for 8 yr. at 6%, payable semiannually

38. $1650 for 10 yr. at 7%, payable semiannually

39. $4800 for 4 yr. 6 mo. at 4%, payable quarterly

40. $6400 for 3 yr. 10 mo. at 5%, payable quarterly

41. $7050 for 6 yr. 8 mo. at 5%, payable semiannually

42. $6500 for 4 yr. 4 mo. 10 da. at 6%, payable quarterly

INTEREST

COMPOUND INTEREST TABLE

Amount of $ 1, at various rates, compound interest, 1 to 20 years

Years	1%	1¼%	1½%	2%	2¼%	3%
1	1.010000	1.012500	1.015000	1.020000	1.025000	1.030000
2	1.020100	1.025156	1.030225	1.040400	1.050625	1.060900
3	1.030301	1.037971	1.045678	1.061208	1.076891	1.092727
4	1.040604	1.050945	1.061364	1.082432	1.103813	1.125509
5	1.051010	1.064082	1.077284	1.104081	1.131408	1.159274
6	1.061520	1.077383	1.093443	1.126162	1.159693	1.194052
7	1.072135	1.090850	1.109845	1.148686	1.188686	1.229874
8	1.082857	1.104486	1.126493	1.171659	1.218403	1.266770
9	1.093685	1.118292	1.143390	1.195093	1.248863	1.304773
10	1.104622	1.132271	1.160541	1.218994	1.280085	1.343916
11	1.115668	1.146424	1.177949	1.243374	1.312087	1.384234
12	1.126825	1.160755	1.195618	1.268242	1.344889	1.425761
13	1.138093	1.175264	1.213552	1.293607	1.378511	1.468534
14	1.149474	1.189955	1.231756	1.319479	1.412974	1.512590
15	1.160969	1.204829	1.250232	1.345868	1.448298	1.557967
16	1.172579	1.219889	1.268986	1.372786	1.484506	1.604706
17	1.184304	1.235138	1.288020	1.400241	1.521618	1.652848
18	1.196148	1.250477	1.307341	1.428246	1.559659	1.702433
19	1.208109	1.266108	1.326951	1.456811	1.598650	1.753506
20	1.220190	1.281935	1.346855	1.485947	1.638616	1.806111

Years	3½%	4%	4½%	5%	6%	7%
1	1.035000	1.040000	1.045000	1.050000	1.060000	1.070000
2	1.071225	1.081600	1.092025	1.102500	1.123600	1.144900
3	1.108718	1.124864	1.141166	1.157625	1.191016	1.225043
4	1.147523	1.169859	1.192519	1.215506	1.262477	1.310796
5	1.187686	1.216653	1.246182	1.276282	1.338226	1.402552
6	1.229255	1.265319	1.302260	1.340096	1.418519	1.500730
7	1.272279	1.315932	1.360862	1.407100	1.503630	1.605782
8	1.316809	1.368569	1.422101	1.477455	1.593848	1.718186
9	1.362897	1.423312	1.486095	1.551328	1.689479	1.838459
10	1.410599	1.480244	1.552969	1.628895	1.790848	1.967151
11	1.459970	1.539454	1.622853	1.710339	1.898299	2.104852
12	1.511069	1.601032	1.695881	1.795856	2.012197	2.252192
13	1.563956	1.665074	1.772196	1.885649	2.132928	2.409845
14	1.618695	1.731676	1.851945	1.979932	2.260904	2.578534
15	1.675349	1.800944	1.935282	2.078928	2.396558	2.759032
16	1.733986	1.872981	2.022370	2.182875	2.540352	2.952164
17	1.794676	1.947901	2.113377	2.292018	2.692773	3.158815
18	1.857489	2.025817	2.208479	2.406619	2.854339	3.379932
19	1.922501	2.106849	2.307860	2.526950	3.025600	3.616528
20	1.989789	2.191123	2.411714	2.653298	3.207136	3.869684

REVIEW

Oral Exercises

Tell quickly the interest at 6 % on :

1. $200 for 1 yr.	**6.** $400 for 6 mo.	**11.** $300 for 60 da.
2. $300 for 2 yr.	**7.** $600 for 4 mo.	**12.** $400 for 15 da.
3. $400 for 3 yr.	**8.** $700 for 2 mo.	**13.** $500 for 30 da.
4. $100 for 2½ yr.	**9.** $200 for 3 mo.	**14.** $600 for 10 da.
5. $200 for 1¼ yr.	**10.** $300 for 4 mo.	**15.** $900 for 20 da.

Tell how you would find the interest ; then estimate it :

16. $175, 1 yr., 6 %	**22.** $550, 60 da., 8 %
17. $250, 2 yr., 4 %	**23.** $450, 30 da., 3 %
18. $150, 6 mo., 8 %	**24.** $200, 20 da., 5 %
19. $350, 60 da., 4 %	**25.** $300, 10 da., 7 %
20. $275, 30 da., 6 %	**26.** $375, 20 da., 6 %
21. $750, 20 da., 4 %	**27.** $450, 4½ mo., 4 %

Written Exercises

1-12. Find the interest in each of the above exercises **16-27.**

Fill the blank spaces in the following :

	PRINCIPAL	RATE	TIME	INTEREST	AMOUNT
13.	$1200	——	30 da.	——	$1206
14.	——	6 %	——	$52.80	$1812.80
15.	$1400	——	3 mo. 18 da.	$21	——
16.	$1450	4½ %	——	——	$1558.75
17.	——	——	3 mo.	$30	$1630
18.	$1350	5 %	1 yr. 4 mo.	——	——
19.	$1290	7 %	——	——	$1560.90
20.	——	——	9 mo. 18 da.	$69.08	$1639.08

21. How long will it take $600 to amount to $1000, with interest at 6 %?

22. At 7 % find the interest on $800 loaned from Mar. 12, 1913 to Nov. 25, 1914.

23. One year Mr. Pierce rented a farm that cost him $5600 for $490. What rate of interest did he earn on his investment?

24. Find the compound interest on $1200 for 4 yr. 6 mo. 12 da., interest payable semiannually at 4 %.

25. Find the exact interest on $21,900 from Dec. 17 to Jan. 31, at 5 %.

26. At 6 % find the amount of $10,000 for 3 yr. 5 mo. 24 da. at simple interest; at compound interest.

27. A man bequeathed to his daughter enough money so that the interest on it at 4½ % would provide an annual income of $1260. Find the amount of the gift.

28. On Jan. 1, 1914, Charles Scott gave George Ryder a negotiable note for $1500 payable in 9 mo. with interest at 4 %. George Ryder sold the note to James Morse. Write the note and indorse it in full. Find the amount due at maturity.

29. On the day a boy was 5 years old, $500 was invested for him at 4 % compound interest. When the boy was 21 years old, what was the amount of the investment?

30. Mr. Butler invested $2400 in business and at the end of 6 yr. 5 mo. withdrew $3632, which sum included the investment and profit. To what average annual rate of interest was his profit equivalent?

31. A note for $800, dated May 12, 1913, had these indorsements: Sept. 18, 1913, $200; Jan. 30, 1914, $20; April 15, 1914, $300. With interest at 6 %, how much was due Sept. 15, 1914?

BANKING

An establishment for the custody, loan, exchange or issue of money, and for facilitating the transmission of funds from one place to another is a **bank**.

If a person intends to use his money soon, he places it in a **bank of deposit**, but if he does not expect to withdraw it for some time, it may be wiser to leave it in a **savings bank**, because the savings bank will pay interest upon *small sums* of money that are left for the *interest term*, usually six months.

Some banks of deposit, however, pay interest on deposits under certain circumstances, usually on large sums.

The chief business of commercial banks or banks of deposit and discount, including **national banks, federal reserve banks, state banks, private banks**, etc., is the receiving of deposits for safe keeping and the lending of money in various ways, as by buying notes at a discount. They also perform certain financial services for business men and corporations.

National banks and federal reserve banks do business under the authority of national law. Besides the ordinary banking business, they issue demand notes, payable to bearer, which circulate as money, and are called respectively **bank notes** and **federal reserve notes**.

A **trust company** is authorized under state law and is usually similar to a bank so far as loans and deposits are concerned, but it does other business peculiar to itself.

Trust companies, being restricted less than banks in regard to the kind of investments they may make, are usually able to pay interest on deposits. They do not issue bank notes.

Money that is deposited in a bank or a trust company is usually payable to the depositor or his order on demand. The demand is made by a written order called a **check**.

STUB	CHECK
No. 460	*No. 460 Rochester, N. Y., Jan. 16, 1915.*
Date Jan. 16, '15.	**Second National Bank of Rochester**
Payable to	
....John Green....	*Pay to the order of John Green ....*
.....For Coal......	*Forty-eight and* $\frac{25}{100}$ *~~~~~~~~~Dollars.*
*Am't. $48.*$\underline{25}$	*$48.*$\underline{25}$ *William Drew.*

The terms **maker, payee, face, negotiable,** and the different kinds of indorsement, etc., apply to checks the same as to promissory notes.

To get this check cashed at the bank John Green, the payee, must indorse it; then, unless he is known at the bank, he must be identified by some one who is known.

The **stub** that remains in the *check book* after the check has been torn out gives a complete record of the check.

After the check has been cashed at the bank, it will be canceled and later returned to Mr. Drew as a **voucher** or **receipt** from John Green since it has his name on the back of it.

When a depositor wishes to draw money for himself from his bank, he may write a check payable to the order of " Self," in which case he must indorse it before he can get it cashed ; or he may make it payable to the order of " Cash," in which case no indorsement is necessary.

A check payable to the payee or " bearer " is usually paid to any one presenting it at the bank on which it is drawn, if the bank is sure that the maker's signature is genuine.

When an authorized officer of a bank upon which a check is drawn writes " good " or " certified " across the face of the check and signs his name, the check becomes a **certified check**, the bank thus guaranteeing its payment.

Opening a Bank Account. — A person wishing to open an account at a bank takes his money there and leaves it with the "cashier" or "receiving teller" together with a deposit slip properly filled out.

He is then given a "bank book" with the amount credited in it, and is furnished with a book of blank checks.

In order that the bank may have his true signature to refer to at

DEPOSIT SLIP

DEPOSITED BY		
..........Samuel J. Slawson..........		
IN THE		
First National Bank		
Malone, N. Y., Jan. 21, 1915.		
Bills	$40	
Coin	14	75
Check on Farmers' Bank	76	17
" " Corn Exch. "	22	38
Total	$153	30

any time, he is required to sign his name in a book of signatures.

When a stranger to the bank officials wishes to open an account, he has to be introduced by some reliable person or he must furnish references.

A deposit slip is handed in each time a deposit is made, and the amount is credited in the depositor's bank book, which he brings with him.

Written Exercises

Supply necessary data and make out a deposit slip for :

1. Bills, $475; check on First National Bank, $125.80.
2. Bills, $50 ; coin, $10 ; check on Mercantile Bank, $75.
3. Bills, $64 ; coin, $5.75 ; checks, $12.50, $24.63, $15.40.
4. Bills, $84 ; coin, $14.78; checks, $30.75, $80, $125.
5. Bills, $275; coin, $10.84 ; checks, $200, $57, $68.24.
6. Bills, $160 ; coin, $8.15; checks, $35.10, $18.40, $15.25.
7. Checks, $300, $154.60, $28.14, $48.20, $96.75, $22.60.
8. Checks, $16.33, $24.87, $125, $240, $600, $375.50.

Write checks and stub records from the following data, supplying other necessary details:

FACE	PAYEE	MAKER	FACE	PAYEE	MAKER
9. $25	H. West	A. Hines	14. $100	B. Kirk	H. Chase
10. $39	Cash	W. Scott	15. $144	F. Long	C. Allen
11. $60	J. Bell	S. Clark	16. $215	J. Cook	D. Colby
12. $48	C. Ford	D. Steele	17. $175	Cash	K. Swift
13. $50	Self	R. Burke	18. $500	H. Case	J. Morse

19. Indorse the check in exercise **13** so that R. Burke may cash it at the bank.

20. Suppose that H. West applies the check in exercise **9** in part payment of a bill at Carroll Brothers' grocery, and that Carroll Brothers deposit it in the bank upon which it was drawn. Write the indorsements that the check will show.

21. A man's balance at the First National Bank was $846.20. Find his balance after checking out $45, $3.75, $75.50, $13.62, $175.25, and $126.10.

22. On May 1 a man's balance at the Franklin National Bank was $142.34. May 3 he deposited $112.50; May 10 he deposited a check for $65; May 24 he deposited $92; May 31 he drew out $125. Find his balance in the bank June 1.

23. John Dixon pays Myron Cox $37.50 per month for rent by check. Supply necessary data and write one of the checks in such a form that Mrs. Cox, who is not known at the bank, may cash it there.

24. A man's balance in the bank at the beginning of a week was $196.50. During the week he deposited $234.60, $490.75, $325.50, $416, $325, and $410.75, and paid by check a bill of goods for $1250 less 3 %, another for $98.50 less 2 %, and $375 for store rent. Find his balance at the end of the week.

BANK DISCOUNT

Lending Money. — Besides lending that part of its own capital that is not needed for other purposes, a bank of deposit and discount is able to lend a large part of the money that is deposited with it, for it is not likely that all the depositors will want the whole of their money at the same time.

Borrowing Money. — A great part of the capital employed in business is borrowed. This money is largely obtained from banks. Banks lend for the most part on notes or on some other form of negotiable paper. Often the names of at least two persons are required on the paper, — the name of the borrower, and the name of a person who becomes responsible with the borrower for the payment of the loan, by *indorsing* the note.

If Mr. Beman wishes to borrow money at a bank and Eugene Dunn agrees to indorse the note, Mr. Beman may make out a promissory note as follows :

$ 1000.$\frac{00}{}$ *Plattsburg, N. Y., Aug. 8, 1914.*

Ninety days after date, for value received,......I.....promise to pay to the order of Eugene Dunn_____

One thousand $\frac{00}{100}$ ~~~~~~~~~~~~~~~~~~~~~~~~Dollars at the Merchants' National Bank.

No. 73.   *Due_____*   *Orson Beman.*

When Eugene Dunn has indorsed the note, Orson Beman may get money on it at the bank, if the officials consider both men reliable. He will not, however, get the whole sum of $ 1000. The bank will take out *interest in advance* for the time the note has yet to run and will give Mr. Beman the balance.

The value of a note at maturity is its **face value**, if it does not bear interest; its **amount**, if it does bear interest.

Interest, collected by a bank in advance, upon the value of a note at maturity is **bank discount**.

The value of a note at maturity less the bank discount is the **proceeds** of the note.

The *number of days* from the time a note is discounted to the time when it legally matures is the **term of discount**.

Written Exercises

1. Find the date of maturity, the term of discount, the bank discount, and the proceeds of the note on page 261, if it was discounted Sept. 16, 1914, at 6 %.

SOLUTION

The **date of maturity** is 90 days after Aug. 8, 1914, or Nov. 6, 1914.
The **term of discount** is from Sept. 16, 1914, to Nov. 6, 1914, or 51 days.
The **bank discount** is the interest on $1000 for 51 days at 6 %, or $8.50.
The **proceeds** is $1000 − $8.50, or $991.50.

The following notes do not bear interest. Find the date of maturity, the term of discount, the bank discount, and the proceeds of each:

NOTE FOR	DATED	TIME	DISCOUNTED	RATE OF DISCOUNT
2. $ 500	Aug. 27	30 da.	Aug. 27	6 %
3. $ 650	Jan. 5	1 mo.	Jan. 6	4½%
4. $ 475	May 1	15 da.	May 1	5 %
5. $ 720	Nov. 18	90 da.	Nov. 19	6 %
6. $ 850	July 28	10 da.	July 28	5 %
7. $2500	Apr. 8	2 mo.	Apr. 15	6 %
8. $3200	Sept. 30	120 da.	Oct. 15	7 %
9. $7500	June 6	3 mo.	June 11	3½%

10. Find the proceeds of a 10-day note for $1000, discounted at $3\frac{1}{2}$ % on the day it was made.

11. A note for $2750, dated May 12 and due in 4 mo., was discounted May 18 at 4 %. Find the proceeds.

12. How much will be realized on a note for $4800, due in 90 days, if it is discounted at 5 % the day it is made?

13. Find the proceeds of a 60-day note for $7250 dated Mar. 12, and discounted April 5 at $4\frac{1}{2}$ %.

14. A 120-day note for $10,500 dated Aug. 15 was discounted Aug. 18 at 5 %. Find the proceeds.

15. What were the proceeds of a note for $15,000, dated July 7 and due in 6 mo., if it was discounted Sept. 21 at $3\frac{1}{2}$ %?

16. Find the bank discount and the proceeds of a 60-day note for $840 with interest at 5 %, discounted 10 days after it was made at 4 %.

SOLUTION

The *interest* at maturity = $7.
The *amount* at maturity = $847.
The **bank discount** = interest at 4 % on $847 for 50 da., or $4.71.
The **proceeds** = $847 − $4.71 = $842.29.

Each of these notes, interest at 6 %, was discounted the day it was made. Find the bank discount and the proceeds:

FACE	TIME	RATE OF DISCOUNT	FACE	TIME	RATE OF DISCOUNT
17. $480	30 da.	6 %	**26.** $2450	90 da.	4 %
18. $525	15 da.	4 %	**27.** $4220	18 da.	5 %
19. $376	60 da.	5 %	**28.** $5760	30 da.	6 %
20. $640	90 da.	7 %	**29.** $3375	60 da.	4 %
21. $296	18 da.	4 %	**30.** $6346	15 da.	7 %
22. $750	60 da.	$3\frac{1}{2}$ %	**31.** $8278	10 da.	5 %
23. $548	24 da.	5 %	**32.** $72.50	24 da.	$3\frac{1}{2}$ %
24. $872	45 da.	$4\frac{1}{2}$ %	**33.** $82.40	90 da.	7 %
25. $685	90 da.	7 %	**34.** $71.95	120 da.	$5\frac{1}{2}$ %

Find the proceeds of each note, discounted at a bank:

NOTE FOR	DATED	TIME	DISCOUNTED	INTEREST RATE	DISCOUNT RATE
35. $400	Apr. 15	3 mo.	Apr. 15	4%	6%
36. $360	Jan. 12	30 da.	Jan. 18	4½%	5%
37. $500	Oct. 30	60 da.	Oct. 31	none	5%
38. $625	Aug. 10	90 da.	Aug. 12	none	½%
39. $480	Nov. 15	3 mo.	Nov. 17	6%	4%
40. $800	May 1	4 mo.	May 1	none	5%
41. $750	Apr. 1	60 da.	Apr. 7	none	4½%
42. $2000	Mar. 12	30 da.	Mar. 14	none	4½%
43. $4000	Dec. 1	6 mo.	Apr. 2	6%	5%
44. $3500	Oct. 1	4 mo.	Dec. 27	6%	6%
45. $4200	Aug. 15	3 mo.	Aug. 17	5%	4%
46. $5000	Nov. 12	1 mo.	Nov. 12	6%	5%
47. $6400	May 1	6 mo.	Aug. 13	4%	5%
48. $8100	Mar. 1	30 da.	Mar. 3	4%	4½%
49. $7500	Oct. 12	60 da.	Oct. 16	5%	4½%

50. Find the face of a 60-day note that, discounted at 6 %
on the day it is made, will realize $9900 in cash.

SUGGESTION. — The proceeds of $1 discounted for 60 days at 6 % = $.99.

51. The proceeds of a note for 3 mo. when discounted at a
bank at 6 % were $ 591. What was its face?

52. Find the face of a 90-day note that, discounted at 4 % on
the day it is made, will realize $1980 in cash.

53. For what sum must a 30-day note be drawn to realize
$199.25 in cash, the rate of discount being 4½ % ?

54. For what sum must I draw my note for 60 days to
obtain $ 2975 in cash, if the rate of discount is 5 % ?

55. For what sum must a man draw a note for 3 mo. 18 da.
to obtain $ 24,700 in cash, if the rate of discount is 4 % ?

56. For what sum must a 90-day note be drawn to realize
$ 19,775 in cash when the rate of discount is 4½ % ?

SAVINGS BANKS

Savings banks pay (or credit) interest to depositors at stated periods, either monthly, quarterly, or semiannually. The time between the dates at which interest is paid is the interest term.

Quarterly interest terms begin Jan. 1, April 1, July 1, and Oct. 1. Semi-annual terms begin Jan. 1 and July 1, or April 1 and Oct. 1.

The bank books of depositors in savings banks must be presented both when deposits are made and when money is drawn out, the amounts being credited or charged as the case may be. Books should also be presented at the bank at the end of each interest term to have interest credited.

Usually savings banks compute interest, *dropping any part of a cent*, on the *dollars only* of the *smallest balance* on deposit during the entire period.

NOTE. — Trust companies and national banks often have savings departments.

The following is an illustrative account of deposits and withdrawals with interest reckoned quarterly at 3 % a year.

DATE	DEPOSITED	DRAWN OUT	INTEREST	BALANCE
1914				
Dec. 15	320			320
1915				
Jan. 1				320
Feb. 6		75		245
April 1			1 83	246 83
June 10	40			286 83
July 1	25		1 84	313 67
Sept. 12		50		263 67
Sept. 20	125			388 67
Oct. 1			1 97	390 64

This shows that the deposit of Dec. 15, 1914, did not begin to draw interest until Jan. 1, 1915, and the interest for the quarter from Jan. 1 to April 1 was computed on the *smallest* balance for the quarter, namely, $245; $ 1.84 is the interest on $246 for 3 mo. and $1.97 is the interest on $263 for 3 mo.

NOTE. — Savings banks allow compound interest in the sense only of paying interest on interest.

266 BANKING

Written Exercises

NOTE. — Usually interest is allowed on deposits made within a few days (3 or 4) of the beginning of an interest term.

Arrange the following as in a savings bank book, and find the balance due Jan. 1, 1914:

1. Interest quarterly (Jan. 1, April 1, etc.) at 4 %.

Deposits: Aug. 8, 1912, $425; Oct. 2, 1912, $100; June 30, 1913, $150; Sept. 3, 1913, $80.

Withdrawals: Sept. 18, 1912, $35; Mar. 19, 1913, $250.

2. Same as exercise 1, except interest semiannually (Jan. 1 and July 1).

3. Interest quarterly (Jan. 1, April 1, etc.) at 4%.

Deposits: Jan. 30, 1913, $85; Feb. 18, 1913, $45; Mar. 8, 1913, $100; Mar. 29, 1913, $50; June 2, 1913, $125.

Withdrawals: April 14, 1913, $60; Sept. 2, 1913, $80.

4. Same as exercise 3, except interest semiannually (Jan. 1 and July 1) at 3%.

5. Interest semiannually (Jan. 1 and July 1) at 3%.

Deposits: Jan. 2, 1913, $250; Jan. 15, 1913, $75; Feb. 3, 1913, $125; Mar. 12, 1913, $175; June 4, 1913, $60.

Withdrawals: Feb. 28, 1913, $10; April 2, 1913, $50.

6. Interest quarterly (Jan. 1, April 1, etc.) at 3½ %. Balance, July 2, 1913, $147.12.

Deposits: July 7, 1913, $15.50; Sept. 12, 1913, $25; Oct. 13, 1913, $60; Nov. 10, 1913, $22.50.

Withdrawals: Oct. 2, 1913, $16; Dec. 12, 1913, $35.

7. Interest semiannually (Jan. 1 and July 1) at 3½ %.

Deposits: May 6, 1910, $85; Oct. 1, 1910, $110; Jan. 16, 1911, $75; Sept. 23, 1911, $120; Mar. 4, 1912, $60; July 1, 1912, $50; Feb. 1, 1913, $44; July 2, 1913, $36.

POSTAL SAVINGS BANKS

For the purpose of providing facilities for depositing savings at interest with the security of the government for repayment, certain post offices have been designated as depositories, or **postal savings banks.**

Who may Deposit. — Deposits may be made by any person 10 years old or over.

Deposits. — Certificates are issued to depositors in fixed denominations of $1, $2, $5, $10, $20, $50, or $100 as evidence of deposits made, the postmaster keeping a duplicate of each.

No one is allowed to deposit more than $100 in any one month or to have more than $500 on deposit or more than one account at any one time.

No account may be opened for less than $1 nor will fractions of a dollar be accepted for deposit, but amounts less than $1 may be *saved for deposit* by the purchase of 10-cent postal savings cards and 10-cent postal savings stamps. A card with nine stamps attached is accepted as a deposit of $1.

Withdrawals. — Deposits may be withdrawn on demand together with the interest that is then due upon them.

Interest. — All deposits bear interest at 2%, payable each year, computed on each savings certificate separately from the first of the month next after the deposit is made.

Compound interest is not allowed on an outstanding certificate and no interest is paid on money that remains on deposit for a part of a year only.

Written Exercises

Find the interest on these postal savings bank deposits:

1. $28 for 2 yr.　　4. $84 for 4 yr.　　7. $154 for 2 yr
2. $54 for 5 yr.　　5. $78 for 7 yr.　　8. $248 for 4 yr.
3. $66 for 3 yr.　　6. $96 for 6 yr.　　9. $406 for 1 yr.

10. In 1912 John North made these deposits in a postal savings bank: Feb. 15, $18; Mar. 30, $22; May 20, $15; July 25, $20; Oct. 30, $25. Find the interest due Jan. 1, 1914.

EXCHANGE

Paying debts or collecting credits in distant places without actually transferring money is **exchange**.

Exchange between two places in the same country is **domestic, or inland, exchange**; exchange between two places in different countries is **foreign exchange**.

Foreign exchange is treated in the Appendix.

DOMESTIC EXCHANGE

The various methods of exchange explained in the following pages are by **postal money order**, by **express money order**, by **telegraphic money order**, by **check**, by **bank draft** (similar to a check), and by **bankers' association money order** (similar to a draft).

A **postal money order** is a written order made by the post-master in one place on the postmaster in another to pay to the person named therein a specified sum of money. It is *negotiable*.

In addition to the *face*, money orders cost as follows:

For Orders From $ .01 to $2.50 . . . .   3 cents.
     From $ 2.51 to $5.00 . . . . . .   5 cents.
     From $ 5.01 to $10.00   . . . . .   8 cents.
     From $10.01 to $20.00   . . . . .   10 cents.
     From $20.01 to $30.00   . . . . .   12 cents.
     From $30.01 to $40.00   . . . . .   15 cents.
     From $40.01 to $50.00   . . . . .   18 cents.
     From $50.01 to $60.00   . . . . .   20 cents.
     From $60.01 to $75.00   . . . . .   25 cents.
     From $75.01 to $100.00   . . . . .   30 cents.

NOTE. — The maximum amount for which a single order may be issued is $100. When a larger sum is to be sent, additional orders must be obtained.

An **express money order** is similar to a postal order, the rates being the same. It is *negotiable*.

NOTE. — A single order for over $50 is not issued, but additional orders may be obtained and when purchased at the same time the rate is the same as for a like amount by postal order; thus, the rate for two 50-dollar orders is 30 ¢, not 36 ¢.

Telegraphic Money Order. — In addition to the regular telegraphic charge for a 15-word message between the two places, telegraph companies make transfers of money between their offices subject to the following charges:

For orders for sums not exceeding $25 . . . . . 25 cents.
Over $25 and not exceeding $50 . . . . 35 cents.
Over $50 and not exceeding $75 . . . . 60 cents.
Over $75 and not exceeding $100 . . . . 85 cents.

Add 25¢ for each $100 or fraction thereof after the first $100 up to and including $3000. Add 20¢ for each $100 or fraction thereof above $3000.

Banks and express companies transfer money for their customers by telegraph when haste is necessary.

Oral Exercises

Find the cost of a postal or an express money order for:

1. $1.50	**4.** $6.00	**7.** $8.50	**10.** $11.20	**13.** $50
2. $7.00	**5.** $4.28	**8.** $5.75	**11.** $21.80	**14.** $66
3. $2.25	**6.** $3.10	**9.** $9.40	**12.** $17.85	**15.** $85

16. Find the cost of telegraphing $50 from New York to Boston, if the telegraphic rate is 30–2 (see page 68).

17. How much will it cost Mr. Drew to telegraph $200 to his son in New Haven, the telegraphic rate being 25–2?

18. How much will it cost to telegraph $250 from Albany to New York, the telegraphic rate being 25–2?

19. How much less will it cost to send $75 by postal money order, including postage, than by telegraph, rate 50–3?

Exchange by Check. — A personal check drawn on a bank in which the drawer has money deposited may be sent by mail. The payee gets it cashed at some bank and this bank collects it from the bank on which it was drawn, sometimes charging the payee a small fee, **exchange,** for collecting.

Certified checks are often required because of the surety of their payment.

Exchange by Bank Draft. — The following illustrates the ordinary **bank draft,** or **bill of exchange,** which is simply a check drawn by one bank on another:

No. 328

Mechanics' Bank

St. Louis, Mo., Dec. 28, 1914.

Pay to the order of   *Clinton Ames* --------------- *$500.00*

Five hundred $\frac{no}{100}$ ~~~~~~~~~~~~~~~~~~~~~~~~~~~~*Dollars.*

To **Commercial Bank,**
      New York.   *Philip Stanley,*
                                 Cashier.

The person who signs a draft is the **drawer**; the one who is directed to pay it, the **drawee**; and the one to whom it is to be paid, the **payee**.

Banks usually keep money on deposit in some other bank, called a **correspondence bank,** in a large money center.

The draft shown above is an order on the Commercial Bank of New York to pay Clinton Ames or his order $500, and to charge the amount to the account of the Mechanics' Bank of St. Louis. By indorsing it Mr. Ames may get it cashed at any bank, which in turn may collect the cash from the New York bank or deposit the draft as a credit to draw on in the New York bank. The latter may charge it to the St. Louis bank's account, if it has one, or it may collect of the St. Louis bank, or deposit the draft there as a credit. In the end, however, the St. Louis bank must pay $500 in credit or money, though in the meantime it has had the use of the $500 paid in by the original purchaser, and has earned a fee charged for exchange.

Bankers' association money orders are bank drafts drawn by certain banks, on a circuit of banks located in important centers.

These orders are payable by any bank in the circuit and since they are in form of New York exchange, they are accepted almost anywhere in the country.

If Robert Ross of Boston wishes to collect a debt of $500 from R. C. Dodge of Chicago, he may draw on Mr. Dodge by means of a **commercial draft.**

$ 500.00   *Boston, Mass., Jan. 8, 1915.*

___At sight pay to the order of Merchants' Bank of Boston

Five hundred $\frac{no}{100}$ ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~Dollars.

Value received, and charge to account of

To R. C. Dodge,   *Robert Ross.*
     225 Wabash Ave., Chicago, Ill.

The Merchants' Bank of Boston cashes this draft, deducting charges for collection, and sends it to some Chicago bank to be collected. The Chicago bank either collects of Mr. Dodge and remits to the Boston bank or, if payment is refused, returns the draft to the Boston bank, which in turn notifies Mr. Ross to "take up the draft" and refund the money advanced on it. Some banks would not cash the draft until collected.

Mr. Ross might make the paper payable to "Self." It would then be merely a demand for payment, becoming a **sight draft** by his indorsement.

Suppose that Mr. Dodge's debt is not due for sixty days. Instead of "at sight" the draft will read "sixty days after sight." Such a commercial draft is a **time draft.**

When presented to Mr. Dodge, if he intends to pay it, he will write across the face in red ink "Accepted," with the date, and sign his name underneath. The draft is then called an **acceptance** and is equivalent to a promissory note, due sixty days after the date when it was accepted.

An acceptance may be sold by indorsing it and discounting it for the time it has to run. In discounting, banks sometimes add several days to the discount term to allow for time to collect; they may also charge exchange.

Time drafts are sometimes made payable at a given time *after date.* Then the acceptor does not write the date of acceptance.

Exchange Market. — Bills of exchange are credits, and when indorsed may be bought and sold. The business of buying and selling them is carried on largely by banks and in the open market. Hence, as with any other commodity, when the supply offered on the market exceeds the demand, the price goes down and exchange is at a **discount,** or **below par,** that is, below the **face value;** but when the demand exceeds the supply, the price goes up and exchange is at a **premium,** or **above par.**

Thus, when the Chicago banks have large sums on deposit in New York banks and wish to use part of their money at home, they will sell sight drafts on New York at favorable rates, probably at par for small amounts, and slightly below par for large amounts.

When the deposits of Chicago banks in New York banks are not sufficient to meet the demands made upon them for New York drafts, they are obliged to send money to New York by express at considerable expense and they must charge something above par for their New York drafts.

How Exchange is Quoted. — The price of domestic exchange (for sight drafts) is quoted either at par, at a certain per cent premium or discount, or at a certain amount of money, premium or discount, for $1000.

The following, taken from a New York newspaper, are market quotations for exchange on New York at:

Boston   . . . . . .   10 ⊄ discount.
Chicago . . . . . .   10 ⊄ premium.
New Orleans   . . . .   commercial,. par; bank, $1 premium.
Savannah . . . . .   buying, 50 ⊄ discount; selling, 75 ⊄ premium.
San Francisco . . . .   sight, 3½ ⊄ premium; telegraph, 7½ ⊄ premium.

Written Exercises

1. At the rates quoted above, how much will New York exchange for $5000 cost in Boston? in Chicago? In which city are the banks more eager to sell New York exchange?

2. Find the cost of a draft on a New York bank for $10,000 in New Orleans; in Savannah.

Find, to the nearest cent, the cost of a draft for:

3. $500 at $\frac{1}{4}\%$ premium   **6.** $2500 at $\frac{1}{8}\%$ premium

4. $475 at $\frac{1}{8}\%$ discount   **7.** $50,360 at $\frac{1}{4}\%$ discount

5. $750 at $\frac{1}{10}\%$ premium   **8.** $115,500 at $\frac{1}{15}\%$ discount

9. Find the cost of a sixty-day draft for $825, exchange being at $1.25 premium.

10. A cotton dealer in Savannah sold some cotton to a New York firm and drew on them at sight for $12,000. A Savannah bank bought the bill of him at a *discount* of 50¢ per $1000. How much did the bank pay for the bill?

11. Thomas Stewart of Kalamazoo draws on William Wood of New York for $3600. Supplying necessary data, write the draft. If the bank in Kalamazoo charges $\frac{1}{8}\%$ for collection, what are the proceeds?

12. Find the proceeds of a 30-day draft for $1600, dated June 3 and discounted June 6 at 6%.

13. What will be the proceeds of a draft for $2000, payable 30 days after date, discounted at 6%, if 6 days are allowed for the collection and the return of the money?

14. The Cleveland Electric Co. sent to C. Avery, their salesman in Nashville, a New York draft for $200, dated June 27, on the First National Bank. The draft was cashed for Mr. Avery by James Hanlon, who deposited it in the Cotton Exchange Bank. This bank forwarded it to the First National Bank of New York as credit. Write the draft and its indorsements.

15. What are the proceeds of a 60-day draft for $1885.50 discounted at 7% for the full term, if the bank charges 80¢ in addition for collection?

16. Henry Adams discounted a draft for $3200 for 63 days at 6% and was charged $\frac{1}{32}\%$ of the face of the draft for exchange. Find the net proceeds of the draft.

17. A commission merchant sold 196 boxes of oranges @ $2.40. After deducting a 2 % commission, and $51.41 for freight and storage, he remitted the proceeds by New York exchange at par. What was the face of the draft?

18. What were the net proceeds of a draft for $1500 at 60 days' sight, if it was accepted Mar. 20 and discounted Mar. 23, at 4 %, and there was an additional charge of 75 ¢ for collection?

19. M. Clay of Charleston sold 200 bbl. tar @ $2.80 and 250 bbl. D resin @ $3.05 to Hines & Co., New York. Hines & Co. remitted with the order a sight draft for $275 on Colfax & Son, Charleston, and authorized M. Clay to draw at 60 days' sight for the balance. Write both drafts, supplying dates.

20. A wholesale dealer sold goods listed at $670.50, discounts 33⅓ %, 5 %, and drew on the buyer at 60 days' sight. The draft was discounted for 66 days at 6 %, and ⅛ % of the face of the draft was charged for exchange. Find the net proceeds.

21. A merchant in Minneapolis sold a New York firm 800 bbl. of flour @ $3.25 on the cars at Minneapolis, terms 60 days' sight draft. On receiving the New York firm's acceptance, he discounted it at the bank for 66 days, at 6 %, and was charged 15 ¢ per $1000 of the face of the acceptance for exchange. Find the net proceeds.

22. Express companies and the U. S. subtreasury charge 75 ¢ per $1000 to transfer money from New York to New Orleans, 50 ¢ to St. Louis, Chicago, and the West, and 25 ¢ to near-by Eastern cities. One week in October when money was in demand for moving the crops, New York banks transferred through these agencies $3,142,000 to New Orleans and the South, $7,495,000 to St. Louis, $3,280,000 to Chicago and the West, and $3,746,000 to near-by Eastern cities. What was the total cost of the week's exchange?

RATIO AND PROPORTION

RATIO

The relation of one number to another of the same kind, expressed by the quotient of the first divided by the second, is the **ratio** of the first number to the second.

Thus, the ratio of 15 to 3 is $15 \div 3$, or $\tfrac{15}{3}$, or 5; the ratio of 3 to 15 is $\tfrac{1}{5}$.

All ratios are abstract numbers.

Thus, the ratio of 3 in. to 9 in. is 3 to 9, or $\tfrac{1}{3}$.

The **ratio sign** is a colon (:).

Thus, the ratio of 15 to 3 may be written $15:3$; the ratio of 3 to 15, $3:15$.

The first number in a ratio is the **antecedent**, and the second number, the **consequent**.

$$3 \div 4 = \tfrac{3}{4} = 3:4 = \frac{\text{dividend}}{\text{divisor}} = \frac{\text{numerator}}{\text{denominator}} = \frac{\text{antecedent}}{\text{consequent}}.$$

The antecedent and the consequent of a ratio are its **terms.**

Since a ratio may be written in the form of a fraction, and since multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction,

Multiplying or dividing both terms of a ratio by the same number does not change its value.

Thus, $4:8$ reduced to lowest terms is $1:2$; also $4:8 = 12:24$.

Oral Exercises

Tell the ratio of:

1. 14 to 7
2. 7 to 14
3. 18 to 6
4. 6 to 18
5. 15 to 30
6. 12 to 60
7. 25 to 100
8. 90 to 100

Reduce to lowest terms:

9. $10:2$
10. $12:6$
11. $16:4$
12. $20:5$
13. $3:27$
14. $4:40$
15. $9:72$
16. $8:64$
17. $\tfrac{12}{24}$
18. $\tfrac{16}{48}$
19. $\tfrac{60}{80}$
20. $\tfrac{75}{100}$

Find the value of each ratio:

1. $\frac{4}{5} : \frac{2}{5}$ 5. $.7 : .8$ 9. $2\frac{1}{2} : 7\frac{1}{2}$ 13. 2 pk. : 2 bu.

2. $\frac{3}{4} : \frac{1}{2}$ 6. $.4 : 10$ 10. $3\frac{1}{3} : 6\frac{2}{3}$ 14. 3 qt. : 2 gal.

3. $\frac{5}{8} : \frac{1}{4}$ 7. $.6 : 3.6$ 11. $10 : 4.5$ 15. 4 ft. : 16 in.

4. $\frac{1}{2} : \frac{1}{8}$ 8. $7.5 : 5$ 12. $7.2 : 80$ 16. 2 yd. : 18 in.

17. What is the ratio of 1 rod to 1 mile?

18. What is the ratio of a long ton to a short ton?

19. In an alloy of 78 oz. of silver and copper there are 6 oz. of silver. Find the ratio of the silver to the copper.

20. To seed a large lawn, 16 lb. of red clover seed were mixed with 56 lb. of orchard grass seed. What was the ratio of red clover seed to orchard grass seed?

21. In a mixed mass of brass and iron weighing 57 lb., there are 15 lb. of iron. Find the ratio of the iron to the brass.

22. In a Bordeaux mixture used to spray a peach orchard, 28 lb. of copper sulphate and 84 lb. of lime were mixed. Find the ratio of the copper sulphate to the lime.

23. What is the ratio of muriate of potash to lime in a fertilizer for potatoes, if it contains 320 lb. of the former to 1 ton of the latter?

24. A solution for preserving eggs was made by mixing 2 qt. of water glass with $4\frac{1}{2}$ gal. of water. In what ratio were the water glass and water mixed?

25. In what ratio were nitrate of soda and acid phosphate mixed in a fertilizer for cucumbers, if 1250 lb. of the first were used to 1 ton of the second?

26. For a certain amount of concrete, $13\frac{1}{2}$ bags of cement and 2 cu. yd. of gravel were used. Find the ratio of the cement to the gravel. (1 bag cement = 1 cu. ft.)

SIMPLE PROPORTION

An equality of two ratios is a **proportion**.

Thus, $4:6 = 8:12$ is a proportion; also $\frac{4}{6} = \frac{8}{12}$.

The double colon (: :) is often used instead of the sign of equality.

The proportion $4:6 = 8:12$, or $\frac{4}{6} = \frac{8}{12}$, or $4:6::8:12$ is read " the ratio of 4 to 6 is equal to the ratio of 8 to 12 " ; or more briefly, " 4 is to 6 as 8 is to 12."

The first and last terms of a proportion are the **extremes**, and the second and third terms the **means**.

, In $4:6 = 8:12$, the extremes are 4 and 12; the means, 6 and 8.

In the proportion $4:6 = 8:12$, what is the product of the extremes? of the means? How does the product of the extremes compare with the product of the means?

Compare the product of the extremes with the product of the means in $1:2 = 4:8$; in $1:2 = 5:10$; in $2:4 = 6:12$.

In any proportion the product of the extremes is equal to the product of the means.

Oral Exercises

In the following, compare the first ratio with the second to find whether or not they are equal; then compare the product of the extremes in each case with the product of the means :

1. $\frac{3}{6} = \frac{4}{8}$	**5.** $2:4 = 5:10$	**9.** $4:8 = 5:10$
2. $\frac{3}{6} = \frac{5}{10}$	**6.** $2:8 = 5:20$	**10.** $3:5 = 9:15$
3. $\frac{3}{6} = \frac{7}{14}$	**7.** $3:6 = 9:18$	**11.** $4:6 = 12:18$
4. $\frac{3}{6} = \frac{6}{12}$	**8.** $2:8 = 6:24$	**12.** $6:9 = 10:15$

13. Is $2:8 = 4:12$ a true proportion? How do you know?

14. Change $6:3 = 8:2$ to a true proportion by changing one of the extremes ; by changing one of the means.

1. If $24 : 7 = 48 : x$, what is the value of x?

SOLUTION

$$24 : 7 = 48 : x$$

Since the product of the extremes equals the product of the means,

$$24\, x = 7 \times 48$$

whence, $x = \dfrac{7 \times \overset{2}{\cancel{48}}}{\cancel{24}} = 14.$

Solve the following proportions:

2.	$15 : x = 6 : 2$	**10.**	$16 : 6 = x : 12$	**18.**	$\frac{1}{2} : \frac{3}{4} = x : 1$
3.	$10 : 5 = x : 7$	**11.**	$25 : x = 5 : 18$	**19.**	$x : \frac{2}{3} = \frac{1}{2} : \frac{1}{4}$
4.	$12 : 8 = 9 : x$	**12.**	$40 : 8 = x : 30$	**20.**	$2 : 3 = \frac{1}{2} : x$
5.	$x : 36 = 4 : 8$	**13.**	$x : 49 = 6 : 14$	**21.**	$\frac{1}{4} : \frac{3}{4} = x : 8$
6.	$30 : 6 = x : 4$	**14.**	$18 : 7 = 72 : x$	**22.**	$\frac{1}{8} : x = \frac{1}{2} : \frac{4}{5}$
7.	$35 : x = 7 : 6$	**15.**	$27 : 6 = 18 : x$	**23.**	$\frac{3}{4} : x = \frac{1}{4} : \frac{1}{3}$
8.	$42 : 7 = 6 : x$	**16.**	$32 : x = 24 : 6$	**24.**	$\frac{5}{6} : \frac{1}{2} = x : \frac{4}{5}$
9.	$75 : x = 5 : 9$	**17.**	$45 : 9 = x : 36$	**25.**	$\frac{1}{6} : \frac{3}{8} = \frac{3}{4} : x$

26. Solve. $3 \text{ tons} : x \text{ tons} = \$15 : \$40$.

SUGGESTION. — Since $\dfrac{3 \text{ tons}}{x \text{ tons}} = \dfrac{3}{x}$ and $\dfrac{\$15}{\$40} = \dfrac{15}{40}$, the number of tons may be found from the proportion $\dfrac{3}{x} = \dfrac{15}{40}$, or $3 : x = 15 : 40$.

Solve the following proportions:

27. $\dfrac{18 \text{ yd.}}{24 \text{ yd.}} = \dfrac{\$x}{\$3.60}$

28. $\dfrac{15 \text{ bu.}}{x \text{ bu.}} = \dfrac{\$37.50}{\$100}$

29. $\dfrac{24 \text{ pt.}}{30 \text{ pt.}} = \dfrac{\$24.20}{\$x}$

30. $\dfrac{4 \text{ men}}{12 \text{ men}} = \dfrac{5 \text{ days}}{x \text{ days}}$

31. $\dfrac{x \text{ ft.}}{5600 \text{ ft.}} = \dfrac{4}{1120}$

32. $\dfrac{250 \text{ lb.}}{600 \text{ lb.}} = \dfrac{x}{5280}$

33. If 4 tons of coal cost $27, how much will 14 tons cost at the same price per ton?

SOLUTION. — Since 14 tons cost more than 4 tons, the fourth term, x dollars, must be greater than the third term, $27; hence, the second term must be 14 because 14 is greater than 4, which must be the first term.

Therefore, $4 : 14 = 27 : x.$
Solving, we have $x = 94\frac{1}{2}.$
Hence, the cost of 14 tons is $94.50.

NOTE. — In proportion problems, the words "at the same rate" are always understood.

Find the cost of :

34. 6 chairs when 3 chairs cost $9.

35. 9 aprons when 2 aprons cost $.98.

36. 6 watches when 4 watches cost $60.

37. 12 mufflers when 9 mufflers cost $8.82.

38. 12 thimbles when 8 thimbles cost $3.92.

39. 10 umbrellas when 3 umbrellas cost $11.25.

40. 24 ladies' waists when 14 waists cost $31.50.

41. 10 iron safes when 3 iron safes cost $127.50.

42. 18 boys' hats when 10 boys' hats cost $12.50.

43. 18 men's hats when 15 men's hats cost $41.25.

44. 12 office desks when 2 office desks cost $84.50.

45. 50 alarm clocks when 12 alarm clocks cost $10.20.

46. 18 sewing machines when 6 sewing machines cost $129.

47. 36 doz. handkerchiefs when 4 doz. handkerchiefs cost $6.

48. If 16 tons of coal can be hauled in 6 loads, how many tons can be hauled in 21 loads?

49. If a 15-pound pressure in a pneumatic tank will raise water 33 ft., to what height will a 20-pound pressure raise it?

50. If a machine can reap $7\frac{1}{2}$ acres of rice in 10 hr., how long will it take it to reap 30 acres?

51. If 24 lb. of maple sugar cost $3.60, how much must be paid for 5 lb. ?

52. If a street car runs 9 mi. in 45 min., how many miles will it run in 1 hr.?

53. If the interest on a sum of money for 6 mo. is $33, what is the interest for 8 mo. at the same rate ?

54. If $600 yields $140 interest in a certain time, how much interest does $750 yield in the same time at the same rate ?

55. If 55 acres of land yield 1430 bu. of wheat, how many bushels do 132 acres yield ?

56. A man bought a 180-acre farm for $12,000 and sold 30 acres of it at cost.   How much did he receive for the part sold?

57. If a woman receives $⅘ for picking 40 qt. of strawberries, how many quarts must she pick to earn $3¾ ?

58. If 100 lb. of skim milk make 18 lb. of cottage cheese, how much skim milk is needed to make 2¼ lb. of cottage cheese ?

59. When the cost of shipping 90 cans of milk is $13½, how many cans of milk can be shipped for $1⅙ ?

60. A grain elevator unloads 2 railway cars of grain in 5 min. How many hours does it take to unload 240 cars ?

61. If 10 sheep require 2⅘ acres of land for their support, how many sheep will 36 acres support ?

62. If it costs 66 ¢ to have a rug of 8¼ sq. yd. cleaned, how much will it cost for a rug of 15½ sq. yd. ?

63. If a manufacturing plant turns out 7500 concrete building blocks in 6 days, how many will it turn out in 20 days ?

64. If a machine makes 900 pins per minute, how many pins will it make in 8 hr. ?

65. If 10 men can dig a ditch in 12 days, how many days will it take 15 men to dig it?

SOLUTION

Let x (the fourth term) represent the number of days it will take 15 men.

Since 15 men take less time than 10 men, the fourth term, x days, must be smaller than the third term, 12 days. Therefore the second term must be smaller than the first term.

$$15 : 10 = 12 : x.$$
$$15\,x = 120.$$

Solve, $\qquad\qquad x = 8.$

Hence, 15 men can dig the ditch in 8 days.

66. If 16 men can do a piece of work in 15 days, how long will it take 20 men to do it?

67. If 16 men can do a piece of work in 15 days, how many men will be required to do it in 10 days?

68. If a force of men working 8 hr. a day can do a piece of work in 20 days, how long will it take them working 10 hr. per day?

69. If it will take 24 men 18 days to repair a street and the work must be done in 16 days, how many men must be employed?

70. A fifty-gang plow turns 1 acre of soil every $4\frac{1}{4}$ min. How many acres can be turned in $8\frac{1}{2}$ hr.?

71. If 150 soldiers consume a certain amount of flour in 9 weeks, how many days would an equal amount last 210 soldiers?

72. If sound travels 6160 ft. in $5\frac{1}{2}$ sec., how many miles does it travel in a minute?

73. A motor-boat race over a course of 136.5 mi. on the Hudson River was run at the rate of 52.5 mi. in 2 hr. How long did it take to run the complete course?

74. On a train of 18 cars, 4050 bbl. of flour are loaded. How many cars must be added so that the train may carry 5175 bbl.?

51. If ... major sugar cost $3.60, how much must ...

52. If a ... runs 9 mi. in 45 min., how many mi...

53. The ... sum of money for 6 mo. is $... at the same rate?

54. ... $140 interest in a certain time. ... in the same time at the sam...

55. ... yield 1430 bu. of whe... yield?

56. ... a 180-acre farm for $12... How much did he receive...

57. If a man receives $... for picking 40... quarts must she pick to earn $...

58. ... lb. of skim milk make 18 lb... skim milk is needed to make 2¼ l...

59. When the cost of shipping 90 c... cans of milk can be shipped for...

60. A grain elevator unloads 2 ra... many hours does it take to un...

61. If 10 sheep require 2½ ac... how many sheep will 36 acres su...

62. If it costs 66¢ to have a... much will it cost for a rug of ...

63. ...

11. A wooden freight car together with its load weighed 90,000 lb. What was the weight of the car alone, if its weight was to the weight of the load as 1 is to 2?

12. In making sausage 1 part of lean beef, 1 part of fat pork, and 2 parts of lean pork were ground together. How much of each was used in making 25 lb. of sausage?

13. A fertilizer for timothy hay was made of 1 part of muriate of potash to 4 parts of acid phosphate. How many pounds of each were used in 2 tons of the fertilizer?

14. A farmer had 420 gal. of diluted kerosene emulsion to spray his orchard. If 1 part of concentrated solution was used to 6 parts of water, how much of the concentrated solution had he?

15. The freight earnings of two railroads on a train load of grain were $2160. One carried the grain 400 mi., the other 500 mi. Find the earnings apportioned to each road.

16. For seeding a 12½-acre field, 237½ lb. of timothy, red clover, and alsike clover seed, mixed in the proportion of 10, 6, and 3, were used. How much of each was required per acre?

17. The total receipts of a coal mining company one year were $16,725,000, and the expenses were to the net earnings as 13 is to 2. What were the expenses? the net earnings?

18. The annual earnings of a certain railroad company are $78,000,000. How much is received from freight charges, from passenger service, and from other sources (such as mail, express, etc.), if they are proportional to 8, 4, and 1?

19. Cement, sand, and stone are mixed in the proportion of 1, 2, and 4 to form concrete. How many barrels of cement and how many cubic yards of sand would be used with 48 cu. yd. of stone? (1 bbl. cement = 4 cu. ft. approximately.)

PARTNERSHIP

When two or more persons agree to combine their money, goods, labor, or skill, in some business enterprise, and to share the profits and losses of the business in certain proportions, they become **partners**, thus forming a **partnership**.

. The partners are collectively called a **firm**, or a **house**.

The investment of a partner is his **capital**.

The capital may be money or anything that has a money value in the business, as goods, labor, skill, experience, the "good will" of the trade, etc.

The gains and losses of a firm are shared in proportion to the capital of each partner.

Written Exercises

1. A and B engaged in business as partners and gained $4000. A's capital was $10,000, and B's was $6000. Find each one's share of the profits.

SOLUTION. — Entire capital = $16,000.

A's share of the capital   = $\frac{10000}{16000}$, or $\frac{5}{8}$;   A's gain = $\frac{5}{8}$ of $4000, or $2500.

B's share of the capital   = $\frac{6000}{16000}$, or $\frac{3}{8}$;   B's gain = $\frac{3}{8}$ of $4000, or $1500.

Test. — A's gain + B's gain = $2500 + $1500 = $4000, the entire gain.

2. Two partners gained $6000 in business one year. Find each one's share, their investments being in the ratio 1 : 4.

3. Apportion a gain of $2500 to two partners in a business, if their investments are $1500 and $3500, respectively.

4. Apportion a loss of $2400 to the three partners in a business, if their investments are $11,000, $15,000, and $6000.

5. A and B owned a strawboard factory, A's investment being $75,000 and B's $45,000. If the net earnings for one year were $11,200, how much did each partner receive?

6. Four men formed a partnership, investing $5600, $5600, $6000, and $6800, respectively. Find each partner's share of a profit amounting to $6000.

POWERS AND ROOTS

RAISING TO POWERS

1. $2 \times 2 = 4$ $2 \times 2 \times 2 = 8$ $2 \times 2 \times 2 \times 2 = 16$

or, $2^2 = 4$ $2^3 = 8$ $2^4 = 16$

What is the product when 2 is taken 2 times as a factor? 3 times? 4 times? 5 times? 6 times?

What is the *second power* of 2? the *third* power of 2? the *fourth* power? the *fifth* power? the *sixth* power?

2. What number is the second power of 3? the third power of 3? the fourth power?

3. Find the second power of 10; the third power.

The number of times a number is to be used as a factor may be indicated by using an **exponent** (page 73).

The product arising from using a number a certain number of times as a factor is a **power** of that number.

4 is the second power of 2, for $4 = 2 \times 2$, or 2^2; 8 is the third power of 2, for $8 = 2 \times 2 \times 2$, or 2^3; 16 is the fourth power of 2; etc.

A number is regarded as the *first* power of itself.

If the side of a *square* is 2, the area is the *second* power of 2; if the side is 3, the area is the *second* power of 3; etc.

Therefore, the *second* power of a number is its **square**.

If the edge of a *cube* is 2, the volume is the *third* power of 2; if the edge is 3, the volume is the *third* power of 3; etc.

Therefore, the *third* power of a number is its **cube**.

285

1. Find the square of :

1   2   3   4   5   6   7   8   9   10   11   12

2. Find the cube of 1; of 2; of 3; of 4; of 5.

3. Find the fourth power of 1; of 2; of 3.

4. 10^2, 10^3, 10^4, 10^5, are read "ten square," "ten cube," "ten fourth power," and "ten fifth power," respectively. Tell the meaning and value of each indicated power.

5. Multiply $\frac{2}{3}$ by $\frac{2}{3}$, or square $\frac{2}{3}$.

6. Square $\frac{1}{2}$; $\frac{3}{4}$; $\frac{5}{6}$; $\frac{6}{5}$; .3; .5; .7; .12; 1.2.

7. Cube $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{3}{4}$; .2; .3; .5; .1; .02; .05.

The square of a fraction may be obtained by squaring both terms; the cube, by cubing both terms; etc.

Raise each of the following to the power indicated :

1. 20^2	**5.** 37^2	**9.** 12^3	**13.** $(\frac{7}{8})^3$	**17.** 1.5^2
2. 25^2	**6.** 72^2	**10.** 21^3	**14.** $(\frac{5}{12})^3$	**18.** 7.5^2
3. 45^2	**7.** 85^2	**11.** 15^3	**15.** $(\frac{15}{28})^2$	**19.** 1.25^2
4. 52^2	**8.** 94^2	**12.** 11^4	**16.** $(\frac{25}{32})^2$	**20.** $.111^2$

Find the area of a square whose side is :

21. 21 in.	**24.** $12\frac{1}{2}$ in.	**27.** 44.7 ft.	**30.** 2.25 ft.
22. 17 yd.	**25.** $18\frac{1}{4}$ in.	**28.** 50.4 ft.	**31.** 7.56 ft.
23. 32 rd.	**26.** $25\frac{3}{4}$ ft.	**29.** 1.19 ft.	**32.** .842 mi.

Find the volume of a cube whose edge is :

33. 14 in.	**35.** 25 in.	**37.** 22 ft.	**39.** 3.4 ft.
34. 18 ft.	**36.** 2.5 in.	**38.** 1.5 yd.	**40.** 12.5 yd.

41. The number of feet a body will fall from rest, in any number of seconds, is 16.08 times the square of the number of seconds. How far will a body fall in 4 sec.? in 15 sec. ?

1. Any integer of two or more figures may be regarded as composed of tens and units. Express 35 as tens and units; 125.

2. You already know the squares of the integers from 1 to 12, inclusive. The square of 13 may be obtained thus:

```
 13          10 + 3
 13          10 + 3
 ──          ──────
 39          30 + 9     (Product of 10 + 3 by 3)
 13         100 + 30    (Product of 10 + 3 by 10)
───         ─────────────
169         100 + 2(30) + 9 = 10² + 2(10 × 3) + 3²
```

The square of any integer composed of tens and units is equal to the square of the tens, plus twice the product of the tens and the units, plus the square of the units.

Oral and Written Exercises -

1. Find the square of the number 14.

SOLUTION. $14^2 = 10^2 + 2(10 \times 4) + 4^2 = 100 + 80 + 16 = 196$.

2-12. Complete this table, then commit it to memory:

NUMBER	SQUARE	NUMBER	SQUARE	NUMBER	SQUARE	NUMBER	SQUARE	NUMBER	SQUARE
1	1	6	36	11	121	16	. . .	21	. . .
2	4	7	49	12	144	17	. . .	22	. . .
3	9	8	64	13	169	18	. . .	23	. . .
4	16	9	81	14	196	19	. . .	24	. . .
5	25	10	100	15	. . .	20	. . .	25	. . .

Give quickly the area of a square whose side is:

13. 3 in. **15.** 11 yd. **17.** 15 in. **19.** 22 yd.

14. 5 ft. **16.** 13 rd. **18.** 17 ft. **20.** 24 in.

Tell the square of: Square as in exercise **1**:

21. 7 **24.** 12 **27.** 21 **30.** 28 **33.** 45 **36.** 120

22. 9 **25.** 14 **28.** 23 **31.** 64 **34.** 81 **37.** 125

23. 8 **26.** 16 **29.** 25 **32.** 35 **35.** 92 **38.** 114

EXTRACTING ROOTS

1. Since $25 = 5 \times 5$, what is one of the two equal factors of 25, or the *square root* of 25? Give the square root of 16 ; 49 ; 36.

2. Find the *cube root* of $2 \times 2 \times 2$, or of 2^3; of 27, or of 3^3; of 64.

One of the *two* equal factors of a number is its **second,** or **square, root**; one of the *three* equal factors, the **third,** or **cube, root.**

The **root sign,** or **radical sign,** is $\sqrt{}$. When placed over a number, it indicates that a root is to be taken.

Thus, $\sqrt{16}$ or $\sqrt[2]{16}$ indicates the square root of 16 ; $\sqrt[3]{8}$, the cube root of 8. The 2 in $\sqrt[2]{16}$ (usually omitted) or 3 in $\sqrt[3]{8}$ is called the **index** of the root.

It follows from p. 285 that if the area of a square is 4, its side is $\sqrt{4}$, or 2 ; if the area is 9, the side is $\sqrt{9}$, or 3 ; etc. That is,

The side of a square equals the square root of its area.

Similarly, *the edge of a cube equals the cube root of its volume.*

Oral Exercises

From the squares memorized, tell the square root of :

1. 16	**4.** 49	**7.** 100	**10.** 121	**13.** 400
2. 36	**5.** 64	**8.** 144	**11.** 225	**14.** 256
3. 25	**6.** 81	**9.** 196	**12.** 169	**15.** 625

ROOTS BY FACTORING

Written Exercises

1. Find the square root of 784 ; the cube root of 216.

SOLUTION.— Factoring and arranging the factors in *two* like groups, we have

$$784 = 2 \times 2 \times 2 \times 2 \times 7 \times 7$$
$$= (2 \times 2 \times 7) \times (2 \times 2 \times 7).$$

Then, $\sqrt{784} = 2 \times 2 \times 7$, or 28.

SOLUTION.— Factoring and arranging the factors in *three* like groups, we have

$$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$$
$$= (2 \times 3) \times (2 \times 3) \times (2 \times 3)$$

Then, $\sqrt[3]{216} = 2 \times 3$, or 6.

Find the square root of: Find the cube root of:

2. 900	**6.** 1024	**10.** 64	**14.** 8000
3. 729	**7.** 1296	**11.** 125	**15.** 1728
4. 676	**8.** 2025	**12.** 512	**16.** 3375
5. 441	**9.** 5184	**13.** 729	**17.** 5832

Find the side of a square whose area is:

18. 324 sq. in.	**21.** 1600 sq. ft.	**24.** 2500 sq. rd.
19. 484 sq. in.	**22.** 1225 sq. ft.	**25.** 3025 sq. rd.
20. 576 sq. in. ·	**23.** 2304 sq. ft.	**26.** 4096 sq. rd.

Find the edge of a cube whose volume is:

27. 216 cu. yd.	**29.** 1000 cu. ft.	**31.** 4096 cu. in.
28. 729 cu. yd.	**30.** 2744 cu. ft.	**32.** 9261 cu. in.

33. A square lot whose area is 4900 sq. rd. has a road running along one side of it. What length of road borders on the lot?

34. The volume of a cubical block of marble is 27,000 cu. in. Find the length of its edge.

SQUARE ROOT

Observe the square roots of these *perfect squares.*

$\sqrt{1} = 1$	$\sqrt{100} = 10$	$\sqrt{10000} = 100$
$\sqrt{25} = 5$	$\sqrt{3600} = 60$	$\sqrt{160000} = 400$
$\sqrt{81} = 9$	$\sqrt{9801} = 99$	$\sqrt{998001} = 999$

How many figures are there in the square root of any perfect square that is expressed by not more than *two* figures? by *four* figures or by *three* figures? by *six* figures or by *five* figures?

The number of figures in the square root of a perfect square is the same as the number of periods of two figures each into which the number can be separated, beginning at units.

The left-hand period may contain only one figure.

1. What is the square root of 576, or what is the side of a square whose area is 576 square units?

$$5'76 \quad \underline{|20}$$
$$20^2 = \qquad 4\ 00 \quad \underline{4}$$
$$2 \times 20 = \quad 40\overline{)1\ 76} \quad \overline{24}$$
$$(40 + 4) \times 4 = 1\ 76$$

Since the number of figures in the square root of a number may be determined by separating the number into periods of two figures each, beginning at units (5'76), the square root of 576 is seen to be composed of *tens* and *units*.

Since the square of tens is hundreds, 5 hundreds must be the square of at least 2 tens. 2 tens or 20 squared is 400, and 400 subtracted from 576 leaves 176. Therefore, the root 20 must be increased by such an amount as will exhaust the remainder.

The square, *A*, already formed from the 576 square units is one whose side is 20 units, but inasmuch as the number of units has not been exhausted, such additions must be made to the square as will exhaust the units and keep the figure a square. The necessary additions are two equal rectangles, *B* and *C*, and a small square, *D*.

Since the square, *D*, is small, the area of the rectangles, *B* and *C*, is nearly 176 square units. This area, divided by the length of the rectangles, will give the width, which is 4 units. The width of the additions is 4 units, and the entire length, including the small square, is 44 units; therefore, the area of all the additions is 4 times 44 units, or 176 square units, which is equal to the entire number of square units to be added. Hence, the side of the square is 24 units, or the square root of 576 is 24.
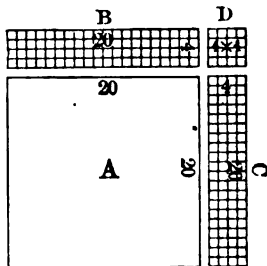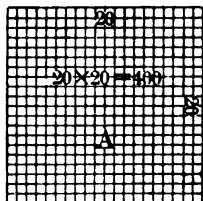
2. Extract the square root of 3844.

$$38'44 \quad \underline{|62}$$
$$36$$
$$2 \times 60 = 120 \overline{)2\ 44}$$
$$\underline{2}$$
$$\overline{122} \quad 2\ 44$$

Separating the number into periods, we find that the root is composed of tens and units. Since the largest square in 38 is 36, the tens of the root cannot be greater than 6 tens, or 60. Writing 6 tens in the root, squaring, and subtracting from 3844, we find there is a remainder of 244.

Since the square of a number composed of tens and units is equal to (*the square of the tens*) + (*twice the product of the tens and the units*) + (*the square of the units*), when the square of the tens has been subtracted, the remainder, 244, is twice the product of the tens and the units, plus the square of the units, or only a little more than twice the product of the tens and the units.

Therefore, 244 divided by twice the tens is approximately equal to the units. 2 × 6 tens, or 120, then, is a *trial divisor*. Dividing 244 by the trial divisor, we find the units' figure to be 2.

Since twice the tens are to be multiplied by the units, and the units also are to be multiplied by the units to obtain the square of the units, to abridge the process the tens and units are first added, forming the *complete divisor*, 122, and then multiplied by the units. Thus (120 + 2) multiplied by 2 = 244. This product is subtracted from 244. There is no remainder. Therefore, the square root of 3844 is 62.

Extract the square root of :

3. 289	**8.** 2809	**13.** 5184	**18.** 3721	**23.** 3249
4. 841	**9.** 2601	**14.** 1156	**19.** 7056	**24.** 2401
5. 676	**10.** 6724	**15.** 6889	**20.** 2116	**25.** 9604
6. 361	**11.** 1936	**16.** 1089	**21.** 9025	**26.** 6241
7. 961	**12.** 8281	**17.** 8836	**22.** 5776	**27.** 9801

Find the side of a square whose area is :

28. 529 sq. in. **31.** 7225 sq. ft. **34.** 7744 sq. rd.

29. 729 sq. in. **32.** 1444 sq. ft. **35.** 4225 sq. rd.

30. 784 sq. in. **33.** 2209 sq. ft. **36.** 4624 sq. rd.

37. The area of a square field is 2025 sq. rd. How many rods of fence are required to inclose it ?

38. I have a lot 144 ft. long and 64 ft. wide. What is its area in square feet ? Find the side of a square lot that has the same area.

39. An officer has 5625 men and wishes to arrange them in a solid square. How many men shall he place on a side ?

1. Extract the square root of 15,625 ; of 1.5625.

```
1'56'25  |125          1'.56'25  |1.25
1                      1
22|  56              22|  56
   44                    44
245|  12 25          245|  12 25
      12 25                12 25
```

After two figures of the root in the first process have been obtained, we find that we have subtracted, in all, 10,000 + 4400, or 14,400, the square of 12 tens, *the part of the root already found.* Regarding 12 tens as the first part of the root, we obtain the units of the root in the usual way.

When there are decimal figures, as in the second process, they are pointed off into periods of two figures each, *beginning at the decimal point.* Annex a decimal cipher when necessary to give the right-hand period two places. The process is then the same as for integers.

Separate the number into periods of two figures each, beginning at units or at the decimal point.

Find the greatest square in the left-hand period, and write its root for the first figure of the required root.

Square this root, subtract the result from the left-hand period, and annex to the remainder the next period for a dividend.

Double the root already found, for a partial divisor, and by it divide the dividend, disregarding the right-hand figure. The quotient, or quotient diminished, will be the second figure of the root.

Annex to the partial divisor for a complete divisor the figure last found, multiply this divisor by the figure of the root last found, subtract the product from the dividend, and to the remainder annex the next period for the next dividend.

Proceed in this manner until all the periods have been used. The result will be the square root sought.

1. When the number is not a perfect square, annex periods of decimal ciphers and continue the process.

2. The square root of a common fraction may be found by extracting the square root of the numerator and the denominator separately or by reducing it to a decimal and then extracting the root.

Extract the square root of:

2. 15,129	**8.** 136,900	**14.** 8.0089	**20.** 4.5369
3. 24,336	**9.** 184,041	**15.** 930.25	**21.** .555025
4. 13,689	**10.** 230,400	**16.** 655.36	**22.** .633616
5. 30,976	**11.** 311,364	**17.** 282.24	**23.** .994009
6. 44,521	**12.** 864,900	**18.** 11.0224	**24.** .00675684
7. 86,436	**13.** 576,081	**19.** 14.5924	**25.** .00767376

Extract the square root and express in lowest terms:

26. $\frac{25}{81}$	**28.** $\frac{121}{400}$	**30.** $\frac{225}{625}$	**32.** $\frac{1225}{2304}$	**34.** $\frac{4096}{9216}$
27. $\frac{64}{225}$	**29.** $\frac{196}{676}$	**31.** $\frac{784}{900}$	**33.** $\frac{2025}{6184}$	**35.** $\frac{1089}{9801}$

Verify the following:

36. $\sqrt{2} = 1.414^+$ **37.** $\sqrt{3} = 1.732^+$ **38.** $\sqrt{5} = 2.236^+$

Extract the square root to three decimal places:

39. 8	**42.** $\frac{1}{3}$, or $\frac{3}{9}$	**45.** .1	**48.** .7854
40. 6	**43.** $\frac{1}{2}$, or $\frac{2}{4}$	**46.** 2.5	**49.** .41265
41. 7	**44.** $\frac{1}{5}$, or .2	**47.** 3.6	**50.** .740063

51. Find, in feet, the side of a square whose area is 1 acre.

SOLUTION

Let x = number of feet in one side of the square.

Then, the area = x^2 square feet. But 1 acre = 43,560 square feet.

Therefore, $x^2 = 43,560.$

Extracting the square root of both members,

         $x = 208.7^+$, the number of feet in the side.

Note that a square 209 feet on a side is only a little more than an acre.

Find, in rods to the nearest tenth, the length of the side of a square whose area is:

52. 10 A.   **54.** 5600 sq. rd.   **56.** .4 A.   **58.** 108,900 sq. ft.

53. 80 A.   **55.** 9578 sq. rd.   **57.** 1.5 A.   **59.** 544,500 sq. ft.

60. Find the length of the side of a square garden containing 99,225 sq. yd.

61. Find the side of a square park that contains 90 acres.

62. How many rods of fence are required to go around a square field containing 40 acres?

63. A football field is 110 yd. by 160 ft. Find, to the nearest foot, the side of a square field having the same area.

64. Find the dimensions, in rods, of a 20-acre rectangular field whose length is twice its width.

SUGGESTION.—Let x = number of rods in width. Then, $2x$ = number of rods in length, and $x(2x)$, or $2x^2$, = number of square rods in area.

65. A farm hotbed has an area of 8 sq. yd. If it is twice as long as it is wide, how many feet long is it?

66. Find the dimensions, in rods, of a 90-acre rectangular field whose length is 4 times its width.

67. If the dimensions of a 5-acre rectangular field are in the ratio 1: 2, what is the length in rods?

68. A large room has a floor space of 972 sq. yd. What are its dimensions, in feet, if the width is to the length as 1 is to 3?

69. How much more fence is required to inclose a field 30 rd. by 120 rd. than a square field of equal area?

70. What are the dimensions of an 8-acre rectangular field, if they are in the ratio 4: 5?

71. At the rate of 3 mi. an hour, how much longer will it take to walk around a rectangular field 80 rd. by 180 rd., than around a square field of equal area?

RIGHT TRIANGLES

A flat surface bounded by three straight lines is a **triangle**.
A triangle that has a right angle is a **right triangle**.

1. Since the longest side, or **hypotenuse**, of this right triangle is 5 units long, how many square units are there in the square described upon the hypotenuse?

2. Since one of the other two sides, or **legs**, is 3 units long and the other 4 units long, how many square units are there in the square described upon each? in both these squares?
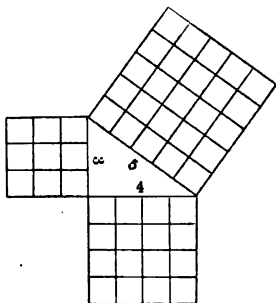
3. How does the number of units in the square of the hypotenuse compare with the number of units in the sum of the squares of the other two sides?

The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides.

Written Exercises

1. Find the hypotenuse of this right triangle.

SOLUTION. $x^2 = 6^2 + 8^2 = 36 + 64.$
    That is, $x^2 = 100.$
    Therefore, $x = 10.$
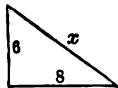
2. Draw a right triangle whose legs are 5 in. and 12 in. Compute the hypotenuse. Measure it. Compare results.

In each right triangle, find the length of the side marked x:

3.   **4.**   **5.**   **6.**
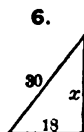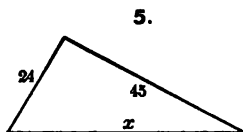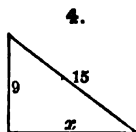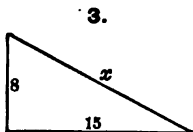
7. Draw an inch square and a straight line connecting two opposite corners, or a *diagonal*. Compute the length of the diagonal to the nearest thousandth of an inch.

8. Draw a rectangle 4 in. by 7½ in. Draw one of its diagonals and compute the area of the square described on it.

9. Draw a 5-inch square and one diagonal. How does the area of the square on the diagonal compare with the area of the 5-inch square?

10. Measure the length and the width of a rectangular room. Measure a diagonal on the floor as accurately as you can. Compute the length of the diagonal to the nearest tenth of a foot.

11. Find the length of a path extending diagonally across a rectangular field 40 rd. long and 30 rd. wide.

12. A field is in the form of a right triangle whose legs are 32 rd. and 24 rd., respectively. How many rods of fence are required to inclose it?

13. A man in a boat crossed a river 48 yd. wide, landing on the opposite shore 64 yd. downstream from his starting point. How far did he go?

14. A 40-foot ladder leans against a wall, with the foot 6 ft. from the base of the wall. Draw a sketch and compute, to the nearest hundredth of a foot, the height of the top of the ladder.

15. Two vessels sailed from the same point, one north at the rate of 15 knots an hour, the other east at the rate of 20 knots an hour. How far apart were they after 6 hr.?

16. How far apart, to the nearest tenth of a rod, are the opposite corners of a square farm that contains 360 acres?

17. Find, to the nearest tenth of a foot, the distance from a lower corner to the opposite upper corner of a room 24 ft. long, 18 ft. wide, and 12 ft. high.

REVIEW

1. Name two kinds of banks in which money may be deposited. State the purpose of each.

2. Make out a deposit slip, supplying necessary data, for: bills, $84; coin, $12.75; check on Merchants' Bank, $125.50.

3. Supplying necessary data, write a check payable to "Self" with the stub properly filled out. What must the maker do to this check before he can get it cashed?

4. Find the bank discount on a 60-day note for $1800, discounted at 4 % on the day it was made, without interest; with interest at 6 %. Upon what is the bank discount reckoned?

5. Find the amount, at the end of 5 yr., of a savings bank deposit of $500, interest quarterly at 4 %, remaining on deposit. How much greater is it than the amount of a note for $500 for the same time, interest at 4 %? Explain.

6. Define exchange. Name six methods of exchange. Supplying necessary data, write a bank draft for $1000; a commercial draft for the same amount.

7. How much more does it cost to send a telegraphic money order for $86, the telegraphic rate being 40–3, than to send a postal money order for a like amount including postage?

8. Find the ratio of 9 to 27; $87\frac{1}{2}$ to 100; 10 % to 1; 35 to 105. Define ratio; antecedent; consequent. How does multiplying or dividing both terms of a ratio by the same number affect the value of the ratio? Explain.

9. Solve: $24 : 72 :: 3 : x$; also, $\frac{1}{3} : \frac{1}{4} :: 1 : x$. Upon what principle does the solution depend? Define proportion; means; extremes.

10. Separate 25 into parts proportional to 2 and 3; to $\frac{1}{2}$ and $\frac{1}{3}$. Explain each process. What is partitive proportion?

11. Raise to the power indicated: 9^3; $.28^2$; $(\frac{3}{4})^4$; $(\frac{7}{12})^3$. Explain each process. Define power; square; cube.

12. Define root. Find the square root of 5329; of 53.29. Explain each step. How many figures are there in the square root of a perfect square of 4 figures? of 3 figures? of 6 figures?

13. Extract the square root, to the nearest thousandth, of: 5; $\frac{5}{8}$, or $\frac{80}{88}$. Explain each.

14. Find the hypotenuse of a right triangle whose sides are 9 ft. and 12 ft., respectively. Explain.

Written Exercises

Raise to the indicated power:

1. 50^2	**4.** 48^2	**7.** $.05^3$	**10.** $(\frac{5}{6})^3$	**13.** $(7\frac{1}{2})^2$
2. 35^2	**5.** 18^3	**8.** 6.5^2	**11.** $(\frac{8}{15})^2$	**14.** $(2\frac{1}{4})^3$
3. 16^3	**6.** 64^2	**9.** $.75^2$	**12.** $(\frac{19}{24})^2$	**15.** $(5\frac{1}{3})^2$

Find the value of each ratio:

16. $42:56$	**18.** $\frac{2}{8}:\frac{4}{5}$	**20.** $80:.4$	**22.** 2 mi. $:8$ rd.
17. $64:96$	**19.** $\frac{3}{4}:\frac{5}{8}$	**21.** $7.5:15$	**23.** 12 oz. $:4$ lb.

Find the square root of:

24. 841	**27.** 1764	**30.** 13,225	**33.** 1900.96
25. 784	**28.** 2304	**31.** 21,025	**34.** .514089
26. 961	**29.** 9604	**32.** 41,616	**35.** .001225

Solve the following proportions:

36. $15:x=6:2$	**38.** $\frac{1}{2}:\frac{3}{4}=x:1$	**40.** $50:75=x:3$
37. $25:x=10:2$	**39.** $x:\frac{2}{3}=\frac{1}{2}:\frac{1}{4}$	**41.** $20:32=5:x$

Find the cost of a draft for:

42. \$3600 at 5 ¢ discount **44.** \$1200 at $\frac{1}{4}\%$ premium

43. \$2400 at 10 ¢ premium **45.** \$1850 at $\frac{1}{10}\%$ discount

46. Find, by factoring, the cube root of: 216; 3375.

Written Exercises

Note. — Read again the directions on page 165 and apply them wherever possible.

1. If 6 tons of coal cost $40.50, how much do 20 tons cost?

2. A warehouse in the form of a square has a ground area of 2304 sq. ft. Find the length of each side.

3. A farmer with his team constructed 7½ mi. of ditch for irrigation in 6 days. At this rate, how many miles of ditch will he construct in 10 days?

4. Find, to the nearest cent, the interest on a postal savings bank deposit of $75 for 4 yr. 3 mo. 15 da.

5. A farm of 240 acres was divided between two children in the ratio 2 : 3. Find each one's share.

6. A baseball infield is a 90-foot square. Find, to the nearest tenth of a foot, the distance from first base to third base.

7. By counting mile posts, a man found that the train upon which he was riding went 5 mi. in 6 min. How many miles per hour was the train moving?

8. Find the side of a square field whose area is 40 acres.

9. Find the date of maturity, the term of discount, the bank discount, and the proceeds of a 90-day note for $5650 with interest at 4%, dated Sept. 12 and discounted Oct. 10 at 6%.

10. For what sum must a note for 120 days be drawn in order to obtain $686 in cash, if the rate of discount is 6%?

11. Three men invested $2200, $1800, and $2000, respectively, in a business. After several years, during which the business had grown to $10,500, the first partner withdrew from the firm. How much was his share worth?

12. The width of a large library building is to the length as 9 is to 13. If it covers an area of 11,700 sq. yd., what are its ground dimensions in feet?

13. A contractor agrees to do a piece of work in 60 days and employs 40 men for that purpose. Before beginning, however, the time is extended 15 days. How many men may he discharge?

14. What are the proceeds of a 30-day draft for $175, discounted at 6% for the full term, if the bank charges 20¢ in addition for collection?

15. The building site of a large box factory forms a right triangle with one side 800 ft. long and the other 550 ft. long. Find, to the nearest tenth of a foot, the length of the hypotenuse.

16. In 1880, 17% of the persons in the United States were illiterate; in 1890, 13.3%; in 1900, 10.7%; in 1910, 7.7%. Express these per cents as ratios having 10,000 as a consequent.

17. A series of photographs for a moving picture was taken in the New York subway at the rate of 500 in 33⅓ sec. How many photographs were there in the series, if it was completed in 7 min.?

18. Suppose that you sell Charles Raymond of your city an automobile for $1850 less 5% for cash, receiving his check on the Second National Bank for the amount. Write the check, supplying the necessary details.

19. Indorse in blank the check in exercise **18** and deposit it to your credit in the Bank of Commerce, together with $75 in bills and $16.63 in coin, filling out a deposit slip in due form.

20. Draw against your deposit in the Bank of Commerce, check No. 1 for $125, payable to "Cash"; draw check No. 2 for $80, payable to the order of "Self" and indorse it; draw check No. 3 for $83.65, payable to the order of Frank Adams.

21. Write the stub record of each check in exercise **20**, the first two for personal expenses and the third for automobile repairs.

MEASURES AND MEASUREMENTS

REVIEW OF DENOMINATE NUMBERS

Carefully review the work given on pages 119–142, and the tables of measures referred to there.

Oral Exercises

1. Reduce to feet: 10 yd. ; 96 in. ; 12 yd. ; 120 in.

2. Reduce to days: 8 wk.; 12 hr.; 11 wk. ; 36 hr.; 6¾ wk.

3. Reduce to pecks: 10 bu.; 88 qt.; 15 bu. ; 10 qt. ; 5½ bu.

4. Reduce to pounds : 8 oz. ; 2 T. ; 12 oz.; 1½ T. ; 2.5 T.

5. Reduce to quarts: 22 pt.; .5 gal.; 30 pt.; ¼ gal. ; 1¾ gal.

Change to the next lower unit :

6. 9 ft.	**10.** 10°.	**14.** 5 qr.	**18.** ⅓ sq. yd.
7. 2 lb.	**11.** 12*s.*	**15.** 20 gr.	**19.** ½ sq. ft.
8. 6 pk.	**12.** 2 A.	**16.** 6⅔ yd.	**20.** ⅔ cu. yd.
9. 7 hr.	**13.** £ 15.	**17.** .4 min.	**21.** ¼ sq. mi.

Change to the next higher unit :

22. 24 pt.	**27.** 96 doz.	**32.** 720′.	**37.** 81 sq. ft.
23. 35 da.	**28.** 160 rd.	**33.** 48 hr.	**38.** 72 sq. in.
24. 45 ft.	**29.** 132 in.	**34.** 220*s.*	**39.** 40 sq. rd.
25. 48 oz.	**30.** 540 sec.	**35.** 144*d.*	**40.** 30 sq. ft.
26. 80 pk.	**31.** 660 min.	**36.** 170 qr.	**41.** 36 cu. ft.

42. To supply the heat for a building 400 lb. of coal are burned daily. At this rate how long would 1 ton of coal last?

43. How many bushels of corn meal are required per day to feed 32 cows 2 qt. apiece, both morning and evening ?

Written Exercises

Reduce to the next lower unit :

1. 152 pk.   **5.** 640′.   **9.** 135 sq. ft.   **13.** 14¾ T.

2. 298 da.   **6.** 460*s*.   **10.** 333 cu. ft.   **14.** 52¼ wk.

3. 587 ft.   **7.** 888 doz.   **11.** 246 qt. (dry)   **15.** 1.15 mi.

4. 696 hr.   **8.** 790 min.   **12.** 450 qt. (liq.)   **16.** 31½ gal.

17–28. In exercises **1–12** reduce to the next higher unit.

Reduce to the lowest of the given units :

29. 48 gal. 3 qt.   **33.** 24 ft. 9 in.   **37.** £ 18 16*s*. 9*d*.

30. 25 lb. 12 oz.   **34.** 18 T. 50 lb.   **38.** 75 A. 60 sq. rd.

31. 32 rd. 14 ft.   **35.** 46 pk. 7 qt.   **39.** 16 bu. 3 pk. 7 qt.

32. 15 da. 20 hr.   **36.** 45° 30′ 15″   **40.** 5 yr. 8 mo. 25 da.

Express in the next higher and the given unit :

41. 173 pt.   **45.** 485 da.   **49.** 745′.   **53.** 1725 sq. in.

42. 376 oz.   **46.** 716 in.   **50.** 954*s*.   **54.** 2310 cu. ft.

43. 539 ft.   **47.** 879 hr.   **51.** 673 c.   **55.** 5269 sq. ft.

44. 796 rd.   **48.** 971 pk.   **52.** 795 qr.   **56.** 7880 sq. rd.

Express :

57. 3⅞ gal. as pints.

58. 8000 ft. as miles and feet.

59. 175 pt. as pecks, quarts, and pints.

60. 4388 min. as days, hours, and minutes.

61. 67° 30′ as the decimal part of a right angle.

62. 305 days as the fractional part of a leap year.

63. 5 fr. 25 c. to the nearest cent in United States money.

64. £ 5 12*s*. 6*d*. to the nearest cent in United States money.

65. 18 M. 50 pf. to the nearest cent in United States money.

Written Exercises

NOTE. — Read again the directions on page 165 and apply them wherever practicable.

1. A man changed 2 5-pound notes and 4 half sovereigns for shillings. How many shillings did he receive?

2. The area of a reservoir is 4.95 sq. mi. Find its area in acres.

3. A cotton-picking machine can pick about 10 lb. of cotton per minute. At this rate, how many tons of cotton can it pick in a day of 12 hr.?

4. How many days would it take to count a million at the rate of 125 per minute?

5. The tidal wave travels about 700 mi. per hour. How many feet does it travel per second?

6. A peanut vender pays $5 for a bag of peanuts containing 110 lb. If there are 2 qt. in a pound and he sells the peanuts at 5¢ a pint, how much does he gain on a bag?

7. On a trial cruise, the scouting cruiser *Chester* steamed 26.53 knots per hour for 4 hr. How many miles did she go?

8. How many rails each 30 ft. long are required to lay 3 mi. of railroad track?

9. A farmer fed each of 33 cows 15 lb. of hay, 30 lb. of silage, 40 lb. of roots, and 8 lb. of grain per day. How many tons of each were fed to all these cows during November?

10. A railroad trackwalker covered 203,670 mi. of track in 31 yr. Making no allowance for days off duty and leap years, how many miles of track did he walk per day?

11. The tank in the tender of a locomotive has a capacity of 7500 gal., but it contains only 875 gal. Find, in barrels and gallons, the amount of water required to fill it.

12. The yacht, *Queen Alexandra*, cost £128,239. Find, to the nearest cent, its cost in United States money.

13. A dealer paid $4.75 for a crate containing 1 bu. 2 pk. of strawberries put up in quart baskets. If he sold them at 15¢ per basket, how much did he gain?

14. Find the value of the 312,500 great gross of brass buttons made in this country one year, at 20¢ per gross.

15. In building a tunnel 5¼ mi. long the work proceeded at an average rate of 12 ft. per day. How many years did it take to complete the tunnel, allowing 300 working days per year?

16. The output of an Australian coal mine was 162,216 tons one year. If the cost of mining was 8s. per ton, what was the total cost in sterling money? in U. S. money, to nearest cent?

17. How long will it take a torpedo to reach a mark 1870 yd. away, if it travels ½ mi. per minute?

18. A battleship carries 135 fathoms of anchor chain weighing 440 lb. per fathom. Find the length of the anchor chain in feet; its weight in tons.

19. The average price of potatoes imported into France from the United States is 15 fr. per bag of 220.5 lb. Find, to the nearest cent, the price per bushel in United States money.

20. The regulation military pace is a step 30 in. long, 2 steps per second. If a company of soldiers should keep this pace for 3 hr. 40 min., how many miles would they march?

21. In 1 hr. 57 min., 198,000 bu. of wheat were put on board a grain steamer. Find, to the nearest bushel, the rate per minute.

22. One winter 900 cakes of ice were cut from a pond. Each cake was 22 in. by 32 in. and averaged 15 in. in thickness. If a cubic foot of ice weighs 58 lb., how many tons of ice were cut?

PLANE FIGURES

A surface such that a straight line joining any two points of it lies wholly in the surface is a **plane surface**.

Any portion of a plane surface bounded by lines, straight or curved, is a **plane figure**.

When a straight line meets another straight line forming two *equal* angles, each angle is a **right angle**. The two lines that form a right angle are **perpendicular** to each other.

TWO RIGHT ANGLES
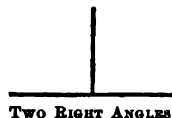
The side on which a figure is assumed to stand is its **base**, and the height of the figure measured on a line perpendicular to the base is its **altitude**.

These figures have different bases but the same altitude:

BASE   BASE   BASE   BASE
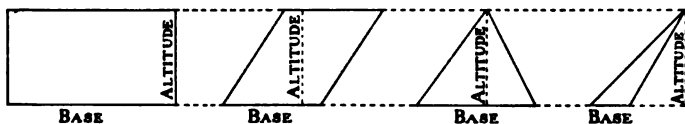
RECTANGLES

The **rectangle**, considered on pages 124 and 125, is the most common form of plane figure. It includes the square.

Since the length of a rectangle may be considered its base and the breadth its altitude, the principle for area as developed on page 124 may be stated as follows:

The area of a rectangle is equal to the product of its base and altitude, expressed in like units.

In this and similar statements, for brevity, we speak of the *product of lines* when we mean the product of the *numbers* that represent them.

From the above it follows that:

Either dimension of a rectangle is the quotient of the area and the other dimension, expressed in corresponding units.

Oral and Written Exercises

Estimate results mentally wherever practicable before doing the written work.

Find the area of a square whose side is:

1. 24 rd.	**3.** $6\frac{1}{2}$ yd.	**5.** 3.25 ft.	**7.** 42 ft. 6 in.
2. 47 ft.	**4.** $9\frac{1}{8}$ ft.	**6.** 87.5 rd.	**8.** 55 yd. 1 ft.

Find the area of a rectangle:

9. 35 ft. by 24 ft.	**12.** $30' \times 10'\ 6''$	**15.** $40'\ 6'' \times 12'\ 4''$
10. 43 rd. by 36 rd.	**13.** $45' \times 15'\ 4''$	**16.** $32'\ 8'' \times 16'\ 6''$
11. 25 yd. by 40 ft.	**14.** $56' \times 20'\ 3''$	**17.** $60'\ 9'' \times 24'\ 10''$

Find the missing dimension of each rectangle:

	Area	Base	Altitude
18.	1680 sq. ft.	—— ft.	20 ft.
19.	1452 sq. ft.	60 ft. 6 in.	—— ft.
20.	1104 sq. yd.	—— yd.	72 ft.
21.	6336 sq. rd.	726 yd.	—— rd.
22.	153 acres	—— rd.	102 rd.
23.	270 acres	240 rd.	—— rd.

24. A rectangular field is 40 rd. long and $22\frac{1}{2}$ rd. wide. Find its value at $60 per acre.

25. A rectangular farm containing 84 acres is 120 rd. long. How wide is the farm?

26. A cement walk 12 rd. long and 5 ft. wide was laid at an expense of 1.12\frac{1}{2}$ per square yard. Find the cost.

27. A city park in the form of a rectangle is 8 rd. wide and contains 21,780 sq. ft. Find its length in feet.

28. If a piece of land 24 rd. long and 20 rd. wide is divided into lots 165 ft. by 66 ft., how many lots are there? How much are they worth at an average price of $175 each?

PARALLELOGRAMS

Lines that cannot meet, however far they are
extended, are **parallel lines**.

A four-sided plane figure whose opposite sides are parallel is
a **parallelogram**.

If the shaded part of this parallelogram is
placed in the position indicated by the dotted
lines, the resulting figure will be a rectangle
of the same area, base, and altitude as the
parallelogram.

*The area of a parallelogram is equal to the product of its base
and altitude, expressed in like units.*

Oral and Written Exercises

Estimate results mentally before solving.

Find the area of each parallelogram :

	BASE	ALTITUDE		BASE	ALTITUDE
1.	27 ft.	18⅜ ft.	**6.**	93 in.	10 ft. 4 in.
2.	20 yd.	16¼ ft.	**7.**	42 ft.	15 ft. 3 in.
3.	39 rd.	14½ rd.	**8.**	12 yd.	26 ft. 9 in.
4.	56 yd.	25¾ yd.	**9.**	64 ft.	18 ft. 6 in.
5.	48 rd.	30⅝ yd.	**10.**	30 rd.	34 yd. 2 ft.

Find the missing dimension of each parallelogram :

	AREA	BASE	ALTITUDE
11.	1875 sq. ft.	—— ft.	8 yd. 1 ft.
12.	2436 sq. yd.	174 ft.	—— yd.
13.	1368 sq. yd.	—— yd.	57 ft.
14.	1632 sq. rd.	374 yd.	—— rd.
15.	6840 sq. ft.	—— ft.	50 ft. 8 in.
16.	180 acres	192 rd.	—— rd.
17.	344 acres	—— rd.	215 rd.

The diagonal of a parallelogram divides it into two equal triangles, as may be shown by cutting a paper parallelogram in this way and applying one triangle to the other.

Then, a triangle is one half of a parallelogram having the same base and altitude and from the principle for the area of a parallelogram we have :
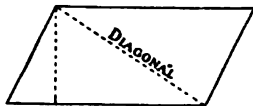
The area of a triangle is equal to one half the product of its base and altitude, expressed in like units.   Hence,

Either dimension of a triangle is twice the quotient of the area and the other dimension, expressed in corresponding units.

Oral and Written Exercises

Estimate, then find the area of each triangle :

	BASE	ALTITUDE		BASE	ALTITUDE
1.	36 ft.	20 in.	**5.**	75 ft.	15 ft. 4 in.
2.	24 yd.	10 ft.	**6.**	28 yd.	26 ft. 6 in.
3.	55⅓ ft.	4½ ft.	**7.**	96 ft.	85 ft. 8 in.
4.	88¼ ft.	64 in.	**8.**	36 rd.	14 yd. 2 ft.

From the given area and dimension, find the other dimension of each triangle :

9. 360 sq. ft., 45 ft.   **12.** 888 sq. ft., 18 ft. 6 in.

10. 264 sq. ft., 8¼ ft.   **13.** 518 sq. yd., 12 yd. 1 ft.

11. 224 sq. yd., 112 ft.   **14.** 684 sq. yd., 50 yd. 2 ft.

15. A field is in the form of a triangle whose base is 36 rd., and altitude, 15 rd.   Find its area in acres.

16. A triangular shelf made to fit a square corner extends 18 in. each way from the corner.   What is the area of the top of the shelf ?

TRAPEZOIDS

A four-sided plane figure that has only two sides parallel is a **trapezoid**.

The parallel sides of a trapezoid are its **bases**, known as the **lower base** and the **upper base**, respectively.

The diagonal of a trapezoid divides it into two triangles whose altitudes are the same as the altitude of the trapezoid and whose bases are the lower and upper base, respectively, of the trapezoid.
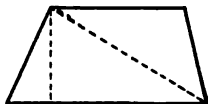
Then, the area of the trapezoid is equal to the sum of the areas of these triangles and from the principle for the area of a triangle, we have :

The area of a trapezoid is equal to one half the product of its altitude and the sum of its bases, expressed in like units.

Oral and Written Exercises

Estimate, then find the area of each trapezoid :

	BASES	ALTITUDE		BASES	ALTITUDE
1.	17 in.; 15 in.	6 in.	8.	43 yd.; 35 yd.	31 ft.
2.	20 ft.; 14 ft.	8 ft.	9.	50 ft.; 42 ft.	14 ft. 6 in.
3.	24 ft.; 12 ft.	16 in.	10.	63 yd.; 51 yd.	16 yd. 1 ft.
4.	33 ft.; 25 ft.	18 ft.	11.	72 ft.; 44 ft.	20 ft. 3 in.
5.	42 ft.; 24 ft.	20 ft.	12.	82 ft.; 38 ft.	31 ft. 8 in.
6.	15 yd.; 31 ft.	16 ft.	13.	57 rd.; 53 rd.	12 rd. 2 yd.
7.	32 yd.; 28 yd.	22 ft.	14.	78 yd.; 62 yd.	25 yd. 18 in.

15. What is the area of a walk, in the form of a trapezoid, 40 ft. long, 4 ft. wide at one end and 6 ft. wide at the other ?

16. A plank 18 ft. long is 8 in. wide at one end and 1 ft. wide at the other end. Find the area of one side of the plank.

17. Find, in acres, the area of a field in the form of a trapezoid whose bases are 24 rd. and 72 rd. and whose altitude is 16 rd.

CIRCLES

A plane figure bounded by a curved line every point of which is equally distant from a point within is a **circle**; the point within is the **center**; and the bounding line, the **circumference**.

A straight line drawn from the center to the circumference of a circle is a **radius**.

A straight line passing through the center of a circle and terminating at both ends in the circumference is a **diameter**.

A diameter is equal in length to two radii.

Finding the circumference of a circle.

If the length of the diameter of a circle is applied, as a measure, to a line equal in length to the circumference, the latter is found to contain this measure nearly 3⅐ times.
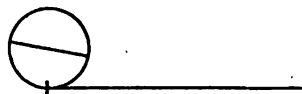
The ratio of the circumference of a circle to its diameter cannot be expressed exactly, but it is proved in geometry to be more nearly 3.1416 than 3⅐.

The symbol for this ratio is the Greek letter π (pī).

The circumference of a circle is equal to π times the diameter, or 2π times the radius.

In this book, unless stated otherwise, the approximate value 3⅐ will be used for π.

Written Exercises

Find the circumference of a circle whose diameter is:

1. 14 in.	**5.** 63 ft.	**9.** 112 in.	**13.** 294 ft.
2. 21 in.	**6.** 56 ft.	**10.** 189 in.	**14.** 371 ft.
3. 35 in.	**7.** 98 ft.	**11.** 203 in.	**15.** 406 ft.
4. 49 in.	**8.** 84 ft.	**12.** 238 in.	**16.** 574 ft.

Find the circumference of a circle whose radius is :

17. 28 in.	**21.** 38½ ft.	**25.** 17.5 rd.	**29.** 53.2 in.
18. 42 ft.	**22.** 105 in.	**26.** 31.5 in.	**30.** 84.7 yd.
19. 70 yd.	**23.** 78¾ ft.	**27.** 36.4 ft.	**31.** 76.3 rd.
20. 91 in.	**24.** 87¼ yd.	**28.** 45.5 yd.	**32.** 94.5 ft.

In exercises **33–48**, use $\pi = 3.1416$.

Find the circumference of a circle whose diameter is :

33. 15 in.	**35.** 6¼ ft.	**37.** 8.5 yd.	**39.** 120 ft.
34. 24 yd.	**36.** 9¼ in.	**38.** 9.5 rd.	**40.** 145 in.

Find the circumference of a circle whose radius is :

41. 3½ in.	**43.** 7.5 ft.	**45.** 34 yd.	**47.** 102 ft.
42. 5⅓ ft.	**44.** 9.5 in.	**46.** 52 rd.	**48.** 138 in.

49. Find the diameter of a circle whose circumference is 44 ft.

SOLUTION. 44 ft. ÷ π = 44 ft. ÷ 3½ = 14 ft., diameter.

Find the diameter of a circle whose circumference is :

50. 22 in.	**52.** 77 ft.	**54.** 132 in.	**56.** 30.8 ft.
51. 66 in.	**53.** 8.8 yd.	**55.** 198 yd.	**57.** 57.2 rd.

58. Find the radius of a circle whose circumference is 8.8 ft.

SOLUTION. 8.8 ft. ÷ 2 π = 8.8 ft. ÷ (2 × 3½) = 1.4 ft., radius.

Find the radius of a circle whose circumference is :

59. 44 in.	**61.** 176 ft.	**63.** 35.2 yd.	**65.** 616 rd.
60. 99 in.	**62.** 198 ft.	**64.** 48.4 yd.	**66.** 770 rd.

67. A wagon wheel is 3¼ ft. in diameter. How far will the wagon have gone when the wheel has made 100 revolutions?

68. A merry-go-round has two rows of seats. One row is 7 ft. from the center and the other 10½ ft. from the center. If John sits in an outer seat and Mary in an inner seat, how much farther than Mary does John ride each revolution?

Finding the area of a circle.

The area of a circle may be illustrated by the accompanying figure. It shows that a circle may be regarded as made up of a large number of triangles, the sum of whose curved bases is the circumference and whose altitude is the radius of the circle.
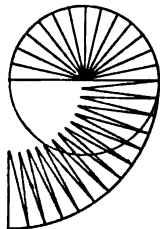
Hence, if we consider these as plane triangles, it follows from the principle for the area of a triangle given on page 308, that:

The area of a circle is equal to one half the product of its circumference and its radius.

The *area* may be found *without finding the circumference.*

For, since the circumference is 2 π times the radius, if we let r stand for the radius, the circumference may be represented by 2 πr, and from the above principle, the area is ½ of 2 πr × r, or πr². Hence,

The area of a circle is π times the square of the radius.

Find the area of a circle whose radius and circumference are :

1. 7 in., 44 in.   **3.** 21 in., 132 in.   **5.** 2.8 ft., 17.6 ft.

2. 14 in., 88 in.   **4.** 35 in., 220 in.   **6.** 4.9 ft., 30.8 ft.

Find the area of a circle whose radius is :

7. 3 ft.	**10.** 12 ft.	**13.** 1.4 rd.	**16.** 24 ft.	**19.** 6.3 in.
8. 7 in.	**11.** 14 ft.	**14.** 2.8 yd.	**17.** 38 in.	**20.** .91 ft.
9. 5 ft.	**12.** 18 in.	**15.** 3.5 rd.	**18.** 56 rd.	**21.** 8.4 yd.

22. Find the radius of a circle whose area is 154 sq. ft.

SOLUTION. — The area = πr², or 154 sq. ft. ; then, r² = 154 ÷ 3½, or 49, and r = √49, or 7; that is, the radius is 7 ft.

Find the radius of a circle whose area is :

23. 616 sq. in.   **24.** 38.5 sq. ft.   **25.** 1386 sq. rd.   **26.** 98.56 sq. yd.

27. Find the area of a table top whose radius is 3½ ft.

28. What is the area of the cross section of a log that is 14 in. in diameter?

29. What is the area of a circular flower bed that is 21 ft. in diameter?

30. A cow is tied to a stake with a rope 17½ ft. long. Over how many square feet of surface can she graze?

31. Find, to the nearest square yard, the area of a circular court 40 ft. in diameter. (Use $\pi = 3.1416$.)

32. A revolving lawn sprinkler throws the water 21 ft. How many square yards of lawn can be watered from one position?

33. The inside diameter of a round barn is 50 ft. How many square yards of concrete are required for the basement floor? (Use $\pi = 3.1416$.)

34. The diameter of a circular reservoir is 120 rd. Find its area to the nearest tenth of an acre.

35. Find, to the nearest hundredth of a rod, the radius of a circular park containing half an acre.

36. A round silo is 15 ft. in diameter, inside measurement. At $1.15 per square yard, find, to the nearest cent, the cost of a concrete floor for the silo.

37. The distance around a circular chimney is 29 ft. 4 in. at the base. How many square feet of surface does it cover?

38. A cement walk 3½ ft. wide surrounds a circular grass plot 35 ft. in diameter. Find the area of the walk.

39. The area inclosed by a circular race track is 61,600 sq. yd. Find the length of the track in miles.

40. The basin of a fountain has a circular rim 1 ft. in width. If the inside measurement of the circumference of the rim is 66 ft., what is the area of the rim?

Miscellaneous Exercises

A rectangular farm was divided into fields as shown in this diagram :
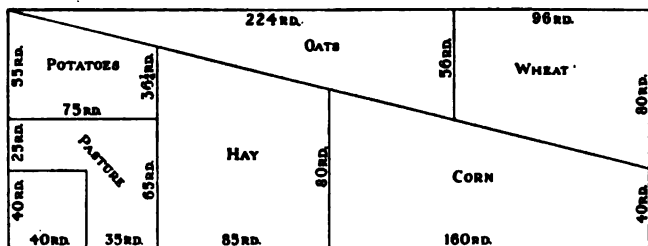
Find the area, in acres and square rods, of the field devoted to each of the following :

1. Corn	**3.** Oats	**5.** Potatoes
2. Wheat	**4.** Hay	**6.** Pasture

7. How many acres did the field in the lower left-hand corner contain?

8. Find the area of the farm from its length and width.

9. Find the area of a circular garden 28 ft. in diameter.

10. Find the base of a triangle whose area is 147 sq. in. and whose altitude is 1 ft. 2 in.

11. A wheel is 3 ft. 6 in. in diameter and has 22 spokes. How far apart are the spokes from center to center at the rim?

12. A city lot is in the form of a trapezoid whose bases are 65 ft. and 5 ft., respectively, and whose altitude is 70 ft. Find the area of the lot.

13. The drive wheel of a traction engine is 6 ft. in diameter. How many revolutions does it make in traveling a mile?

14. An open tank 6 ft. long, $2\frac{1}{2}$ ft. wide, and $2\frac{3}{4}$ ft. deep (inside dimensions) was lined with sheet lead weighing $3\frac{1}{2}$ lb. per square foot. Find the cost of the lead at 6¢ per pound.

PRACTICAL APPLICATIONS

SCALE DRAWINGS

A drawing whose linear dimensions are in a certain ratio to the corresponding dimensions of the thing represented is a **scale drawing**.

Maps, architects' plans, etc., are *drawn to a scale*.

If a line 1″ long represents a line 3″ long, the *scale* is said to be ⅓, or 1 to 3.

What is the meaning of " Scale: 1 in. to 4 rd."?

Oral and Written Exercises

Draw a line to represent the given length to the given scale:

1. 15 in., scale ⅙
2. 16 in., scale ¼
3. 20 in., scale ⅖
4. 18 in., scale ⅔
5. 24 in., scale ⅜

6. 20′, scale 1″ to 4′
7. 15′, scale 2″ to 5′
8. 12′, scale 3″ to 4′
9. 14′, scale 4″ to 7′
10. 16′, scale 5″ to 8′

When ¼″ represents 1′, what length in feet is represented by

11. 3 in.? 13. 8 in.? 15. 1½ in.? 17. 1¾ in.? 19. 6¼ in.?
12. 5 in.? 14. 6 in.? 16. 3¼ in.? 18. 5½ in.? 20. 8¾ in.?

If the scale is ¼″ to 1′, what length in feet is represented by

21. ⅛″? 23. ⅝″? 25. ¹⁄₁₆″? 27. ⁹⁄₁₆″? 29. ¹³⁄₁₆″?
22. ⅜″? 24. ⅞″? 26. ⁵⁄₁₆″? 28. ¹¹⁄₁₆″? 30. ¹⁵⁄₁₆″?

When the scale is ½″ to 1′, how many inches represent

31. 5 ft.? 33. 3 ft. 6 in.? 35. 12 ft. 9 in.?
32. 9 ft.? 34. 8 ft. 3 in.? 36. 21 ft. 6 in.?

37. A rectangle, 16 in. by 32 in., is to be represented by a drawing, scale 1 to 8. Make the drawing.

38. A rectangular drawing 2″ × 4½″ represents a village lot. If the scale is 1″ to 24 rd., what are the dimensions of the lot?

Plot a rectangular lot:

39. 24 rd. by 36 rd., scale 1 in. to 4 rd.

40. 15 rd. by 20 rd., scale 2 in. to 5 rd.

41. 42 rd. by 60 rd., scale 1 in. to 6 rd.

42. 64 rd. by 80 rd., scale 3 in. to 16 rd.

43–46. Find in acres the area of each lot in exercises **39–42**.

Using the scale $\frac{1}{4}''$ to 1′, draw the floor plan of a:

47. Barn, 30′ × 48′   **50.** Den, 12′ 6″ × 11′

48. House, 32′ × 28′   **51.** Parlor, 18′ 6″ × 17′ 9″

49. Garage, 36′ × 34′   **52.** Chamber, 14′ 9″ × 13′ 3″

53–55. Using a ruler divided to sixteenths of an inch, find from this floor plan, the thickness of the walls of the house; the width of the front door; of the double door between the parlor and the dining room.
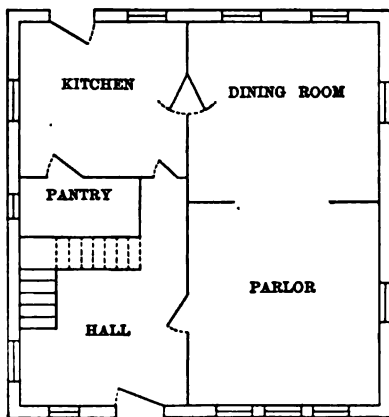
SCALE: $\frac{1}{16}''$ TO 1′

Find the dimensions of the:

56. Parlor.

57. Pantry.

58. Kitchen.

59. Dining room.

60. House (outside).

61. Represent (scale $\frac{1}{8}''$ to 1′) the floor of a room, 15′ × 18′, having a rug on it, 12′ × 15′, with a margin of $1\frac{1}{2}$′ on each side.

62. Represent (scale $\frac{1}{2}''$ to 1′) the top of a table, 2′ 6″ × 4′; the top of a desk, 2′ 3″ × 3′ 6″.

63. Make a drawing, to some convenient scale, to represent a picture and frame, 28″ × 24″, the frame being 2″ wide.

LATHING AND PLASTERING

Lathing is often done by the **bundle** or by the **square yard**.

Laths are 4 feet long and are sold in bundles of 50 or 100.

In this book 1 bundle means 100 laths. Parts of a bundle are not sold.

A bundle is estimated to cover 5 square yards of surface.

Plastering is usually computed by the **square yard**.

Estimates for plastering, which often include the lathing, are figured to the *nearest* square yard in final results.

There is no regular custom in regard to allowances for openings and base-board. Therefore, if allowance is to be made, the fact should be stated in the contract.

Since in all good work plastering extends to the floor, this book does not consider the baseboard, and exact allowances for openings (if any) are stated in the exercises.

Written Exercises

1. How much does it cost to lath 569 sq. ft. of surface at 54¢ a bundle for material and labor?

2. Find the cost of lathing and plastering a room $18' \times 16'$ and $12'$ high at 34¢ per square yard.

SOLUTION

Distance around the room, $2 \times (18 + 16')$ = $68'$
Area of the walls, (12×68) sq. ft. = 816 sq. ft.
Area of the ceiling, (18×16) sq. ft. = 288 sq. ft.
        Total area of walls and ceiling, 1104 sq. ft.
    The total area in square yards, $(1104 + 9)$ sq. yd. $= 122\frac{2}{3}$ sq. yd., or to the nearest square yard, 123 sq. yd.
    Cost at 34¢ per square yard, $123 \times \$.34 = \$41.82$.

3. How many bundles of lath must be purchased to lath the walls and ceiling of a room $18' \times 14'$ and $9'$ high?

4. At 23¢ a square yard, find the cost of plastering the walls and ceiling of a room $18' 6'' \times 16'$ and $10'$ high, deducting for 3 windows each $3' \times 6'$ and 2 doors each $3' \times 7'$.

5. At 22 ¢ per square yard put on, find the cost of metal lath for a ceiling 20′ 3″ × 18′ 8″.

At 32 ¢ per square yard for material and labor, find the cost of lathing and plastering the walls and ceiling of a room :

	LENGTH	WIDTH	HEIGHT		LENGTH	WIDTH	HEIGHT
6.	12′	11′	10′	**10.**	14′ 6″	13′ 6″	9′ 4″
7.	14′	13′	10′	**11.**	15′ 8″	12′ 4″	9′ 6″
8.	16′	14′	9′ 6″	**12.**	18′ 9″	16′ 8″	9′ 6″
9.	18′	15′	9′ 6″	**13.**	20′ 3″	18′ 4″	9′ 8″

14. Find how many square yards of surface there are to be covered with plaster in the walls and ceiling of a room 15′ 6″ × 14′ 9″ and 9′ 6″ high, deducting 107 sq. ft. for openings.

At 36 ¢ per square yard for material and labor, find the cost of lathing and plastering each of these rooms :

	ROOM	LENGTH	WIDTH	HEIGHT	ALLOWANCE
15.	Hall	14′	9′	9′ 6″	9 sq. yd.
16.	Parlor	18′	16′	9′ 6″	12 sq. yd.
17.	Library	16′	15′ 6″	9′ 6″	10 sq. yd.
18.	Dining room	15′	14′ 4″	9′ 6″	9 sq. yd.
19.	Kitchen	14′	12′ 8″	9′ 6″	10 sq. yd.
20.	Bedroom	12′	10′ 3″	9′ 4″	6 sq. yd.
21.	Bedroom	12′	9′ 9″	9′ 4″	5 sq. yd.
22.	Bathroom	10′	7′ 6″	9′ 4″	4 sq. yd.

23. Find the cost, at 48 ¢ per square yard for material and labor, of plastering the four brick walls of a schoolroom 36′ × 32′ and 15′ high, deducting ½ the area of 9 windows each 7′ × 4′ and of 2 doors each 7′ 6″ × 4′.

24. A mason estimated the cost of 100 sq. yd. of plastering thus: material, $15.85; labor, $14.50; cartage, etc., $2.65. How much must he charge for plastering the walls and ceiling of a room 18′ square and 9¼′ high to make a profit of 10 % ?

PAINTING AND CALCIMINING

Painting and calcimining are usually estimated by the **square yard**.

In this book areas to be painted or calcimined are found to the nearest square yard.

Written Exercises

Allowing 1 gal. of varnish to 240 sq. ft. (two coats) of floor, find the quantity of varnish needed for a floor:

1. $24' \times 20'$ 2. $20' \times 15'$ 3. $15' \times 12'$ 4. $25' \times 18'$

At 19 ¢ per square yard for material and labor, find the cost of painting a surface of the given dimensions:

5. $28' \times 24\frac{1}{2}'$ 8. $33' \times 26'\ 4''$ 11. $28' \times 25'\ 3''$

6. $29' \times 27\frac{1}{4}'$ 9. $34' \times 29'\ 6''$ 12. $30' \times 28'\ 4''$

7. $32' \times 28\frac{3}{4}'$ 10. $36' \times 30'\ 9''$ 13. $40' \times 36'\ 9''$

Deducting 2 sq. yd. for each opening, find the cost of labor, at 6 ¢ per square yard, of calcimining the walls and ceiling of a room:

	LENGTH	WIDTH	HEIGHT	OPENINGS		LENGTH	WIDTH	HEIGHT	OPENINGS
14.	$10'$	$8'$	$8'$	4	20.	$15'$	$15'$	$10'$	8
15.	$12'$	$9'$	$9'$	3	21.	$16'$	$9'\ 6''$	$10'$	4
16.	$12'$	$11'$	$8\frac{1}{2}'$	3	22.	$12'$	$10'\ 6''$	$9'\ 6''$	4
17.	$14'$	$12'$	$8\frac{3}{4}'$	4	23.	$15'$	$12'\ 8''$	$9'\ 6''$	5
18.	$16'$	$16'$	$9\frac{1}{4}'$	6	24.	$18'$	$16'\ 4''$	$9'\ 8''$	7
19.	$18'$	$15'$	$9\frac{3}{8}'$	7	25.	$20'$	$19'\ 9''$	$10'\ 4''$	9

26. Find the cost of painting the roof of this barn at 16 ¢ per square yard.

27. How much does it cost to paint the ends and sides of the barn at $12\frac{1}{2}$ ¢ per square yard?
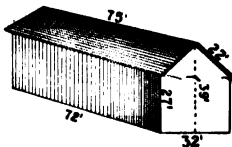
Wall paper is sold in **single rolls** 8 yards long or in **double rolls** 16 yards long. It is usually 18 inches wide.

Fractional parts of a roll are not sold. "Roll" means "single roll."
Imported papers differ as to the width and the length of the roll.
· Borders or friezes are sold by the *linear yard*. They vary in width.

In practice it is not always possible to compute the exact cost of papering, but the following method is approximately correct when allowance is made for openings.

Allowance should be made for loss in matching the pattern.

The number of vertical strips required is double the number of yards in the perimeter of the room.

The number of rolls needed is the quotient of the required number of strips divided by the number of whole strips that can be cut from a roll.

Written Exercises

1. How many rolls of plain paper are required for a ceiling $18' \times 12'$, if the strips run crosswise of the room?

2. At 45 ¢ per yard, find the cost of the border for a room $16' \ 6'' \times 15'$.

3. How many rolls of plain paper are needed to paper the walls of a room $16' \times 14'$ and $8'$ high, if no allowances are made?

4. A room $21' \times 18'$ has 3 windows and 2 doors. How many strips of paper are necessary to paper the walls of the room, if 2 strips are deducted for each opening?

5. At 45 ¢ per roll, how much will it cost to paper the walls of a room $15' \times 12'$, if the strips are $7' \ 6''$ long and 6 strips are deducted for openings?

6. At 55 ¢ per double roll, what is the cost of the paper for the walls and ceiling of a room $15'$ square, if the wall strips are $7' \ 4''$ long and 10 strips are allowed for openings? Find the cost of the border for this room at 80 ¢ per yard.

Find the cost of the paper for the walls of each room, the distance from the baseboard to the border being 7' 9'':

	ROOM	LENGTH	WIDTH	ALLOWANCE	PRICE PER ROLL
7.	Hall	12'	8'	8 strips	20¢
8.	Hall	12'	12'	10 strips	20¢
9.	Parlor	20'	18'	14 strips	25¢
10.	Library	21'	19'	12 strips	22¢
11.	Dining room	18'	15'	12 strips	21¢
12.	Kitchen	15'	14'	10 strips	14¢
13.	Bathroom	10'	9' 6''	4 strips	16¢
14.	Bedroom	12' 4''	11' 8''	6 strips	15¢
15.	Bedroom	11' 9''	11' 3''	4 strips	15¢
16.	Bedroom	14' 5''	12' 7''	6 strips	16¢

17. Find the cost of the paper for the walls and ceiling of a room 18' 6'' square, the wall strips being 9' 3'' long and 12 strips being allowed for openings. The wall paper is 90¢ per double roll, ceiling paper 30¢ per roll.

CARPETING

Carpeting is sold by the **linear yard** and it varies in width, being usually either 1 yard or ¾ of a yard wide.

Ingrains are 1 yard wide, while Wiltons, Brussels, moquettes, Axminsters, and some others are ¾ of a yard wide.

The width of carpeting should be kept in mind to avoid unnecessary waste in *turning under*, for the salesman never splits a strip. Consequently, while strips are usually laid lengthwise of the room, there is often a saving of material in laying them crosswise.

Unless otherwise stated, the exercises in this book require the strips to run lengthwise of the room.

There is often waste in matching the pattern, but no allowance is made for the first strip because the other strips are matched to it.

Matting, oilcloth, and linoleum are sold both by the **linear yard** and the **square yard**. They are of various widths.

Written Exercises

1. Find the cost, at $1.25 a yard, of a carpet, strips ¾ yd. wide, necessary for a room 16′ × 15′, allowing ⅙ yd. on each strip (except the first) for matching.

Solution

Since the carpeting is ¾ yd., or 2¼′, wide and the strips run lengthwise of the room, the number of strips required = 15′ ÷ 2¼′, or 6⅔; that is, 7 strips must be bought. Since ⅙ yd., or ½′, is allowed on each strip, except the first, for matching, we have

$$1 \text{ strip} \qquad\qquad = \quad 16'$$
$$6 \text{ strips (each } 16\tfrac{1}{2}' \text{ long)} \quad = \quad \underline{99'}$$

That is, the total length of carpeting, $\overline{115'} = 38\tfrac{1}{3}$ yd.

Then, the cost = 38⅓ × $1.25, or $47.92, to the nearest cent.

Find the number of yards of unfigured matting, 1 yd. wide, required to cover a floor, strips running the more economical way:

2. 11′ × 10′	**5.** 14′ × 13′	**8.** 14′ 3″ × 12′	**11.** 13′ × 11′ 6″
3. 12′ × 11′	**6.** 16′ × 15′	**9.** 15′ 6″ × 14′	**12.** 15′ × 12′ 4″
4. 12′ × 12′	**7.** 15′ × 12′	**10.** 16′ 8″ × 12′	**13.** 18′ × 16′ 8″

Allowing ⅙ yd. on each strip (except the first) for matching, find the number of yards of carpet 27″ wide needed for a floor:

14. 12′ × 12′	**17.** 16′ × 14′	**20.** 14′ × 12′ 6″	**23.** 15′ 6″ × 14′
15. 14′ × 12′	**18.** 15′ × 15′	**21.** 15′ × 13′ 4″	**24.** 16′ 4″ × 15′
16. 15′ × 13′	**19.** 18′ × 16′	**22.** 18′ × 17′ 9″	**25.** 21′ 3″ × 20′

26. At $2 per linear yard, find the cost of enough unfigured linoleum 2 yd. wide for a floor 22′ × 18′.

27. At $1.75 per yard, find the cost of Brussels carpeting for a floor 18′ 9″ × 17′ 6″, allowing in all 1¼ yd. for matching.

28. Measure your room at home and find how many yards of plain ingrain carpet are required to cover the floor, the strips running the most economical way.

The unit of lumber measure is the **board foot**.

It is a board or a part of a board 1 inch (or less) in thickness and having a surface area on one side of 1 square foot.

The number of board feet in a board 1 inch (or less) in thickness equals the number of square feet of surface in one side of the board.

Thus, a board 1 ft. wide and 15 ft. long contains 15 sq. ft., or 15 ft. *board measure,* if the board is 1 in. or less in thickness. "Foot" is often used for "board foot" and "M" for "thousand board feet."

The number of board feet in a board or a timber more than 1 inch in thickness equals the product of the number of square feet in one side by the number of inches in the thickness.

Thus, a board 6 ft. long, 10 in. wide, if it is 1 in. thick contains (6 × ⅚) sq. ft., or 5 board feet; but if it is 2 in. thick it contains 2 × 5, or 10, board feet; if 1½ in. thick, 1½ × 5, or 7½, board feet.

Oral and Written Exercises

Estimate, then find the number of feet in boards 1 inch or less in thickness that have the following lengths and widths :

1. $4' \times 3''$	10. $9' \times 8''$	19. $18' \times 14''$
2. $16' \times 3''$	11. $20' \times 8''$	20. $18' \times 16''$
3. $18' \times 3''$	12. $7\frac{1}{2}' \times 8''$	21. $18' \times 18''$
4. $15' \times 4''$	13 $12' \times 10''$	22. $18' \times 20''$
5. $14' \times 4''$	14. $18' \times 10''$	23. $12' \times 16''$
6. $20' \times 4''$	15. $15' \times 10''$	24. $16' \times 15''$
7. $10' \times 6''$	16. $12' \times 12''$	25. $15' \times 16''$
8. $14' \times 6''$	17. $16' \times 12''$	26. $20' \times 18''$
9. $11' \times 6'$	18. $12' \times 14''$	27. $16' \times 16''$

28. How many board feet are there in a 2-inch plank 12 ft. long and 1¼ ft. wide ?

29. Find the cost of 12 boards, $10' \times 8'' \times \frac{7}{8}''$, at $36 per M.

30. At $32 per M, find the cost of 18 planks, $16' \times 9'' \times 2\frac{1}{2}''$.

31. Find, to the nearest cent, the cost of 18 sticks of timber, each $14'$ long and $6''$ square at $28.50 per M.

Find the number of feet in the following timbers:

Number of Pieces	Size	Length	Number of Feet
32. 24	$2'' \times 12''$	$12'$	——
33. 15	$2'' \times 6''$	$14'$	——
34. 250	$2'' \times 4''$	$10'$	——
35. 112	$8'' \times 8''$	$16'$	——
36. 64	$2'' \times 8''$	$14'$	——
37. 80	$2'' \times 12''$	$12'$	——
38. 125	$4'' \times 6''$	$18'$	——
39. 75	$6'' \times 8''$	$18'$	——
40. 48	$4'' \times 8''$	$12'$	——
41. 84	$8'' \times 10''$	$18'$	——
42. 112	$2'' \times 14''$	$18'$	——

Find, to the nearest cent, the cost of each item, and foot the following lumber bills at $28 per M:

43. 10 sills, $4'' \times 8'' \times 20'$
7 posts, $6'' \times 6'' \times 22'$
2 posts, $6'' \times 6'' \times 18'$
3 beams, $3'' \times 8'' \times 18'$

44. 12 ties, $4'' \times 6'' \times 30'$
Ridge, $3'' \times 12'' \times 28'$
45 beams, $3'' \times 8'' \times 22'$
24 rafters, $2'' \times 8'' \times 20'$

45. 10 plates, $4'' \times 6'' \times 25'$
14 ties, $4'' \times 6'' \times 28'$
Girder, $8'' \times 12'' \times 32'$
10 posts, $4'' \times 8'' \times 24'$

46. 70 joists, $2'' \times 10'' \times 22'$
20 joists, $2'' \times 10'' \times 16'$
18 joists, $2'' \times 10'' \times 13'$
16 beams, $3'' \times 8'' \times 20'$

47. At $30 per thousand feet, what is the cost of the boards for a fence $6\frac{1}{4}$ rd. long and 5 boards high, each board being 6 in. wide and 1 in. thick?

FLOORING

Flooring varies in width and thickness according to the purpose for which it is designed. It is usually matched so that the tongue of one board is made to fit the groove of another. Although narrower widths result from the planing and matching of boards, the width in the rough is the width recognized as a basis of sale.
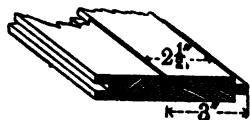
Thus, flooring 3 in. wide means unmatched boards 3 in. wide. By planing and matching, these boards are narrowed to a face width of about 2¼ in., which is the width covered when laid.

In estimating the quantity of lumber required for a floor, add to the area to be covered ⅓ of itself for narrow flooring (3 in. or less in width) and ⅕ for wider flooring.

NOTE. — Give answers involving cost to the nearest cent.

Written Exercises

Find the cost, at $32 per M, of flooring, 5″ wide and ⅞″ thick, for a room:

1. $15' \times 12'$ 3. $12' \times 9' 6''$ 5. $18' 6'' \times 15'$
2. $16' \times 16'$ 4. $15' \times 14' 6''$ 6. $12' 6'' \times 5' 6''$

Find the cost, at $44 per M, of flooring, 3″ wide and ⅝″ thick, for a room:

7. $18' \times 15'$ 9. $15' 3'' \times 6'$ 11. $16' \times 12' 6''$
8. $16' \times 14'$ 10. $14' 6'' \times 10'$ 12. $12' \times 10' 9''$

13. How many pieces of inch flooring 14' long and 2½″ face width, laid crosswise of the room, are required for a floor $15' \times 14'$? Find the cost at $54 per M.

14. Find the cost of the floor of a schoolroom $50' \times 32'$ made of matched lumber 3″ wide and 1″ thick, at $48 per M for the lumber and $15 per M for labor.

ROOFING

The unit of roofing is the **square** of 100 square feet.

Common shingles are about 16 inches long and average 4 inches in width. When laid "4 inches to the weather," the exposed surface of 1 shingle is (4 × 4) sq. in., or ⅑ sq. ft.

It takes 900 shingles, then, to cover 1 square, but to allow for waste 1000 shingles per square are often estimated.

Shingles are put up in **bunches** of 250 shingles each.

A fractional part of a bunch cannot be bought.

Written Exercises

1. Find how many bunches of shingles are required to shingle the gable roof of the barn on page 319, estimating 1000 shingles per square.

Find the number of bunches of shingles, laid 4″ to the weather, required for both sides of a gable roof, no allowance:

	LENGTH	RAFTER		LENGTH	RAFTER		LENGTH	RAFTER
2.	20′	10′	**5.**	36′	15′	**8.**	36′	17′ 3″
3.	26′	12′	**6.**	34′	16′ 6″	**9.**	42′	18′ 4″
4.	32′	14′	**7.**	32′	15′ 6″	**10.**	46′	19′ 6″

11. A slag roof 32′ × 45′ cost $93.60. Find the cost per square.

12. At $10 per square for material and labor, find the cost of covering the hip roof shown in the cut with asbestos shingles; with tile at $16 per square.
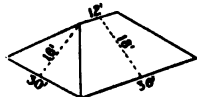
13. Find the cost of the material for a roof 50′ × 24′ at $28 per M for the roof boards, 1″ thick, and $5.75 per square for the tin to cover them.

14. At 55¢ per hour, how much does a slater receive for slating a roof 50′ × 32′, if he covers 2 squares in a day of 8 hr.?

PAVEMENTS, ROADS, AND SIDEWALKS

Estimates for paving, road making, and sidewalk building are usually based on the cost of a **square foot** or a **square yard**.

Written Exercises

1. Find the cost of building a sidewalk 110 ft. long and 5 ft. wide, at 20 ¢ per square foot.

At 12½ ¢ per square foot, find the cost of a walk :

2. $500' \times 5'$	6. $825' \times 6'$	10. $540' \times 7' 6''$
3. $650' \times 4'$	7. $680' \times 8'$	11. $750' \times 6' 4''$
4. $475' \times 6'$	8. $560' \times 5\frac{1}{2}'$	12. $840' \times 5' 3''$
5. $725' \times 4'$	9. $720' \times 6\frac{1}{2}'$	13. $930' \times 5' 8''$

14. How much did it cost to construct a macadam road ½ mi. long and 36 ft. wide at $.75 per square yard?

15. A city street containing 8450 sq. yd. was paved with bricks, 48 to the square yard. How much did the bricks cost at $22 per M?

16. What is the cost of asphalt paving for 405 ft. of street 30 ft. wide, if the concrete foundation cost 75 ¢ per square yard and the asphalt $1.85 per square yard?

17. Each property owner on a street paid ½ of the cost of the paving in front of his property. How much did it cost a man owning 72 ft. of frontage, if the paving was 48 ft. wide and cost $2.75 per square yard?

18. It took 5 bbl. of cement at $2.15 per barrel and 4 loads of gravel at $1.05 a load for a walk 18 yd. by 5 ft. Find, to the nearest cent, the cost of the material per square foot.

19. Find the cost of paving a driveway 36 ft. long and 9 ft. wide with bricks at $27.50 per M laid, each brick having an exposed surface $9'' \times 3''$.

Tell how you would solve each exercise ; then solve:

1. At $ 600 per acre, find the cost of the village lot represented in the diagram.
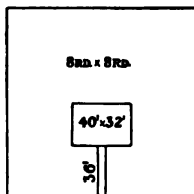
2. Find, to the nearest tenth, the number of square yards of concrete required for the floor of the cellar shown ; its cost at $ 1.15 per square yard.

3. At 15 ¢ per square foot, find the cost of the concrete walk 6 ft. wide, extending from the cellar to the front of the lot.

4. A fence with posts 12 ft. apart was built at each side and at the back of the lot.   Find the cost of the posts required at 30 ¢ each ; of the woven wire at 9 ¢ per foot.

5. At 12 ¢ per square yard for sod and 11 ¢ per square yard for labor, find the cost of sodding the front of the lot as far back as the house line, deducting 24 sq. yd. for the walk.

6. At $ 30 per M, find the cost of a plank, 16' × 8'' × 1½''.

7. Find the price per linear foot of a macadam road costing $ 8448 per mile.

8. If cobblestone gutters cost 25 ¢ per linear foot, how much will the 2 gutters for a mile of road cost ?

9. How many bundles of lath must be purchased to lath the walls and ceiling of a living room, 14' × 16' and 9' 6'' high?

10. Using the scale 1'' to 3', draw the floor plan of a kitchen 12' × 15', with a cupboard 2' 3'' deep extending across one end.

11. If 1 gal. of paint covers 650 sq. ft., how much paint is required to cover one side of a tight board fence 6 ft. 3 in. high and 78 ft. long ?

At $1.25 per yard, find the cost, to the nearest cent, of unfigured carpet, 27″ wide, required for each room :

12. Hall, 12′ × 13′ 6″   **15.** Bedroom, 13′ × 14′ 6″

13. Library, 16′ × 18′   **16.** Dining room, 14′ × 15′

14. Parlor, 15′ × 15′ 6″   **17.** Living room, 15′ × 16′ 6″

18. How many feet of inch flooring are required for a floor 20′ × 16′, if ¼ of its area is allowed for matching and waste?

19. Find how many bunches of common shingles must be bought to cover a roof 80′ × 32′, estimating 4 bunches per square.

20. At 6¢ per square yard, find the cost of calcimining the walls and ceiling of a room 13′ × 12′ and 9′ 6″ high, deducting 88 sq. ft. for openings.

21. A plank 16 ft. long and 2 in. thick was 12 in. wide at one end and 6 in. at the other end. Find its cost at 4½¢ per board foot.

22. Find the cost, at 24¢ per square yard, of plastering the walls and ceiling of a room 14′ × 15′ and 9′ 6″ high, allowing 108 sq. ft. for openings.

23. The pasting table used by a paper hanger consisted of 4 ¾-inch white pine boards 6 ft. long and 10 in. wide. How many feet of lumber did it contain?

24. Each side of a gable roof is 40′ long and 33′ wide. How many pieces of slate 10″ wide, laid in courses lengthwise of the roof and 5½″ to the weather, are required to cover it?

25. How many bricks 8¼ in. long and 2 in. thick, laid on edge, are required to pave a street 2310 ft. by 30 ft.?

26. How much cheaper is it to paper the walls of a room, 14′ 6″ × 12′ 6″ with double rolls at 60¢ each that cut 7 strips per roll, than with single rolls at 30¢ each that cut 3 strips per roll, if 10 strips are deducted for openings?

SOLIDS

RECTANGULAR SOLIDS

The **rectangular solid** considered on pages 126–128 is the most common solid. It includes the cube.

From the principle for volume (page 127) it follows that :

Either dimension of a rectangular solid is the quotient of the volume divided by the product of the other two dimensions, expressed in corresponding units.

Written Exercises

1. Find the area of the entire surface of a rectangular solid $10'' \times 8'' \times 4''$.

Find the area of the entire surface of a cube whose edge is :

2. 8 ft.	**4.** 15 in.	**6.** $6\frac{1}{2}$ ft.	**8.** 10 ft. 6 in.
3. 7 ft.	**5.** 18 in.	**7.** $9\frac{1}{3}$ ft.	**9.** 15 ft. 4 in.

Find the volume of a rectangular solid :

10. $2' \times 4' \times 16'$	**13.** $7' \times 4' \times 9'\ 6''$	**16.** $8'\ 8'' \times 9' \times 24'$
11. $6' \times 9' \times 22'$	**14.** $9' \times 3'\ 8'' \times 18'$	**17.** $12' \times 6'\ 10'' \times 36'$
12. $8' \times 5' \times 35'$	**15.** $12' \times 14'\ 9'' \times 9'$	**18.** $25' \times 42' \times 15'\ 2''$

Find the missing dimension from the given volume and two given dimensions :

19. 504 cu. ft.; $18' \times 4'$	**21.** 1584 cu. ft.; $32' \times 5'\ 6''$
20. 672 cu. ft.; $24' \times 8'$	**22.** 3420 cu. ft.; $45' \times 6'\ 4''$

23. If a bale of hay is 4 ft. long, $2\frac{1}{4}$ ft. wide, and 2 ft. thick, how much space is required for storing 150 bales ?

24. What is the depth of a box 6 ft. long and 4 ft. wide, if it contains 27,648 cu. in. ?

25. How many square feet of copper are required to line the sides and bottom of a tank, $6'\ 4''$ long, $4'$ wide, and $3'\ 6''$ high, inside dimensions, if $2\frac{1}{3}$ sq. ft. are allowed for seams, etc. ?

PRISMS AND CYLINDERS

Any plane figure bounded by straight lines is a **polygon.**
The total length of the bounding lines of a polygon is its **perimeter.**

A solid whose sides are parallelograms and whose two ends, or **bases,** are equal polygons, parallel to each other, is a **prism.**

TRIANGULAR
PRISM

SQUARE
PRISM

Prisms are named from the shape of their bases, — *triangular, square, rectangular,* etc.

A rectangular solid is a prism.

The perpendicular distance between the bases of a prism is its **altitude.**

The parallelograms taken together form the **lateral surface** of the prism.

A solid bounded by a uniformly curved surface and having for its bases circles that are parallel to each other is a **circular cylinder.**

There are other kinds of cylinders, but in this book " cylinder " means " circular cylinder."

CYLINDER

Finding the lateral surface of a prism or a cylinder.
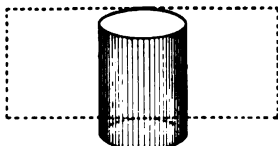
If a piece of paper is wrapped over the lateral surface of a prism or a cylinder, being made to fit it exactly, and if it is then unfolded and flattened out, it will be found to be a rectangle whose altitude is the altitude of the solid and whose base is the perimeter of the base of the solid.

The lateral surface of a prism or a cylinder is equal to the product of its altitude and the perimeter of its base.

Estimate, then find the lateral surface of a prism having its altitude and the perimeter of its base in order as given :

1. 3 ft.; 10 in.	**5.** 12 ft.; 8¼ ft.	**9.** 36′; 7′ 3″
2. 4 ft.; 1½ ft.	**6.** 18 ft.; 14 ft.	**10.** 28′; 9′ 9″
3. 6 ft.; 3.5 ft.	**7.** 32 ft.; 6¾ ft.	**11.** 42′; 10′ 4″
4. 9 ft.; 2¾ ft.	**8.** 27 ft.; 8⅜ ft.	**12.** 54′; 15′ 6″

13. What is the lateral surface of a square prism whose altitude is 16 ft., if each side of its base is 12½ ft. ?

14. Find the lateral surface of a cylinder having an altitude of 4½ ft. and a circumference of 15½ ft.

Find the lateral surface of each cylinder, having the given altitude (Alt.) and diameter (Diam.) or radius of base :

ALT.	DIAM.	ALT.	RADIUS	ALT.	RADIUS
15. 6 ft.	7 ft.	**20.** 14 ft.	3 ft.	**25.** 25 ft.	14 ft.
16. 5 ft.	21 ft.	**21.** 15 ft.	7 ft.	**26.** 49 ft.	15 ft.
17. 8 ft.	14 ft.	**22.** 28 ft.	5 ft.	**27.** 36 ft.	21 ft.
18. 7 ft.	25 ft.	**23.** 32 ft.	3½ ft.	**28.** 45 ft.	3.5 ft.
19. 9 ft.	28 ft.	**24.** 42 ft.	5⅓ ft.	**29.** 60 ft.	4.2 ft.

30. How many square feet of lawn are rolled by one revolution of a roller 3 ft. long and 2 ft. 4 in. in diameter ?

Making no allowance for lapping, find the number of square feet of sheet iron in a piece of stovepipe :

31. 3′ long, 6″ in diam.   **33.** 2′ long, 3″ in diam.

32. 3′ long, 5″ in diam.   **34.** 2½′ long, 4″ in diam.

35. Find the entire surface (lateral surface and surface of the two bases) of a cylinder, 9 in. high and 7 in. in diameter.

36. Find how many square inches of tin are required for a tomato can 4⅝ in. high and 4¼ in. in diameter, making no allowance for seams.

Finding the volume of a prism or a cylinder.

A prism or a cylinder 1 inch high contains as many cubic inches as there are square inches in its base ; a prism or a cylinder 2 inches high contains twice as many cubic inches as there are square inches in its base; one 3 inches high contains 3 times as many cubic inches ; and so on. Hence,
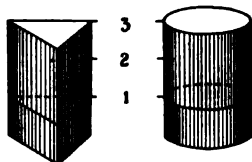
The volume of a prism or a cylinder is equal to the product of its altitude and the area of its base.

Oral and Written Exercises

1. What is the volume of a triangular prism 5 in. high, if its base has an area of 36 sq. in.?

2. What is the volume of a cylinder 8 in. high, having a base 24 sq. in. in area?

Estimate, then find the volume of each prism or cylinder :

	AREA OF BASE	ALTITUDE		AREA OF BASE	ALTITUDE
3.	25 sq. ft.	6 ft.	**12.**	120 sq. ft.	24 ft.
4.	34 sq. ft.	8 ft.	**13.**	112 sq. ft.	12¼ ft.
5.	28 sq. ft.	4¼ ft.	**14.**	135 sq. ft.	15¾ ft.
6.	36 sq. ft.	18 in.	**15.**	148 sq. ft.	18¾ ft.
7.	40 sq. ft.	15 in.	**16.**	180 sq. ft.	20 ft. 4 in.
8.	52 sq. ft.	21 in.	**17.**	196 sq. ft.	21 ft. 3 in.
9.	48 sq. ft.	8⅓ ft.	**18.**	208 sq. ft.	24 ft. 6 in.
10.	56 sq. ft.	20 ft.	**19.**	225 sq. ft.	22 ft. 4 in.
11.	63 sq. ft.	18 ft.	**20.**	240 sq. ft.	25 ft. 7 in.

21. What is the volume of a prism with a base 15 in. square, if its altitude is 13½ in.?

22. Find the volume of a cylinder having a diameter of 14 in. and an altitude of 10½ in.

Find the volume of a prism that is a cube having an edge:

23. 12 ft.	**26.** 8½ in.	**29.** 21 ft.	**32.** 3.6 ft.
24. 15 ft.	**27.** 7¾ in.	**30.** 24 ft.	**33.** 4.2 ft.
25. 18 ft.	**28.** 9.5 in.	**31.** 32 ft.	**34.** 3.8 ft.

Estimate, then find the volume of each cylinder:

ALTITUDE	DIAMETER	ALTITUDE	RADIUS	ALTITUDE	RADIUS
35. 7 in.	3 in.	**40.** 12 ft.	7 ft.	**45.** 18 ft.	14 ft.
36. 9 in.	7 in.	**41.** 14 ft.	9 ft.	**46.** 21 ft.	15 ft.
37. 8 in.	3½ in.	**42.** 16 ft.	3¼ ft.	**47.** 25 ft.	21 ft.
38. 6 in.	14 in.	**43.** 8 in.	1¾ ft.	**48.** 20 ft.	4.2 ft.
39. 7 in.	16 in.	**44.** 9 in.	2⅛ ft.	**49.** 28 ft.	5.6 ft.

50. How many cubic feet of silage will a cylindrical silo 14 ft. in diameter and 30 ft. high hold?

51. Find the depth of a box 6 ft. long and 4 ft. 6 in. wide that contains 8 cu. yd.

52. Find the number of cubic inches of metal in a piece of round shafting 3½ in. in diameter and 8 ft. long.

53. The area of the base of a marble column is 126 sq. in. and its altitude is 16 ft. Find its volume in cubic feet.

54. How many cubic feet of gas will a cylindrical tank hold that is 9 ft. 6 in. high and 21 ft. in diameter?

55. A cylindrical oil tank is 42 ft. in diameter and contains 88,160 cu. ft. How high is it?

Find the altitude of each cylinder:

VOLUME	DIAMETER OF BASE	VOLUME	RADIUS OF BASE
56. 462 cu. in.	14 in.	**58.** 2464 cu. ft.	9 ft. 4 in.
57. 704 cu. in.	16 in.	**59.** 3850 cu. ft.	8 ft. 9 in.

60. Find the number of cubic feet of concrete in the wall of a cylindrical bin in a grain elevator, the wall being 7 in. thick, 54 ft. high, and 44 ft. outside circumference.

PYRAMIDS AND CONES

A solid whose base is a polygon and whose faces are triangles meeting at a point is called a **pyramid**.

The triangles form the **lateral surface** of the pyramid; the point where they meet is the **vertex**.

Pyramids, like prisms, are named from their bases, as *triangular, square, hexagonal* (six-sided), etc.

The perpendicular distance, as AB, from the vertex to the base of a pyramid is its **altitude**.

The altitude, as AC, of one of the triangles of a pyramid is the **slant height** of the pyramid.

A solid whose base is a circle and whose surface tapers uniformly to a point, the *vertex*, is a **circular cone**.

In this book "cone" means "right circular cone."

The vertex of a cone is sometimes called its **apex**.

A cone may be thought of as a pyramid whose base is a circle and whose lateral surface is made up of an infinite number of infinitely narrow triangles.

The **altitude** and **slant height** of a cone correspond to the altitude and slant height of a pyramid.
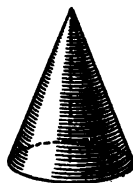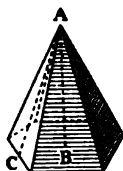
Finding the lateral surface of a pyramid or a cone.

The lateral surface of a pyramid is composed of triangles, and that of a cone may be assumed to be made up of an infinite number of triangles. In each case the altitude of the triangles is the slant height of the solid and the sum of the bases of the triangles is the perimeter of the base of the solid.
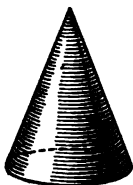
Hence, from the principle for the area of a triangle, we have:

The lateral surface of a pyramid or a cone is equal to one half the product of its slant height and the perimeter of its base.

Oral and Written Exercises

Find the lateral surface of a pyramid or a cone :

	SLANT HEIGHT	PERIMETER OF BASE		SLANT HEIGHT	PERIMETER OF BASE
1.	8 in.	4½ ft.	**4.**	3¾ ft.	25 ft. 4 in.
2.	9 in.	5¼ ft.	**5.**	5¼ ft.	34 ft. 8 in.
3.	4⅔ ft.	19½ ft.	**6.**	9¾ ft.	48 ft. 8 in.

7. What is the lateral surface of a pyramid whose base is 6 ft. square and whose slant height is 5½ ft. ?

Find the lateral surface of each of these cones :

	SLANT HEIGHT	RADIUS OF BASE		SLANT HEIGHT	RADIUS OF BASE
8.	7 in.	2 in.	**11.**	5 ft. 3 in.	3 ft.
9.	9 in.	3½ in.	**12.**	7 ft. 4 in.	8½ ft.
10.	6 ft.	2⅛ ft.	**13.**	6 ft. 9 in.	4⅔ ft.

14. The roof of a summerhouse is in the form of a square pyramid whose base is 12 ft. square and slant height 8 ft. Find the cost of painting the roof at 15¢ per square yard.

Finding the volume of a pyramid or a cone.

If a hollow pyramid is used as a measure, it will be found that just three measures of sand, or some liquid, will fill a hollow prism of equal base and altitude, that is, the volume of the pyramid is one third that of the prism.
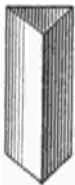
Similarly, by using a hollow cone and a hollow cylinder of equal base and altitude, it may be discovered that the volume of the cone is one third that of the cylinder.

From the principle for the volume of a prism or a cylinder :

The volume of a pyramid or a cone is equal to one third the product of its altitude and the area of its base.

Oral and Written Exercises

Estimate, then find the volume of each pyramid or cone :

	ALTITUDE	AREA OF BASE		ALTITUDE	AREA OF BASE
1.	4½ ft.	9 sq. ft.	8.	24 ft.	125 sq. ft.
2.	12 ft.	20 sq. ft.	9.	28¼ ft.	144 sq. ft.
3.	18 ft.	32 sq. ft.	10.	34⅛ ft.	207 sq. ft.
4.	21 ft.	45 sq. ft.	11.	18 ft. 6 in.	186 sq. ft.
5.	6.5 ft.	27 sq. ft.	12.	20 ft. 4 in.	216 sq. ft.
6.	5.8 ft.	36 sq. ft.	13.	25 ft. 3 in.	228 sq. ft.
7.	8.7 ft.	54 sq. ft.	14.	30 ft. 8 in.	252 sq. ft.

15. Find the volume of a pyramid whose base is 10 ft. square and whose altitude is 20 ft.

16. What is the volume of a cone, if the diameter of its base is 6 ft. and its altitude is 9 ft. ?

Find the volume of each cone :

	ALTITUDE	RADIUS OF BASE		ALTITUDE	RADIUS OF BASE
17.	6 in.	3½ in.	24.	20 ft.	10 ft. 6 in.
18.	9 in.	2¼ ft.	25.	27 ft.	16 ft. 4 in.
19.	12 ft.	4¾ ft.	26.	36 ft.	18 ft. 8 in.
20.	14 ft.	7½ ft.	27.	22.4 ft.	20 ft. 3 in.
21.	21 ft.	12 ft.	28.	18.9 ft.	21 ft. 8 in.
22.	24 ft.	21 ft.	29.	15¾ ft.	19 ft. 4 in.
23.	28 ft.	24 ft.	30.	21⅛ ft.	22 ft. 9 in.

31. A cylinder is 7 in. in diameter and 10 in. high. What is the volume of a cone of like dimensions?

32. Find the altitude of a cone, having a volume of 1725 cu. in. and a base whose area is 115 sq. in.

33. The volume of a cone is 1848 cu. in. The radius of the base is 10½ in. Find the altitude.

Find the missing number for each pyramid or cone :

	ALTITUDE	AREA OF BASE	VOLUME
34.	4.5 in.	76 sq. in.	——— cu. in.
35.	——— ft.	64 sq. ft.	51.2 cu. ft.
36.	5.9 ft.	——— sq. ft.	495.6 cu. ft.
37.	——— ft.	125 sq. ft.	1500 cu. ft.
38.	18 ft.	245 sq. ft.	——— cu. ft.
39.	24 ft.	——— sq. ft.	8640 cu. ft.
40.	——— ft.	231.5 sq. ft.	6019 cu. ft.
41.	12½ ft.	116 sq. ft.	——— cu. ft.
42.	15¼ ft.	——— sq. ft.	1830 cu. ft.
43.	——— ft.	138 sq. ft.	1978 cu. ft.
44.	——— ft.	125½ sq. ft.	2256 cu. ft.

45. A conical pile of grain is 3 ft. high and the radius of the base is 3½ ft. What is its volume ?

46. An ice-cream cup in the form of a cone is 3 in. in diameter at the top and 4 in. deep. Find its capacity.

47. A pile of coal in the shape of a cone is 30 ft. high and the radius of the base is 21 ft. Find its volume.

48. What is the volume of a marble pyramid whose base is 4 ft. square and whose altitude is 8 ft. ?

49. What is the volume of a granite cone the diameter of whose base is 6 ft. and whose altitude is 8 ft. ?

50. The top of the Washington monument is a pyramid with a base 34½ ft. square and altitude 25 ft. Find its volume.

51. A conical pile of gravel is 5 ft. high and contains 64½ cu. ft. Find the diameter of its base.

52. How many cubic feet of marble will be chipped from a block of marble 4′ × 4′ × 6′ in making a cap for a monument, if the cap forms a square pyramid 4′ on a side and 5½′ high ?

A solid bounded by a curved surface every point of which is equally distant from a point within, called the *center*, is a sphere.

A straight line passing through the center of a sphere and terminating at both ends in the surface is its **diameter**.

The distance from the center to the surface is the **radius** of a sphere.

A circle of a sphere whose plane passes through the center is a **great circle** of the sphere.

A great circle divides a sphere into two equal parts called **hemispheres**. The circle is the **base** of each hemisphere.

The circumference of a great circle of a sphere is the **circumference** of the sphere. It is the greatest distance around the sphere.
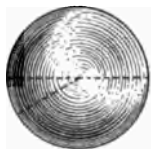
Finding the surface of a sphere.

The length of a waxed cord sufficient to cover the convex surface of a hemisphere, when carefully wound as shown in the picture, is just twice the length of the cord required to cover the base of the hemisphere; then, the area of the convex surface of a hemisphere is twice the area of its base, or it is the area of two great circles of the sphere.
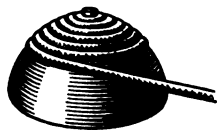
Hence, from the principle for the area of a circle:

The surface of a sphere is equal to 4 great circles, or to 4 πr^2.

If we represent the diameter of a sphere by d, then, since $4 r^2 = (2 r)^2 = d^2$:

The surface of a sphere is equal to π times the square of the diameter, or πd^2.

Using 3.1416 for π, find the surface of each sphere :

RADIUS	DIAMETER	RADIUS	DIAMETER
1. 7 in.	**8.** 42 in.	**15.** $1\frac{1}{4}$ ft.	**22.** 2 ft. 3 in.
2. 5 in.	**9.** 45 in.	**16.** $3\frac{1}{2}$ ft.	**23.** 2 ft. 9 in.
3. 6 in.	**10.** 49 in.	**17.** $2\frac{1}{8}$ ft.	**24.** 4 ft. 4 in.
4. 14 in.	**11.** 56 in.	**18.** .75 ft.	**25.** 3 ft. 3 in.
5. 21 in.	**12.** 60 in.	**19.** 2.5 ft.	**26.** 3 ft. 8 in.
6. 25 in.	**13.** 63 in.	**20.** 4.5 ft.	**27.** 4 ft. 3 in.
7. 35 in.	**14.** 70 in.	**21.** 5.5 ft.	**28.** 5 ft. 4 in.

29. Find, to the nearest square inch, the amount of leather in the cover of a baseball 9 in. in circumference.

30. At 12¢ a square foot, find, to the nearest cent, the cost of gilding a sphere 28 in. in diameter.

31. At 50¢ a square yard, find the cost of painting the dome of an observatory in the form of a hemisphere 21 ft. in diameter.

Finding the volume of a sphere.

As indicated in the figure, a sphere may be divided into a great number of solids that are essentially pyramids. The sum of the bases of these pyramids is the surface of the sphere, and the altitude of each pyramid is the radius of the sphere.

From the principle for the volume of a pyramid and that for the surface of a sphere, representing the radius by r and the surface by $4 \pi r^2$:
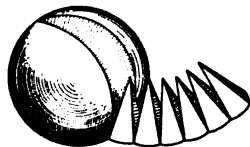
The volume of a sphere is equal to one third the product of its radius and its surface, or $\frac{4}{3} \pi r^3$.

Find the volume of a sphere whose radius is:

1. 3 in.	**4.** 28 in.	**7.** 10½ ft.	**10.** 3 ft. 6 in.
2. 6 in.	**5.** 35 in.	**8.** 15¾ ft.	**11.** 5 ft. 3 in.
3. 9 in.	**6.** 42 in.	**9.** 17.5 ft.	**12.** 8 ft. 9 in.

Using $\pi = 3.1416$, find the volume of:

13. A baseball; $r = 1\frac{1}{2}''$ **16.** A toy balloon; $r = 5''$

14. A glass ball; $r = \frac{1}{2}''$ **17.** A croquet ball; $r = 2''$

15. A cannon ball; $r = 3''$ **18.** A bowling ball; $r = 4''$

19. Find the volume of a granite sphere that is 3 ft. 6 in. in diameter.

20. A globe at the top of a flagstaff is 7 in. in diameter. Find its volume.

21. Find, to the nearest cubic inch, the amount of air required to inflate a basket ball, 10 in. in diameter.

22. How many cubic inches of water does a spherical aquarium 14 in. inside diameter contain, when it is half full?

23. How much marble is chipped from a 3-foot cube in making a sphere 3 ft. in diameter?

24. The basin of a fountain is a hemisphere 21 ft. in diameter. How many cubic feet of water will it hold?

25. If a cubic foot of marble weighs 168 lb., what is the weight of a marble sphere 5 ft. 3 in. in diameter?

26. If a cubic foot of steel weighs 490 lb., what is the weight, to the nearest pound, of a hollow steel ball whose outside diameter is 12 in., the thickness of the shell being 1 in.?

27. What part of the volume of a sphere 6 in. in diameter is the volume of a cone 6 in. in diameter and 6 in. high?

28. What part of the volume of a cylinder 4 in. in diameter and 4 in. high is the volume of a sphere 4 in. in diameter?

PRACTICAL APPLICATIONS

WOOD MEASURE

A pile of 4-foot wood, 8 feet long, and 4 feet high is a cord (cd.). A cord contains, therefore, **128 cubic feet**.

Wood is often cut into shorter lengths and sold as stove wood. A pile of such wood is considered a cord, if its surface measure on one side is 32 square feet, without regard to the length of the sticks.

Oral and Written Exercises

Estimate each result mentally before solving.

Change to cubic feet :   Change to cords :

1. $11\frac{1}{2}$ cd.   4. $24\frac{1}{8}$ cd.   7. 384 cu. ft.   10. 1216 cu. ft.

2. $18\frac{1}{4}$ cd.   5. $27\frac{3}{4}$ cd.   8. 800 cu. ft.   11. 1536 cu. ft.

3. $20\frac{1}{2}$ cd.   6. $32\frac{5}{8}$ cd.   9. 896 cu. ft.   12. 2144 cu. ft.

Find the number of cords of 4-foot wood in a pile :

13. $12' \times 4'$ 16. $24' \times 5'$ 19. $36' \times 4'$ 22. $56' \times 5'$

14. $24' \times 3'$ 17. $28' \times 8'$ 20. $40' \times 5'$ 23. $64' \times 7'$

15. $16' \times 8'$ 18. $30' \times 6'$ 21. $48' \times 6'$ 24. $72' \times 8'$

Find the number of cords of stove wood in a pile :

25. $16' \times 4'$ 27. $25' \times 8'$ 29. $42' \times 8'$ 31. $65' \times 6'$

26. $21' \times 8'$ 28. $34' \times 6'$ 30. $46' \times 4'$ 32. $70' \times 8'$

33. What is the value of 10 trees that average 64 cu. ft. of wood each, when such wood sells at $6.50 per cord ?

34. Find the value of a pile of stove wood 56 ft. long and 6 ft. high at $3.50 per cord.

35. A farmer sold 6 loads of 4-foot wood, each 8 ft. long and $4\frac{1}{2}$ ft. high. How much did he receive for it at $6 a cord ?

36. How many cords of 4-foot pulp wood can be piled in 2 tiers each 6 ft. high on a platform car 34 ft. long ?

CAPACITY EQUIVALENTS

Water in large quantities is measured by the barrel of $31\frac{1}{2}$ gallons, by the cubic foot, and by the 1000 gallons.

The following **equivalents** may be *memorized:*

Accurate	Approximate
1 gallon = 231 cu. in.	1 bushel = $1\frac{1}{4}$ cu. ft.
1 bushel = 2150.42 cu. in.	1 cu. ft. = $7\frac{1}{2}$ gal.

A **heaped bushel**, used for apples, potatoes, etc., contains about $1\frac{1}{2}$ cu. ft.
A cubic foot of water weighs $62\frac{1}{2}$ lb.; a **gallon**, approximately $8\frac{1}{4}$ lb.
In this book, use approximate equivalents unless otherwise stated.

Written Exercises

Using accurate equivalents, change to cubic inches:

1. 16 gal.   **3.** 34 bu.   **5.** 45 gal.   **7.** $32\frac{1}{2}$ bu.   **9.** $105\frac{2}{3}$ gal.

2. 28 gal.   **4.** 27 bu.   **6.** 54 gal.   **8.** $45\frac{1}{2}$ bu.   **10.** $136\frac{1}{2}$ gal.

Change to cubic feet:

11. 36 bu.   **13.** 75 gal.   **15.** 148 bu.   **17.** 165 gal.   **19.** 272 bu.

12. 64 bu.   **14.** 90 gal.   **16.** 152 bu.   **18.** 240 gal.   **20.** 336 bu.

Find the equivalent in bushels of :

21. 70 cu. ft.   **23.** 135 cu. ft.   **25.** 220 cu. ft.   **27.** 450 cu. ft.

22. 85 cu. ft.   **24.** 200 cu. ft.   **26.** 345 cu. ft.   **28.** 625 cu. ft.

Find the equivalent in gallons of :

29. 78 cu. ft.   **31.** 120 cu. ft.   **33.** 250 cu. ft.   **35.** $87\frac{1}{3}$ cu. ft.

30. 94 cu. ft.   **32.** 184 cu. ft.   **34.** 308 cu. ft.   **36.** $97\frac{1}{2}$ cu. ft.

Find the accurate equivalent in gallons of :

37. 4158 cu. in.   **39.** 11,550 cu. in.   **41.** 18,942 cu. in.

38. 6237 cu. in.   **40.** 15,015 cu. in.   **42.** 23,793 cu. in.

Finding the capacity of bins, boxes, tanks, cisterns, etc.

Find the weight of each quantity of water :

1. 12 gal.   **3.** 45 gal.   **5.** 78 gal.   **7.** 117 gal.   **9.** 357 gal.

2. 27 gal.   **4.** 63 gal.   **6.** 96 gal.   **8.** 135 gal.   **10.** 501 gal.

Find the capacity in cubic feet of a box that will hold :

11. 36 bu.   **13.** 74 bu.   **15.** 17½ bu.   **17.** 236 bu.   **19.** 520 bu.

12. 52 bu.   **14.** 85 bu.   **16.** 29¼ bu.   **18.** 342 bu.   **20.** 475 bu.

Find the capacity in bushels of a rectangular bin :

21. $10' \times 6' \times 3'$ **23.** $14' \times 8' \times 7\frac{1}{2}'$ **25.** $15' \times 12' \times 10' 6''$

22. $12' \times 8' \times 5'$ **24.** $16' \times 9' \times 8\frac{1}{8}'$ **26.** $20' \times 14' \times 12' 9''$

Find the exact capacity in gallons of a rectangular tank :

27. $21' \times 11' \times 5'$ **29.** $14' \times 12' \times 5\frac{1}{2}'$ **31.** $14' \times 12' \times 7' 4''$

28. $22' \times 9\frac{1}{8}' \times 7'$ **30.** $11' \times 10' \times 3\frac{1}{4}'$ **32.** $21' \times 16' \times 8' 3''$

Find the capacity in barrels of a rectangular cistern :

33. $11' \times 7' \times 6'$ **35.** $10' \times 7' \times 4\frac{1}{2}'$ **37.** $14' \times 12' \times 7' 6''$

34. $15' \times 8' \times 7'$ **36.** $12' \times 9' \times 5\frac{1}{4}'$ **38.** $16' \times 14' \times 8' 3''$

Find the exact weight of water that will just half fill a rectangular vat :

39. $6' \times 5' \times 3\frac{1}{2}'$ **41.** $7' \times 6\frac{3}{4}' \times 4\frac{1}{2}'$ **43.** $10' \times 5' \times 4' 6''$

40. $8' \times 6' \times 4\frac{1}{4}'$ **42.** $9' \times 5\frac{3}{4}' \times 3\frac{1}{8}'$ **44.** $11' \times 6' \times 5' 4''$

Find the capacity in gallons of each cylindrical receptacle :

ALTITUDE	DIAMETER OF BASE		ALTITUDE	RADIUS OF BASE
45. 7 ft.	12 in.		**50.** 7 ft.	8 ft.
46. 3½ ft.	36 in.		**51.** 4 ft.	3 ft. 6 in.
47. 4⅔ ft.	24 in.		**52.** 8 ft.	5 ft. 3 in.
48. 5¼ ft.	48 in.		**53.** 6 ft.	2 ft. 4 in.
49. 10 ft.	84 in.		**54.** 9 ft.	5 ft. 10 in.

55. How many pounds of water will a 3-quart can hold?

56. If an earthen jar, 3′ 6″ deep and 2′ in diameter, is ⅔ full of water, how many gallons does it contain?

57. If a ton of coal occupies 36 cu. ft. of space, how many tons will a bin 12′ × 4′ 6″ × 4′ hold?

58. A standpipe is 35 ft. in diameter and 100 ft. high. How many barrels of water will it hold?

59. A mow of hay is 38′ × 18′ × 15′. Estimating 475 cu. ft. per ton, find how many tons of hay there are in the mow.

60. A cylindrical silo is 14 ft. in diameter and 36 ft. high. How many tons of silage will it hold, if 1 cu. ft. of silage weighs 40 lb.?

61. Ice is .92 as heavy as water. Find how high an ice house 20 ft. square must be built to store 184 tons of ice, making no allowance for packing.

62. How many quarts of water will a hemispherical kettle hold, if its inside diameter is 12 in.? (Use $\pi = 3.1416$.)

63. A grain bin in the form of a triangular prism, built in a square corner, extends 7½ ft. each way. If it is 6 ft. deep, how many bushels of grain will it hold?

64. A cylindrical cistern has a base area of 38½ sq. ft. How deep is it, if it holds 55 bbl. of water?

65. A car 40 ft. long, 8 ft. 6 in. wide, and 8 ft. high, inside measurements, was half full of shelled corn. How many tons of shelled corn did it contain?

66. If iron is 7.21 times as heavy as water, what is the weight, to the nearest tenth of a pound, of a 6-inch cannon ball?

67. A rectangular grain bin 6 ft. by 8 ft. has a bottom in the form of an inverted pyramid 3 ft. in altitude. If the total depth of the bin is 15 ft., how many bushels will it hold?

EXCAVATION AND MASONRY

Excavation is estimated to the nearest **cubic yard**.

Brickwork is estimated by the **thousand** (M) **bricks**.

Common bricks, laid in mortar, are usually reckoned at 22 per cubic foot. In this book, "bricks" means "common bricks," unless otherwise stated.

Stonework is estimated by the **perch** $(24\frac{3}{4}$ cu. ft.$)$, by the **cubic foot**, or by the **cubic yard**.

In *estimating the labor* of laying stone and brick no allowance is made for openings, and the corners are usually *counted twice* by taking the total length of wall equal to the perimeter of the wall *measured on the outside*. This is to offset the extra work of building around openings and corners.

In *estimating the material* used, however, some allowance is made for openings and the corners are *counted only once*.

NOTE. — Sometimes masons estimate the number of common bricks, laid in mortar, as follows, observing the above regulations as to allowance and corners according to whether the estimate is made for *labor* or for *material* :

7 bricks per square foot of wall surface, if the wall is 1 brick thick; 14, if it is 2 bricks thick; 21, if it is 3 bricks thick; and so on.

Concrete walls are estimated to the nearest **cubic yard**.
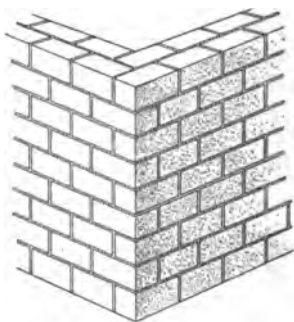
Written Exercises

Find the cost, at 42¢ per cubic yard, of the excavation for a cellar :

1. $27' \times 25' \times 4'$
4. $33' \times 30' \times 5\frac{1}{2}'$
7. $45' \times 38' \times 4' \ 6''$
2. $30' \times 27' \times 6'$
5. $39' \times 36' \times 4\frac{3}{4}'$
8. $36' \times 30' \times 7' \ 3''$
3. $36' \times 33' \times 7'$
6. $42' \times 40' \times 6\frac{3}{8}'$
9. $42' \times 33' \times 5' \ 9''$

Find, to the nearest perch, the masonry in a wall :

10. $30' \times 3\frac{1}{2}' \times 8'$
12. $50' \times 2\frac{2}{8}' \times 12'$
14. $48' \times 2' \ 3'' \times 7'$
11. $45' \times 2\frac{1}{4}' \times 9'$
13. $65' \times 3\frac{3}{4}' \times 14'$
15. $56' \times 2' \ 9'' \times 8'$

Find the cost, at $6.25 per cubic yard, of a concrete retaining wall:

16. $25' \times 3' \times 9'$ **19.** $54' \times 2\frac{3}{4}' \times 10'$ **22.** $60' \times 2' 4'' \times 19'$

17. $27' \times 2' \times 5'$ **20.** $45' \times 3\frac{1}{2}' \times 12'$ **23.** $63' \times 2' 8'' \times 27'$

18. $30' \times 3' \times 6'$ **21.** $81' \times 2\frac{3}{4}' \times 14'$ **24.** $90' \times 2' 9'' \times 18'$

25. At 35¢ per cubic yard, find the cost of removing a bed of sand the average measurements of which are $19' 6'' \times 16' \times 6' 9''$.

26. A wall 80' long, 4' high, and 10'' thick was built of concrete blocks each 24'' by 8'' by 10''. Find the cost of the wall at 28¢ per block laid.

27. How many cubic yards of earth were removed in digging a cylindrical well 7 ft. in diameter and 24 ft. deep?

28. At $8.40 per M, what is the cost of labor for building the brick foundation walls of a house $38' \times 32'$, the walls being 8' high and 1' thick?

<div align="center">SOLUTION</div>

Since in estimating labor, the corners are counted twice, the total length of wall is equal to the perimeter of the foundation measured on the *outside;* $2 \times (38' + 32') = 140'$.

Then, the volume $= (140 \times 8 \times 1)$ cu. ft., or 1120 cu. ft.

The number of bricks $= 1120 \times 22$, or 24,640, or 24.64 M.

Hence, the cost of labor $= 24.64 \times \$8.40$, or $206.98, to the nearest cent.

At $7.75 per M, find the cost of labor for laying the brick foundation walls, 8' high and 1' thick, of a house:

29. $35' \times 30'$ **32.** $45' \times 35\frac{1}{2}'$ **35.** $50' 4'' \times 42'$

30. $28' \times 22'$ **33.** $36' \times 28\frac{1}{4}'$ **36.** $46' 9'' \times 40'$

31. $40' \times 34'$ **34.** $30' \times 25\frac{3}{4}'$ **37.** $52' 8'' \times 38'$

38. The gable of a house is in the form of a triangle whose base is 24' and altitude, 12'. Find the number of bricks required to brick it up, if the wall is 2 bricks thick.

39. Find the cost, at $8 per M, of the bricks for the foundation walls of a house 34′ × 30′, the walls being 7′ 9″ high and 1′ thick, allowing 48 cu. ft. for openings.

<div align="center">SOLUTION</div>

Since in estimating material, the corners are counted only once, and since the wall is 1′ thick, the total length of wall is taken as 4 × 1′, or 4′, less than the perimeter measured on the outside; 2 × (34′ + 30′) − 4′ = 124′.

Then, allowing 48 cu. ft. for openings, we have the volume, (124 × 7¾ × 1) cu. ft. − 48 cu. ft. = 913 cu. ft.

The number of bricks, 913 × 22 = 20,086, or 20.086 M.

The cost of bricks, 20.086 × $8 = $160.69, to the nearest cent.

At $10 per M, find the cost of the bricks needed for the foundation walls, 8′ 6″ high and 1′ thick, of a house:

40. 26′ × 24′   **43.** 35½′ × 25′   **46.** 44′ × 36′ 2″

41. 32′ × 80′   **44.** 40⅓′ × 36′   **47.** 38′ × 38′ 8″

42. 38′ × 35′   **45.** 36¼′ × 34′   **48.** 46′ × 40′ 9″

49. How much will it cost, at $17 per thousand bricks laid, to build 4 brick piers each 8′ by 6′ by 18′ high?

50. A house is 42′ by 36′. How many bricks are required for the walls, 22′ high and 1′ thick, if 248 cu. ft. are allowed for openings?

51. Find, to the nearest perch, the amount of stone required to build the foundation walls for a barn 60′ by 40′, if the walls are to be 4′ high and 2½′ thick.

52. The concrete foundation walls for a house 36′ by 32′ are 8′ high and 1′ thick. If the corners are counted only once and 42 cu. ft. are deducted for openings, what is the cost at $7 per cubic yard?

53. What is the cost, at $7.75 per M, of laying the brick walls of a house 40′ by 34′ and 22′ high, if the lower 12′ of the walls are 1′ thick, and the upper 10′ only 8″ thick?

REVIEW

Written Exercises

Find the area of each of the following figures :

SQUARES	RECTANGLES	TRIANGLES
1. 26′ × 26′	**5.** 32 rd. by 26 rd.	**9.** Base 32′; alt. 18′
2. 45′ × 45′	**6.** 45 ft. by 96 in.	**10.** Base 62′; alt. 3.5′
3. 5′ 6″ × 5′ 6″	**7.** 36 yd. by 35 ft.	**11.** Base 56′; alt. 78½″
4. 12′ 8″ × 12′ 8″	**8.** 48 rd. by 66 yd.	**12.** Base 88′; alt. 4′ 6″

From the given area and one dimension, find the other dimension of each parallelogram :

13. 936 sq. ft., 36 ft.   **16.** 3888 sq. ft., 18 yd.

14. 1470 sq. yd., 35 yd.   **17.** 1162 sq. rd., 77 yd.

15. 5472 sq. rd., 456 rd.   **18.** 225 acres, 120 rd.

19–24. Regarding the numbers in exercises **13–18** as applying to triangles, find the other dimension in each exercise.

Find the area of each trapezoid :

	BASES	ALTITUDE		BASES	ALTITUDE
25.	24 in.; 18 in.	9 in.	**28.**	43 ft.; 37 ft.	35 ft. 3 in.
26.	16 ft.; 25 ft.	12 ft.	**29.**	32 yd.; 66 ft.	30 yd. 2 ft.
27.	32 yd.; 28 yd.	25 ft.	**30.**	48 rd.; 40 rd.	25 rd. 4 yd.

Fill the blanks in regard to circles :

	RADIUS	CIRCUMFERENCE		DIAMETER	AREA
31.	42.7 ft.	—— ft.	**35.**	84 ft. .	—— sq. ft.
32.	—— in.	132 in.	**36.**	—— yd.	2464 sq. yd.
33.	61¼ yd.	—— yd.	**37.**	12.6 rd.	—— sq. rd.
34.	—— rd.	26.4 rd.	**38.**	—— yd.	75.46 sq. yd.

Find the area of the entire surface of a cube whose edge is :

39. 9 ft.   **40.** 16 in.   **41.** 5.5 ft.   **42.** 4 ft. 3 in.

Find the volume of a rectangular solid:

43. $3' \times 4' \, 6'' \times 12'$ **44.** $8' \times 3' \, 3'' \times 5' \, 6''$ **45.** $12' \times 4' \, 8'' \times 6' \, 8''$

Find the lateral surface of a prism, having the perimeter of the base and the altitude in order as given:

46. 6 ft.; $4\frac{1}{2}$ ft. **48.** 36 ft.; $16\frac{2}{3}$ ft. **50.** $45'$; $12' \, 4''$

47. 8 ft.; $7\frac{3}{4}$ ft. **49.** 40 ft.; 25.5 ft. **51.** $56'$; $16' \, 9''$

Find the lateral surface of a cylinder, having the altitude and the radius of the base in order as given:

52. 7 ft.; 5 ft. **54.** 12 ft.; $3\frac{1}{4}$ ft. **56.** 28 ft.; 5.5 ft.

53. 9 ft.; 7 ft. **55.** 16 ft.; $4\frac{2}{3}$ ft. **57.** 35 ft.; $5\frac{1}{4}$ ft.

58–63. In exercises **52–57**, find the volume of each cylinder.

Find the lateral surface of a cone:

SLANT HEIGHT	PERIMETER OF BASE		SLANT HEIGHT	RADIUS OF BASE
64. 10 in.	2 ft.		**67.** 3 ft. 4 in.	1 ft. 9 in.
65. $2\frac{1}{4}$ ft.	$5\frac{1}{2}$ ft.		**68.** 4 ft. 9 in.	3 ft. 6 in.
66. $3\frac{2}{3}$ ft.	$4\frac{1}{2}$ ft.		**69.** 7 ft. 6 in.	4 ft. 8 in.

Find the surface of a sphere whose radius is:

70. 49 in. **72.** 2.8 ft. **74.** $5\frac{1}{4}$ ft. **76.** $31\frac{1}{2}$ ft.

71. 77 in. **73.** 4.2 ft. **75.** 84 ft. **77.** $45\frac{1}{2}$ ft.

78–85. In exercises **70–77**, find the volume of each sphere.

Find the volume of a pyramid: Find the volume of a cone:

	ALTITUDE	AREA OF BASE		ALTITUDE	RADIUS OF BASE
86.	$1\frac{1}{2}$ ft.	$28\frac{1}{2}$ sq. ft.	**91.**	6 ft.	$3\frac{1}{2}$ ft.
87.	4.5 ft.	120 sq. ft.	**92.**	7 ft.	3.6 ft.
88.	10.8 ft.	150 sq. ft.	**93.**	27 ft.	$8\frac{1}{4}$ ft.
89.	$12\frac{1}{2}$ ft.	162 sq. ft.	**94.**	30 ft.	$10\frac{1}{2}$ ft.
90.	$35\frac{1}{4}$ ft.	204 sq. ft.	**95.**	9.6 ft.	$5\frac{1}{4}$ ft.

Tell how you would solve each problem, then solve it :

96. If a wagon wheel is 3 ft. 6 in. in diameter, what is the length of its tire ?

97. How many cubic yards of earth were removed in excavating for a cellar 24′ × 21′ 6″ × 6′ ?

98. Clay is 1.2 times as heavy as water. Find the weight of a cubic yard of clay.

99. At 25 ¢ per square yard for sod and labor, find the cost of sodding a piece of ground 4 rd. long and 3 rd. wide.

100. A cylindrical well is 7 ft. in diameter and 21 ft. deep. How many barrels of water does it contain when it is half full?

101. How many pieces of flooring, 16′ long and 2½″ wide when laid, are required for the floor of a room 15′ × 16′ ?

102. Lead is 11.35 times as heavy as water. At 4½ ¢ per pound, find, to the nearest cent, the value of a cubic foot of lead.

103. A dome in the form of a hemisphere 28 ft. in diameter was covered with sheet copper. How many square feet were covered with copper ?

104. How many bundles of lath must be purchased to lath the walls and ceiling of a room 15′ × 18′ and 9′ 6″ high ?

105. How many cords of wood can be piled in a shed in 8 tiers, each 12 ft. long, 8 ft. high at one end and 6 ft. high at the other ?

106. A rectangular bin 5½′ × 5′ × 4′ is filled with shelled corn. Find the value of the corn at 64 ¢ per bushel.

107. At 30 ¢ per yard, find the cost of plain matting 1 yd. wide required for a floor 18′ × 14′, strips running crosswise.

108. A concrete cellar floor is 24 ft. 8 in. long and 22 ft. 6 in. wide. Find its cost at $1.20 per square yard.

109. At 25 ¢ per square yard, find the cost of painting the walls and ceiling of a room 14 ft. by 16 ft. and 9 ft. 6 in. high, making no allowance for openings.

110. At $35 per M, find the cost of 75 planks, each 16 ft. long, 10 in. wide, and 2¼ in. thick.

111. How many yards of Brussels carpet 27″ wide are required for a room 17′ 4″ by 22′ 6″, allowing 1¼ yd. for matching?

112. At $6 per square for shingles and labor, find the cost of shingling a gable roof, each side of which is 42′ by 18′ 6″.

113. Brass is 8.5 times as heavy as water. Find, to the nearest pound, the weight of 9 doz. brass balls each 2 in. in diameter.

114. A canvas tent was made in the form of a cone, the diameter of the base being 7 ft. and the slant height 9 ft. How many square yards of canvas were used, allowing 1 sq. yd. for seams and waste?

115. A room is 18′ 9″ × 16′ 8″. At 75 ¢ per double roll, what is the cost of papering the side walls, if the strips are 8′ 6″ long and 10 strips are deducted for openings?

116. The excavation for a large reservoir was let out to contractors in sections. One section, 135⅛ ft. by 102½ ft., was dug 6 ft. 9 in. deep. Find the cost at 19½ ¢ per cubic yard.

117. What is the cost of asphalt paving for ½ mi. of street 60 ft. wide, if the concrete foundation cost 90 ¢ per square yard and the asphalt $2.15 per square yard?

118. A brick building is 90′ by 42′. Find the cost, at $7.50 per M, of labor for laying the bricks, the walls being 32′ high and 20″ thick.

119. At 25 ¢ a square yard, find the cost of plastering the walls and ceiling of a room 14′ × 15′ and 9′ 6″ high, deducting for 1 door 3′ × 7′ and 2 windows 3′ × 6′.

120. At $8.25 per M, find the cost of the bricks for the foundation walls of a house 36′ by 30′, the walls being 8′ high and 1′ thick, allowing 40 cu. ft. for openings.

EFFICIENCY — SHORT METHODS

Preparation. — The eighth-year pupil has mastered the fundamental processes. He should be able to recognize instantly the forty-five addition combinations and their inverses in subtraction, the multiplication combinations and the inverse facts of division. He should know the factors, multiples, and parts of numbers.

Accuracy is of the first importance. An inaccurate result is not only worthless, but may cause great loss in business.

The student, then, should not only do his work carefully, but should use various tests and checks to insure accuracy.

Rapidity. — Next in importance to accuracy, and an outgrowth of it, is rapidity. With practice the accurate computer becomes rapid. The student who is rapid as well as careful is more accurate than the one who loiters over his work.

Rapidity is increased by the use of such short methods as are practical and generally applicable. The student who has a thorough knowledge of numbers and their relations will be able to devise short cuts as his work progresses.

Efficiency. — Accuracy and rapidity mean efficiency.

As with the pianist, so with the computer, constant practice is necessary for skill or efficiency.

Constant daily drill in rapid accurate work, then, is of the utmost importance.

The price of efficiency is practice.

ADDITION

Speed in adding may be greatly increased by the *method of grouping*, considered on page 18.

Written Exercises

Add upward and *test* each result by adding downward:

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
3⎫	7⎫	2	5	1	3	6	2	4	6	8	6
6⎭	4⎭	8	6	9	4	6	5	6	5	1	4
7⎫	5⎫	8	4	8	3	7	3	3	9	2	2
3⎭	3⎬	3	3	4	8	3	9	8	3	6	9
5⎫	2⎭	6	3	9	4	4	6	2	3	4	7
5⎭	9	4	8	2	7	6	4	8	4	7	3

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.	23.	24.
8⎫	9⎫	4	5	9	4	5	8	2	3	4	4
2⎭	1⎭	4	7	1	6	3	2	3	7	3	8
6⎫	8	2	6	3	6	2	3	5	6	3	5
4⎭	7⎫	9	4	5	5	6	9	4	5	7	5
2⎫	5⎭	3	1	2	7	5	7	4	7	4	7
4⎬	4⎫	5	3	4	3	4	6	2	8	9	7
4⎭	6⎭	2	6	7	9	6	4	9	4	·1	4

25.	26.	27.	28.	29.	30.	31.	32.	33.	34.	35.	36.
7⎫	6⎫	4	5	7	9	7	6	3	5	4	8
3⎭	5⎭	6	7	7	1	9	6	5	7	6	7
8	3⎫	7	8	3	5	5	2	2	9	8	3
9	8⎬	2	2	4	6	5	8	8	1	9	4
5⎫	4⎭	1	3	5	9	8	1	4	5	5	5
5⎭	3⎫	5	6	1	4	3	2	7	3	7	1
7⎫	1⎬	5	1	6	3	9	7	4	2	4	6
8⎭	6⎭	9	8	5	3	3	9	6	9	7	6 .

Since it is often necessary to add two two-figure numbers, it is important that the student drill on tables similar to the following in order that he may be able to tell the sum of such numbers at sight.

TABLE I

Tell sums quickly by rows; then by columns:

	a.	b.	c.	d.	e.	f.	g.	h.	i.	j.
1.	32	74	48	56	83	58	92	45	67	49
	20	40	30	50	40	20	70	30	50	10
2.	54	63	29	81	32	14	65	49	75	61
	70	50	20	70	30	20	40	20	60	20
3.	45	38	64	27	54	82	37	43	42	89
	40	40	70	50	30	60	50	30	50	30
4.	91	76	31	45	63	78	17	67	96	84
	70	60	80	20	40	30	90	30	30	20
5.	88	51	37	56	18	61	49	51	78	58
	60	30	90	40	80	90	80	70	20	60

TABLE II

Tell sums quickly by rows; then by columns:

	a.	b.	c.	d.	e.	f.	g.	h.	i.	j.
1.	65	72	48	39	28	35	54	36	48	63
	6	8	4	2	4	7	8	6	3	9
2.	27	52	89	49	99	76	99	64	29	38
	5	9	2	8	3	6	5	7	8	4
3.	47	63	67	47	29	46	74	28	54	45
	6	8	4	8	5	7	6	3	9	8
4.	37	39	24	59	74	87	93	83	38	27
	7	5	9	6	9	6	8	7	5	9
5.	49	78	97	38	46	98	79	95	57	98
	3	6	7	6	8	8	5	9	6	7

TABLE III

In this table combine the processes of tables I and II.

Thus, in exercise 1 *a*, 51 + 30 = 81 ; 81 + 8 = 89.

Tell sums quickly by rows ; then by columns :

	a.	*b.*	*c.*	*d.*	*e.*	*f.*	*g.*	*h.*	*i.*	*j.*
1.	51	48	54	28	93	56	23	25	72	49
	38	37	61	64	26	91	68	47	81	22
2.	28	46	69	82	37	61	49	53	38	74
	76	97	54	23	65	42	98	64	75	49
3.	55	91	38	67	57	85	43	27	59	27
	39	28	29	42	54	67	39	65	32	93
4.	47	63	81	72	32	28	65	24	56	48
	23	85	18	39	98	67	18	63	36	75
5.	71	49	48	27	58	26	29	44	25	84
	28	63	16	74	65	42	63	67	75	89

Oral Exercises

Tell sums quickly :

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**	**9.**	**10.**
32	26	57	49	28	66	54	35	46	59
18	25	36	24	17	33	44	38	29	38

11.	**12.**	**13.**	**14.**	**15.**	**16.**	**17.**	**18.**	**19.**	**20.**
39	43	93	48	75	65	77	93	81	62
59	26	17	85	63	47	54	28	32	43

21.	**22.**	**23.**	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
75	89	65	52	79	83	69	93	83	95
29	32	46	37	24	39	47	78	66	69

31.	**32.**	**33.**	**34.**	**35.**	**36.**	**37.**	**38.**	**39.**	**40.**
94	77	43	89	88	59	98	85	67	99
85	74	59	26	69	47	97	64	·49	89

Add by grouping and test, timing yourself on the last row of exercises :

1.	2.	3.	4.	5.	6.	7.	8.	9.
15	32	26	57	83	450	682	716	954
32	48	81	34	29	762	354	249	665
24	57	13	68	54	835	473	560	436
61	16	38	75	36	593	521	393	572
46	23	45	82	75	241	945	452	340
54	64	54	36	91	156	278	878	781
63	75	12	49	46	918	887	631	827

10.	11.	12.	13.	14.	15.	16.	17.	18.
26	16	43	38	55	681	915	841	735
14	33	27	52	48	734	472	269	988
57	64	52	16	39	942	386	474	497
63	27	64	43	72	450	654	653	512
85	42	39	74	64	833	467	782	604
78	51	78	69	28	878	593	567	823
42	78	81	25	76	267	828	396	376
81	95	46	87	13	549	245	930	851

19.	20.	21.	22.	23.	24.	25.	26.	27.
41	23	58	64	32	843	760	591	925
38	57	72	89	79	582	159	467	184
63	62	16	52	16	195	845	173	718
29	34	34	95	45	657	602	615	267
57	48	27	53	57	374	954	388	439
16	56	45	76	24	460	493	724	356
82	71	87	21	51	711	377	942	642
98	88	51	47	93	236	631	856	573
74	19	68	35	88	928	286	209	898

Add by grouping and test, timing yourself on exercises **34–39**:

28.	29.	30.	31.	32.	33.
$1.50	$2.25	$12.40	$45.00	$37.75	$506.10
3.25	4.50	36.85	18.36	28.64	650.25
2.75	7.82	60.38	32.55	52.07	124.65
5.42	5.03	25.16	69.27	16.40	438.19
4.07	3.16	84.34	27.83	83.56	327.52
6.51	1.87	78.00	94.72	54.82	291.07
8.69	9.48	59.73	71.90	95.08	745.50

34.	35.	36.	37.	38.	39.
$5.75	$4.09	$25.30	$18.25	$30.64	$214.50
3.16	1.82	47.28	35.40	16.58	527.64
2.28	3.65	60.49	62.74	75.39	750.28
8.47	9.50	18.62	27.82	44.05	345.83
4.39	2.78	39.65	39.91	81.71	472.91
1.03	7.47	72.84	81.76	52.83	633.06
6.54	6.13	55.73	44.37	67.97	969.35
7.92	8.34	91.07	53.19	93.42	801.47

40.	41.	42.	43.	44.	45.
$2.47	$1.68	$43.35	$68.74	$53.18	$108.78
5.09	3.42	27.75	81.45	18.79	327.46
8.51	7.75	82.63	27.60	60.45	683.64
5.32	6.01	19.48	79.58	27.30	219.35
7.63	2.80	38.56	92.83	72.23	734.50
1.74	5.93	54.81	13.96	34.54	455.87
6.98	8.34	80.20	54.37	59.06	746.93
8.20	9.77	61.02	36.00	41.82	962.02
9.15	4.16	62.94	45.29	85.91	871.19
3.86	3.29	75.17	28.13	68.37	590.21
4.57	7.50	96.69	80.72	90.70	689.78

In some business forms, numbers to be added are written in a horizontal line. To copy and arrange them in a column involves a waste of time and labor. Hence, it is of importance that the student should practice adding numbers arranged horizontally.

SUGGESTION. — Add from left to right or from right to left, *being careful that the units added are of the same order.*

Test results by adding in the opposite direction.

Written Exercises

Without copying, add by horizontal addition, and test:

1. 30, 7, 15, 8, 67, 41, 82, 11, 42, 20, 28, 34, 27, 46, 72.
2. 13, 24, 17, 63, 45, 72, 9, 37, 26, 50, 36, 64, 43, 81, 22.
3. 54, 27, 65, 89, 70, 42, 78, 20, 56, 34, 62, 18, 53, 27, 64.
4. 26, 45, 37, 53, 88, 62, 91, 76, 30, 14, 66, 27, 84, 46, 53.
5. 40, 16, 52, 24, 51, 33, 65, 97, 73, 46, 80, 36, 75, 95, 64.
6. 32, 27, 18, 53, 49, 71, 95, 70, 84, 66, 48, 23, 57, 29, 81.
7. 68, 40, 91, 34, 55, 47, 29, 15, 72, 80, 37, 64, 36, 58, 75.
8. 10, 35, 64, 21, 46, 59, 83, 77, 30, 63, 98, 55, 42, 28, 87.
9. 53, 60, 47, 33, 86, 24, 79, 14, 88, 50, 46, 91, 30, 63, 29.
10. 85, 14, 26, 47, 38, 72, 160, 150, 327, 483, 190, 268, 922.
11. 70, 45, 18, 32, 67, 53, 800, 245, 368, 709, 436, 914, 640.
12. 41, 39, 764, 310, 225, 158, 509, 684, 446, 817, 293, 717.
13. 8, 15, 42, 7, 23, 6, 50, 230, 416, 392, 615, 530, 721, 959.
14. 9, 17, 58, 24, 6, 5, 49, 562, 348, 509, 672, 836, 490, 999.

15. How many yards of muslin are there in ten pieces containing, respectively, 33, 35, 34, 32, 33, 37, 38, 35, 36, and 38 yards?

16. Find the cost, at $ 3.50 a yard, of ten pieces of broadcloth containing 44, 48, 50, 46, 52, 54, 48, 45, 47, and 52 yards.

Add horizontally, then vertically; test by finding whether the sums of the respective totals agree:

	23.	**24.**	**25.**	**26.**	**27.**	**28.**	**29.**	**30.**
17.	265 +	788 +	764 +	422 +	533 +	871 +	367 +	628 =
18.	431 +	356 +	253 +	586 +	217 +	254 +	508 +	275 =
19.	687 +	435 +	564 +	243 +	957 +	240 +	482 +	591 =
20.	329 +	234 +	213 +	133 +	516 +	166 +	625 +	736 =
21.	845 +	927 +	975 +	329 +	768 +	139 +	996 +	469 =
22.	521 +	276 +	432 +	115 +	275 +	752 +	843 +	957 = ___
	+	+	+	+	+	+	+	=

	37.	**38.**	**39.**	**40.**	**41.**	**42.**
31.	\$3.52 +	\$2.87 +	\$7.16 +	\$2.70 +	\$3.05 +	\$1.63 = \$
32.	6.28 +	5.99 +	.94 +	9.50 +	1.72 +	9.74 =
33.	5.41 +	8.64 +	2.43 +	.44 +	7.30 +	5.20 =
34.	4.75 +	4.31 +	7.10 +	2.85 +	8.98 +	.75 =
35.	7.64 +	9.18 +	5.81 +	6.48 +	.55 +	7.68 =
36.	2.32 +	2.17 +	.17 +	2.00 +	5.26 +	4.15 = ___
	+	+	+	+	+	=

43. A lumber merchant's sales for a week:

DAYS	HEMLOCK	SPRUCE	PINE	ASH	OAK	MISCEL-LANEOUS	TOTALS
Monday	\$ 225.18	\$ 114.30	\$ 125.00	\$ 113.94	\$ 32.42	\$ 104.16	\$
Tuesday	301.68	175.93	84.75	37.62	47.88	73.24	
Wednesday	307.16	204.87	205.50	84.73	56.24	30.71	
Thursday	204.37	226.18	54.45	99.18	75.37	86.49	
Friday	302.15	250.24	227.90	29.75	40.14	24.63	
Saturday	419.98	310.76	332.65	154.48	92.56	115.59	
Totals							

SUBTRACTION

For speed in subtraction, the student must rely on practice.

Oral Exercises

Subtract, giving results instantly :

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
35	54	67	43	89	72	61	97	75	99
10	20	40	30	60	40	50	70	30	20

11.	12.	13.	14.	15.	16.	17.	18.	19.	20.
14	33	22	44	61	54	42	75	94	82
7	6	3	.8	5	9	8	7	5	6

Subtract the tens of one number from the whole of the other and then subtract the units, thus in exercise **21**, "75, 35, 26."

21.	22.	23.	24.	25.	26.	27.	28.	29.	30.
75	52	61	46	88	72	55	99	64	72
49	36	29	18	25	37	29	63	28	37

31.	32.	33.	34.	35.	36.	37.	38.	39.	40.
48	67	81	56	44	91	73	85	95	63
19	42	33	27	16	59	25	68	79	45

41-100. Subtract each number rapidly from 100 :

94	39	52	75	88	41	25	67	59	91
65	51	11	22	43	69	32	72	93	89
77	19	38	44	27	61	84	92	12	54
29	42	63	82	16	34	53	73	95	46
57	23	37	45	62	76	97	13	58	87
48	56	81	78	14	64	36	28	71	35

101-204. Give the difference between each number and the number that follows it in the same row ; in the same column.

Oral and Written Exercises

Subtract quickly, timing yourself on exercises 41-50.

Solve mentally when possible.

1. 365 120	**11.** $2.15 1.67	**21.** 1739 1465	**31.** $25.75 13.90	**41.** 14.125 8.031
2. 418 256	**12.** $5.25 1.18	**22.** 4126 2340	**32.** $42.50 12.75	**42.** 275 16.82
3. 630 452	**13.** $4.50 2.55	**23.** 5310 4475	**33.** $60.82 49.91	**43.** 5.0085 1.2038
4. 509 316	**14.** $6.09 1.34	**24.** 6200 3529	**34.** $75 37.50	**44.** 4200 152.5
5. 200 149	**15.** $8.20 3.82	**25.** 3045 1152	**35.** $33.25 17.49	**45.** 608.15 138.25
6. 715 318	**16.** $7.14 4.20	**26.** 2480 1195	**36.** $51.43 25.52	**46.** 59 49.536
7. 841 502	**17.** $3.43 2.58	**27.** 8224 5330	**37.** $86.10 49.85	**47.** 3701.4 1911.5
8. 432 333	**18.** $6.30 4.38	**28.** 9000 3047	**38.** $32 15.72	**48.** .43698 .35721
9. 921 625	**19.** $5.15 3.27	**29.** 7139 4282	**39.** $90.15 38.35	**49.** 800.05 523.84
10. 660 571	**20.** $9 4.82	**30.** 4100 1175	**40.** $46.70 16.80	**50.** 9 1.2705

MULTIPLICATION

Certain short methods in multiplication, some of which have been given earlier in the book, are important because they are often applicable. A few are given here. Others, not so generally usable, are omitted.

The student should review multiplication by 10, 100, 1000, etc., considered on page 36.

Multiplication by parts of 1, 10, 100, 1000, etc., is given on page 370.

Multiplication, when the multiplier is near 10, 100, 1000, etc.

Written Exercises

1. Multiply 476 by 98.

$$98 = 100 - 2$$
$$100 \times 476 = 47600$$
$$2 \times 476 = \underline{952}$$
$$98 \times 476 = \overline{46648}$$

It is apparent that 98 times the multiplicand is 2 times less than 100 times the multiplicand. Hence, multiply 476 by 100, and from this product subtract 2 times 476. The result is 46,648.

2. Multiply 375 by 103.

$$103 = 100 + 3$$
$$100 \times 375 = 37500$$
$$3 \times 375 = \underline{1125}$$
$$103 \times 375 = \overline{38625}$$

It is apparent that 103 times the multiplicand is 3 times more than 100 times the multiplicand. Hence, multiply 375 by 100, and to this product add 3 times 375. The result is 38,625.

NOTE. — A method similar to that illustrated in exercise **1** or **2** may be employed, when the multiplier is near 10, 1000, and so on.

Find each product :

3. 9×345 **8.** 101×3.64 **13.** $103 \times \$ 37.25$

4. 11×544 **9.** 96×147.9 **14.** $998 \times \$ 62.75$

5. 98×406 **10.** 102×516.8 **15.** $105 \times \$ 58.49$

6. 99×738 **11.** $104 \times .0756$ **16.** $1001 \times \$ 85.50$

7. 97×827 **12.** 999×5.384 **17.** $1008 \times \$ 73.68$

Multiplication, when the multiplier is 11, 22, 33, 44, etc.

Oral and Written Exercises

1. Multiply 568 by 11.

$$
\begin{array}{r}
568 \\
11 \\
\hline
568 \\
568 \\
\hline
6248
\end{array}
$$

By observing the ordinary process we discover that each partial product contains the same figures as the multiplicand and in the same order, the figures of one being set under those of the other, one place to the left, and their sum taken for the complete product. We may do the same thing by inspection, omitting the partial products and writing the complete product from right to left from the figures of the multiplicand, thus:

Write **8**;
8 + 6 = 14, write **4**;
6 + 5 + 1 (carried) = 12, write **2**; $11 \times 568 = 6248$
5 + 1 (carried) =   6, write **6**.

To multiply by 11, *write the units' figure of the multiplicand for the units' figure of the product; then add the units to the tens, the tens to the hundreds, and so on, carrying when necessary.*

Multiply by 11, writing the product by inspection :

2. 84	**4.** 61	**6.** 876	**8.** 234	**10.** 2745
3. 79	**5.** 35	**7.** 498	**9.** 502	**11.** 8479

12. Write by inspection the product of 357 and 22.

SOLUTION. — Proceed as though the multiplier were 11, multiplying each sum by 2 before adding the figure to be carried, thus: 2 × 7 = 14, write **4**; 2 × (7 + 5) + 1 (carried) = 25, write **5**; 2 × (5 + 3) + 2 (carried) = 18, write **8**; 2 × 3 + 1 (carried) = 7, write **7**. That is, 22 × 357 = 7854.

NOTE. — Before carrying, multiply by 3 when the multiplier is 33; by 4 when the multiplier is 44; by 7 when the multiplier is 77; and so on.

Multiply by 22 :

13. 96	**15.** 72	**17.** 690	**19.** 757	**21.** 1028
14. 58	**16.** 47	**18.** 882	**20.** 365	**22.** 3739

23–82. In exercises **13–22**, multiply by 33; 55; 88; 66; 99; 44.

Multiplication, when one part of the multiplier is a multiple of another part.

Written Exercises

1. Multiply 2748 by 213.

```
2748
 213
8244
57708
585324
```

The partial product by 3, the units' figure of the multiplier, is 8244.

Since 21 is 7 times 3, the partial product by 21 contains 7 times as many tens as the first partial product contains units; hence, multiplying 8244 by 7, we obtain 57,708 tens for the second partial product, which is written with its right-hand figure in tens' place. Adding the partial products, we find that the entire product is 585,824.

NOTE. — Any figure of the multiplier may be used in obtaining the first partial product.

Multiply :

2.
```
  8346
   436
 33384
300456
3638856
```

3.
```
   4912
   5607
  34384
 275072
27541584
```

Find products :

4. 142 × 370

5. 153 × 524

6. 255 × 652

7. 412 × 285

8. 164 × 487

9. 427 × 823

10. 612 × 509

11. 832 × 476

12. 639 × 715

13. 321 × 1245

14. 416 × 3014

15. 355 × 2570

16. 182 × 5338

17. 624 × 7132

18. 816 × 2683

19. 549 × 6052

20. 721 × 4795

21. 648 × 8321

22. 1206 × 3175

23. 1503 × 4781

24. 4016 × 2096

25. 2018 × 5514

26. 3505 × 7328

27. 7042 × 6157

28. 3204 × 8265

29. 9027 × 5341

30. 8048 × 7982

Cost of articles sold by the hundred or by the thousand.

Prices of articles sold *by the hundred* or *by the thousand* are often given " per C " or " per M," respectively.

The student should review division by 10, 100, 1000, etc., considered on page 51.

Written Exercises

1. Find the cost of 560 posts at $ 17 per C.

SOLUTION. $560 \div 100 = 5,6$ C; hence, the cost of the posts $= 5.6 \times \$ 17$, or $ 95.20.

2. Find the cost of 1264 ft. of lumber at $ 28 per M.

SOLUTION. $1264 \div 1000 = 1.264$ M.
Cost of the lumber, $1.264 \times \$ 28 = \$ 35.392$, or $ 35.39.

Find, to the nearest cent, the cost of :

3. 625 staves at $ 2.40 per C.

4. 450 cucumbers at $ .50 per C.

5. 725 berry baskets at $ .45 per C.

6. 1650 lb. pork at $ 15.50 per cwt.

7. 2125 vegetable crates at $ 18 per C.

8. 6150 feet of birch flooring at $ 50 per M.

9. 7125 feet of plain oak flooring at $ 90 per M.

10. 8240 feet of hard pine flooring at $ 75 per M.

11. 6480 feet of rock maple flooring at $ 55 per M.

12. 8175 feet of quartered oak flooring at $ 125 per M.

13. A stationer bought 12,500 sheets of paper.   Find its cost at $ 3.20 per M.

14. A flat roof 45' 4" by 27' was covered with tin.   Making no allowance for lapping, find the cost of the tin at $ 6.75 per square.

15. At $ 4.50 per M, find the cost of the laths that must be purchased for the walls and ceiling of a room 12' by 15' and $9\frac{1}{2}$' high, making no allowance for openings.

Cost of articles sold by the ton of 2000 pounds.

1. How much will 3450 lb. of hay cost at $18 per ton ?

SOLUTION. — Since 1 ton of hay costs $18, 1000 lb., or ½ of a ton, will cost ½ of $18, or $9. Since 3450 lb. = 3.450 thousand pounds, or 3.45 half tons, 3450 lb. of hay will cost 3.45 × $9, or $31.05.

Find, to the nearest cent, the cost of :

2. 1250 lb. of bran at $25 per ton.

3. 2580 lb. of straw at $16 per ton.

4. 1500 lb. of meal at $35 per ton.

5. 9362 lb. of ice at $4.25 per ton.

6. 4358 lb. of fertilizer at $24 per ton.

7. 1085 lb. of pea coal at $3.50 per ton.

8. 3625 lb. of stove coal at $6.50 per ton.

9. In a certain time four cows ate 3725 lb. of bran. Find its value at $22 per ton.

10. A cow ate 7280 lb. of silage in a year. Find the value of the silage eaten at $2.75 per ton.

11. At $26 per ton, what is the cost of the corn meal used annually at a goldfish farm, if the fish require 3600 lb. per month ?

12. On 19 acres of land, a farmer used 508 lb. of quicklime per acre. Find the cost of the quicklime at $5.25 a ton.

13. In mixing fertilizer, a farmer used 500 lb. of acid phosphate per ton of fertilizer. At $14.15 per ton, find the cost of the acid phosphate required for 5½ tons of the mixture.

14. Find the total cost of the following loads of coal at $6.25 per ton : 3849 lb., 2935 lb., 4832 lb., 2364 lb., 2837 lb., 4378 lb., and 1352 lb.

DIVISION

There are not many short methods in division that are of importance to pupils of the eighth year. Rapidity in division is increased, however, by practice and by familiarity with products and multiples. Pupils should use short division with as large divisors as possible, thereby saving the time of writing products and remainders.

Division by parts of 1, 10, 100, 1000, etc., is given on page 873.

Written Exercises

Find quotients, timing yourself on the second column :

1. 2495 + 5	**17.** 39.816 + 86	**33.** $602.14 + 119
2. 27.16 + 4	**18.** .82732 + 52	**34.** $718.68 + 818
3. 760.9 + 7	**19.** 51,675 ÷ 75	**35.** $980.10 + 242
4. $27.36 + 8	**20.** $732.00 + 48	**36.** $898.56 + 416
5. $35.01 + 9	**21.** $633.07 + 59	**37.** 768,800 + 3000
6. $49.83 + 11	**22.** $925.44 + 64	**38.** 677,500 + 2710
7. $18.84 ÷ 12	**23.** 734.56 + 400	**39.** 586,696 + 1298
8. $85.12 ÷ 16	**24.** 5027.5 + 250	**40.** 778.848 + 8416
9. 19,960 + 20	**25.** 234,848 + 328	**41.** 862,722 + 3507
10. 32,160 + 15	**26.** 340,704 + 546	**42.** 770,296 + 5276
11. .16982 + 14	**27.** 513,996 + 609	**43.** 7159.12 + 1642
12. 37,575 + 25	**28.** 408,595 + 437	**44.** 960.872 + 2684
13. 29,920 ÷ 40	**29.** 4181.44 + 584	**45.** 675,957 + 4863
14. 88,512 + 16	**30.** 57.7005 + 715	**46.** 732,768 ÷ 1796
15. 21,816 + 18	**31.** 62651.6 + 982	**47.** 90732.6 + 2894
16. 50,688 + 22	**32.** 852.489 + 869	**48.** 845,053 + 4519

ALIQUOT PARTS

The parts of a number that exactly divide it are aliquot parts of the number.

Thus, 10 and 20 are $\frac{1}{10}$ and $\frac{1}{5}$, respectively, of 100 and they are aliquot parts, because each is contained an exact number of times in 100, but 30, which is $\frac{3}{10}$ of 100, is not an aliquot part, because it is not an exact divisor of 100. It is, however, a *multiple* of an aliquot part.

Similarly, 2, 5, and 3$\frac{1}{3}$ are aliquot parts of 10, but 6$\frac{2}{3}$ is a multiple of the aliquot part 3$\frac{1}{3}$.

Also, 25, 50, 12$\frac{1}{2}$, 33$\frac{1}{3}$, and 16$\frac{2}{3}$ are aliquot parts of 100.

And, 250, 500, 125, 333$\frac{1}{3}$, and 166$\frac{2}{3}$ are aliquot parts of 1000.

Many of the ordinary business computations are much abbreviated by the use of aliquot or other simple parts.

Memorize this table of aliquot parts of $1 and their multiples:

50 ¢ = $ $\frac{1}{2}$ = $.50	5 ¢ = $ $\frac{1}{20}$ = $.05
25 ¢ = $ $\frac{1}{4}$ = $.25	12$\frac{1}{2}$ ¢ = $ $\frac{1}{8}$ = $.12$\frac{1}{2}$
75 ¢ = $ $\frac{3}{4}$ = $.75	37$\frac{1}{2}$ ¢ = $ $\frac{3}{8}$ = $.37$\frac{1}{2}$
20 ¢ = $ $\frac{1}{5}$ = $.20	62$\frac{1}{2}$ ¢ = $ $\frac{5}{8}$ = $.62$\frac{1}{2}$
40 ¢ = $ $\frac{2}{5}$ = $.40	87$\frac{1}{2}$ ¢ = $ $\frac{7}{8}$ = $.87$\frac{1}{2}$
60 ¢ = $ $\frac{3}{5}$ = $.60	6$\frac{1}{4}$ ¢ = $ $\frac{1}{16}$ = $.06$\frac{1}{4}$
80 ¢ = $ $\frac{4}{5}$ = $.80	33$\frac{1}{3}$ ¢ = $ $\frac{1}{3}$ = $.33$\frac{1}{3}$
10 ¢ = $ $\frac{1}{10}$ = $.10	66$\frac{2}{3}$ ¢ = $ $\frac{2}{3}$ = $.66$\frac{2}{3}$
30 ¢ = $ $\frac{3}{10}$ = $.30	16$\frac{2}{3}$ ¢ = $ $\frac{1}{6}$ = $.16$\frac{2}{3}$
70 ¢ = $ $\frac{7}{10}$ = $.70	83$\frac{1}{3}$ ¢ = $ $\frac{5}{6}$ = $.83$\frac{1}{3}$
90 ¢ = $ $\frac{9}{10}$ = $.90	8$\frac{1}{3}$ ¢ = $ $\frac{1}{12}$ = $.08$\frac{1}{3}$

From the parts of $1 (100 ¢) given in the table make a similar table of the corresponding parts of 100; thus, 50 is $\frac{1}{2}$ of 100; 25 is $\frac{1}{4}$ of 100; etc. Memorize the table you have made.

Construct the parts of 1, 10, 100, 1000, thus:

Parts	Or 1	Or 10	Or 100	Or 1000
$\frac{1}{2}$	.5	5	50	500
$\frac{1}{4}$	.25	2.5	25	250
$\frac{1}{8}$	.125	1.25	12.5	125

and so on.

MULTIPLICATION BY PARTS

Oral and Written Exercises

1. Multiply 48 by 12½.   **2.** Multiply 32 by 37½.

$$8)\overline{4800}$$
$$\,600$$

$$\frac{3}{8} \text{ of } \cancel{3200} = 1200$$

$100 \times 48 = 4800$; $12\frac{1}{2}$ is $\frac{1}{8}$ of 100. $100 \times 82 = 3200$; $37\frac{1}{2}$ is $\frac{3}{8}$ of 100.
Then, $12\frac{1}{2} \times 48 = \frac{1}{8}$ of 4800, or 600.   Then, $37\frac{1}{2} \times 32 = \frac{3}{8}$ of 3200, or 1200.

To multiply by a part of 100, *multiply by* 100 *and take such a fractional part of the result as the given multiplier is of* 100.

When the multiplier is an *aliquot* part of 100, proceed as in exercise **1.**

Multiply, first estimating products mentally:

3. 64 by 25	**8.** 144 by 12½	**13.** $5.20 by 37½
4. 55 by 60	**9.** 267 by 33⅓	**14.** $4.92 by 83⅓
5. 72 by 75	**10.** 414 by 16⅔	**15.** $6.48 by 62½
6. 96 by 6¼	**11.** 360 by 37½	**16.** $8.85 by 33⅓
7. 84 by 8⅓	**12.** 624 by 66⅔	**17.** $9.52 by 87½
18. 72 by .66⅔	**19.** 72 by 6⅔	**20.** 72 by 666⅔

$$\frac{2}{3} \text{ of } \cancel{72} = 48$$

$$\frac{2}{3} \text{ of } \cancel{720} = 480$$

$$\frac{2}{3} \text{ of } \cancel{72000} = 48000$$

To multiply by the parts of 1, 10, or 1000, employ a method similar to that used in multiplying by the parts of 100.

21. 56 by 75	**27.** 465 by 33⅓	**33.** $4.50 by 500
22. 72 by .25	**28.** 392 by .62½	**34.** $3.76 by 12.5
23. 78 by 1⅔	**29.** 556 by 750	**35.** $7.52 by 250
24. 81 by .3⅓	**30.** 752 by .125	**36.** $9.04 by 875
25. 88 by 1¼	**31.** 848 by 3.75	**37.** $6.42 by 166⅔
26. 96 by .75	**32.** 681 by 6.6⅔	**38.** $8.46 by 333⅓

Finding the cost when the price is an aliquot or other simple fractional part of one dollar.

NOTE. — In the following exercises, estimate the result mentally before performing the solution.

In finding cost, give answers to the nearest cent. (See note on page 98.)

Oral and Written Exercises

1. Find the cost of 22 doz. eggs @ 33⅓ ¢.

SOLUTION

At $1 a dozen, 22 doz. eggs would cost $22.

Then, at $.33⅓, or $⅓, a dozen, they cost ⅓ of $22, or $7.33.

Find the cost of 35 articles at each of the given prices:

2. 20 ¢	6. 40 ¢	10. 5 ¢	14. 12½ ¢	18. 62½ ¢
3. 50 ¢	7. 60 ¢	11. 80 ¢	15. 33⅓ ¢	19. 66⅔ ¢
4. 25 ¢	8. 75 ¢	12. 6¼ ¢	16. 16⅔ ¢	20. 83⅓ ¢
5. 30 ¢	9. 90 ¢	13. 8⅓ ¢	17. 37½ ¢	21. 87½ ¢

22–41. Find the cost of 112 articles at the prices given in exercises 2–21.

42. Find the cost of 50 yd. of carpet @ $1.12½ ; @ $.87½.

$50	The cost @ $1 would be $50.	$50
6.25	The cost @ $1.12½ is $50 + ⅛ of $50.	6.25
$56.25	The cost @ $.87½ is $50 − ⅛ of $50.	$43.75

Find the cost of 26 articles at each of the given prices:

43. $1.50	47. $.90	51. $1.33⅓	55. $.83⅓	59. $2.37½
44. $1.25	48. $.75	52. $1.16⅔	56. $.87½	60. $2.83⅓
45. $1.30	49. $.80	53. $1.12½	57. $.91⅔	61. $2.87½
46. $1.75	50. $.95	54. $1.66⅔	58. $.93¾	62. $2.08⅓

63–82. Find the cost of 85 articles at the prices given in exercises 43–62.

Find the cost of :

83. 48 sleds @ $.75

84. 72 towels @ $.12½

85. 60 aprons @ $.16⅔

86. 24 footballs @ $1.50

87. 60 baseballs @ $1.25

88. 96 toy watches @ $.40

89. 32 hair brushes @ $.37½

90. 100 lace collars @ $.33⅓

91. 120 tennis balls @ $.37½

92. 60 purses @ $.62½

93. 48 shears @ $.83⅓

94. 25 knives @ $.66⅔

95. 72 mufflers @ $.87½

96. 108 fur caps @ $3.50

97. 125 toboggans @ $3.25

98. 120 hammocks @ $2.87½

99. 144 sacks flour @ $1.33⅓

100. 152 hat brushes @ $1.37½

101. 124 Indian suits @ $1.66⅔

102. 240 boys' sweaters @ $1.33⅓

103. Find the cost of 64 lb. of rice at 6¼ ¢ per pound.

104. How much did a farmer receive for a crate containing 30 doz. eggs, if he sold them for 37½ ¢ per dozen ?

105. How much must a fruit dealer pay for a box containing 85 grapefruits at 8⅓ ¢ each ?

106. A grocer bought a cheese weighing 40 lb. for 16⅔ ¢ per pound.   Find the cost of the cheese.

107. How much will 25 pairs of skates cost at $2.75 per pair ?

108. A haberdasher bought 15 dozen neckties at 37½ ¢ each and sold them at 50 ¢ each.   What was his profit ?

109. One year Mr. Wend sold 54 bu. of timothy seed at $4.25 per bushel.   How much did he receive for the seed ?

110. Mr. Odell sold 755 bu. of potatoes at 62½ ¢ per bushel. How much did he receive for them ?

111. A contractor put down 2898 sq. ft. of cement walk at $1.12½ per square yard.   How much did he receive for it ?

DIVISION BY PARTS

Oral and Written Exercises

1. Divide 275 by $12\frac{1}{2}$.

$$\begin{array}{r} 2.75 \\ 8 \\ \hline 22.\cancel{00} \end{array}$$

Since $12\frac{1}{2} = \frac{1}{8}$ of 100, we divide by 100 and then by $\frac{1}{8}$; that is, we divide by 100 and multiply by 8.

2. Divide 348 by $37\frac{1}{2}$.

$$3.48 \div \frac{3}{8} = \overset{1.16}{\cancel{3.48}} \times \frac{8}{\cancel{3}} = 9.28$$

Since $37\frac{1}{2} = \frac{3}{8}$ of 100, we divide by 100 by pointing off two places, and divide the result by $\frac{3}{8}$; that is, we multiply 3.48 by $\frac{8}{3}$, using cancellation.

To divide by a part of 100, *divide by* 100 *and then by the fraction that shows what part the given divisor is of* 100.

When the divisor is an *aliquot* part of 100, proceed as in exercise **1**.

Divide, first estimating the quotient mentally :

3. 16 by ,25
4. _24 by 20
5. 39 by 30
6. 65 by 75
7. 56 by 80

8. 75 by $6\frac{1}{4}$
9. 51 by $8\frac{1}{3}$
10. 47 by $12\frac{1}{2}$
11. 63 by $33\frac{1}{3}$
12. 72 by $37\frac{1}{2}$

13. $8.45 by $16\frac{2}{3}$
14. $2.16 by $66\frac{2}{3}$
15. $3.75 by $62\frac{1}{2}$
16. $5.25 by $83\frac{1}{3}$
17. $9.59 by $87\frac{1}{2}$

18. 847 by $.87\frac{1}{2}$

$$\overset{1\overset{.}{2}1}{\cancel{847}} \times \frac{8}{7} = 968$$

19. 847 by $8\frac{3}{4}$

$$\overset{12.1}{\cancel{84.7}} \times \frac{8}{7} = 96.8$$

20. 847 by 875

$$\overset{.121}{\cancel{.847}} \times \frac{8}{7} = .968$$

To divide by the parts of 1, 10, or 1000, employ a method similar to that used in dividing by the parts of 100.

21. 173 by .25
22. 354 by 75
23. 268 by $8\frac{1}{3}$
24. 495 by $.8\frac{1}{3}$
25. 576 by $6\frac{2}{3}$

26. 381 by $16\frac{2}{3}$
27. 718 by $3.3\frac{1}{3}$
28. 275 by 500
29. 589 by 87.5
30. 948 by 750

31. $32.50 by $83\frac{1}{3}$
32. $19.75 by .125
33. $49.14 by 3.75
34. $82.25 by .625
35. $7320 by $666\frac{2}{3}$

Do orally as many of the following exercises as possible :

1. How many 50-cent vases can be bought for $5 ?

2. How many 25-cent balls can be bought for $8 ?

3. How many 12½-cent collars can be bought for $4 ?

4. How many 33⅓-cent neckties can be bought for $9 ?

5. How many 16⅔-cent doilies can be bought for $12 ?

6. At 87½ ¢ each, how many articles will $28 buy ?

Suggestion. — The number of articles that $28 will buy at 87½ ¢ each is equal to the number of times that $28 contains $⅞.

7. At $1.33⅓ each how many articles will $84 buy ?

Suggestion. — The number of articles that $84 will buy = $84 ÷ $1⅓.

Find the number of articles that can be bought for :

8. $21 at 75 ¢ each.

9. $18 at 37½ ¢ each.

10. $40 at 62½ ¢ each.

11. $112 at 66⅔ ¢ each.

12. $252 at 87½ ¢ each.

13. $77.50 at $1.25 each.

14. $87.75 at $1.12½ each.

15. $94.50 at $1.16⅔ each.

16. If 12 cases of canned corn cost $36, and the price per can was 12½ ¢, how many cans of corn did each case contain ?

17. How many days, at $4.50 per day, must a man work to earn enough money to buy a suit of clothes costing $30 ?

18. A wagonload of apples sold for $25.20. If the price per bushel was 60 ¢, how many bushels were there in the load ?

19. When berries cost 16⅔ ¢ per basket, how many baskets can be bought for $16.50 ?

20. How many pound cans of coffee were there in five cases that cost $240, if the wholesale price was 33⅓ ¢ per pound ?

21. The receipts for tickets for a football game were $266.50. If the tickets cost 25 ¢ each, how many were sold ?

COMPARISON OF TERMS

The application of the principles of simple multiplication to subjects closely related is worthy of attention. Besides the incidental gain coming from a review of these subjects, there is a special advantage in associating the familiar terms with the somewhat less familiar terms used in percentage and its applications. Following are some corresponding terms:

In multiplication,	*multiplier*	×	*multiplicand*	=	*product*
In percentage,	*rate*	×	*base*	=	*percentage*
In profit and loss,	*rate of profit (loss)*	×	*cost* ·	=	*gain (loss)*
In commission,	*rate of commission*	×	*amount of sales*	=	*commission*
In discount,	*rate of discount*	×	*list price*	=	*discount*
In interest,	*(rate × time)*	×	*principal*	=	*interest*

PERCENTAGE PRACTICE

Since the multiplier and the multiplicand are factors of the product in each of the above topics, *any one of the three terms may be found when the other two are given.*

Oral and Written Exercises

Tell how you would solve, then solve, finding the missing term:

MULTIPLIER	MULTIPLICAND	PRODUCT
1. Rate, 5%	Base, 840	Percentage, ——
2. Rate of dis., 12%	List price, ——	Discount, $2.88
3. Rate of gain, ——	Cost, $1500	Gain, $375
4. Rate, 12½%	Base, ——	Percentage, $250
5. Rate of com., 5%	Amt. of sales, $4725	Commission, ——
6. Rate of loss, ——	Cost, $36.80	Loss, $5.52
7. Rate of dis., 33⅓%	List price, ——	Discount, $25.20
8. Rate of gain, 37½%	Cost, $64.80	Gain, ——
9. Rate of com., ——	Amt. of sales, $8420	Commission, $294.70
10. Rate of loss, 28%	Cost, ——	Loss, $90.93

11. A dealer sold oilcloth marked $.75 per yard at a discount of 20 %. What was the selling price ?

12. A train that was running 48 mi. an hour increased its speed 8¾ %. How many miles an hour was it then running ?

13. A grocer made 15¢ by selling a pound of tea for 60¢. What per cent of the cost did he gain ?

14. An electric toaster listed at $6.30 was sold at 16⅔ % off. Find the selling price.

15. How much did a merchant pay for 25 doz. pencils listed at 50¢ per dozen, discount 30 % ?

16. Gloves that were bought for $1.12½ per pair were retailed at a gain of 33⅓ %. What was the selling price?

17. An agent sold a farm for $6750, receiving $236.25 commission. What was the rate of commission ?

18. A city lot that cost $525 was sold at a loss of 8⅓ %. What was the selling price ?

19. A barrel of flour sold at the grocery for 3½ ¢ per pound, which was 140 % of the cost. Find the cost.

20. If in a certain town 3640 of the 4160 inhabitants are miners, what per cent of the population are miners?

21. A commission merchant sold 5375 bu. of oats at 45¢ a bushel. Find his commission at 2½ %.

22. How many tons will 520 bags of coffee, weighing 132 lb. each, weigh when roasted, if coffee loses 15 % of its weight in roasting ?

23. Find the gain per cent on a gross of brooms bought for $34.56 and sold at $.28 each.

24. A broker bought flour at $5.50 a barrel, receiving $110 brokerage. If the brokerage was 2 %, how many barrels of flour did he buy ?

Since in interest the *principal*, the *rate*, and the *time* are factors of the *interest*, any one of the *four terms may be found when three are given or can be determined.*

Review as much of the chapter on Interest as may be necessary in solving the following exercises. Employ short methods that are applicable. *Use cancellation whenever possible.*

Written Exercises

Find the interest :   Find the amount :

	PRINCIPAL	TIME	RATE		PRINCIPAL	TIME	RATE
1.	$780	2⅓ yr.	3%	9.	$1500	3 yr. 2 mo.	5%
2.	$535	1¾ yr.	4%	10.	$1450	2 yr. 8 mo.	4%
3.	$860	3½ yr.	6%	11.	$1675	3 yr. 8 mo.	5%
4.	$585	3 mo.	7%	12.	$1936	4 yr. 3 mo.	4%
5.	$950	8 mo.	3%	13.	$25.50	2 yr. 4 mo.	3%
6.	$736	9 mo.	4%	14.	$86.40	1 yr. 9 mo.	4½%
7.	$672	15 da.	3½%	15.	$72.70	3 yr. 6 mo.	3½%
8.	$996	60 da.	4½%	16.	$96.42	2 yr. 2 mo.	5½%

Find the interest on :

17–19. $560 for 1 yr. 2 mo. 12 da. at 6% ; at 5% ; at 4%

20–22. $750 for 2 yr. 1 mo. 15 da. at 3% ; at 4% ; at 7%

23–25. $625 for 3 yr. 4 mo. 24 da. at 6% ; at 7% ; at 4%

26–28. $885 for 2 yr. 4 mo. 15 da. at 3% ; at 5% ; at 8%

29. Mr. Harvey had a mortgage of $4275 on his house. How much interest did he pay on the mortgage in 3 yr. 6 mo. at 5¼ % ?

30. Mr. Brown borrowed $3650 from a loan association, paying the agent ¾% commission. How much did the loan cost him, if he kept the money 1 yr. 8 mo., interest at 6 %?

Fill the blank spaces in the following:

	Principal	Rate	Time	Interest	Amount
31.	$1470	4 %	1 yr. 8 mo.	——	——
32.	$1680	——	2 yr. 9 mo.	——	$1841.70
33.	$1578	6 %	——	$118.35	——
34.	——	3 %	2 yr. 4 mo.	$132.30	——
35.	$1476	7 %	——	——	$1699.86
36.	——	4 %	4 yr. 3 mo.	$283.56	——
37.	$1896	——	1 yr. 6 mo.	——	$2023.98
38.	$1974	5 %	——	$180.95	——
39.	——	6 %	3 yr. 4 mo.	——	$2484
40.	$2050	——	4 yr. 6 mo.	$645.75	——
41.	$2175	3 %	——	——	$2414.25
42.	——	4 %	4 yr. 9 mo.	$467.40	——
43.	$2280	——	1 yr. 7 mo.	——	$2496.60
44.	$2472	$3\frac{1}{2}$ %	——	$252.35	——
45.	$2544	——	3 yr. 5 mo.	——	$2935.14

46. The amount, at maturity, of a note for $2400 was $2706. How long did the note run, if the rate of interest was 5 % ?

47. John Moore bought a house for $7250. He paid $2000 cash and gave a note for the rest payable in 3 yr. 4 mo. with interest at $4\frac{1}{2}$ %. Find the amount of the note at maturity.

48. Which yields the greater yearly income, $1250 deposited in a savings bank with interest compounded quarterly at 4 % per annum, or $1250 loaned at 5 % per annum? how much greater?

49. A man draws the interest on his money at the bank as it falls due. He thus receives semiannually $28.25, interest being at $2\frac{1}{2}$ % annually. How much money has he on deposit and in how many years will it be equaled by the interest?

BUSINESS FORMS

ACCOUNTS

A record of business transactions is an **account**.
The business of the day is usually recorded, as the transactions occur, in a **daybook**. The records of the several transactions are afterward transferred, or **posted**, into another book, a **ledger**, where they are arranged in groups, each containing the transactions that belong to one class, as, for example, those relating to one person or firm.

These groups, or **ledger accounts**, headed by the name of the person, are arranged so that the amounts charged against him, **debits**, appear on the *left* side and those in his favor, **credits**, on the *right* side.

A person who owes a debt is a **debtor**, and a person to whom a debt is owed is a **creditor**.

The difference between the sum of the debits and the sum of the credits of an account is the **balance**.

Computing the balance, *entering* it on the lesser side, and *ruling* up the account, is known as **balancing** the account.

Some business men balance accounts at stated times, as once a month, the balance being *brought down* to continue the account as shown on page 380. An account is also balanced when it is settled. This is called **closing** the account.

The following are a few abbreviations in common use:

Company, Co.	Credit or creditor, Cr.	Paid, pd.
Balance, bal.	Debit or debtor, Dr.	Received, rec'd.
Account, acc't.	Merchandise, mdse.	Payment, pay't.

The symbol # placed *after* a number stands for *pounds*, but if placed *before*, it is an abbreviation for the word "number."

Thus, 24 # means "24 pounds," but # 24 means "number 24."

The following form illustrates a balanced ledger account with George Fox, the balance being brought down to continue the account into the next month :

George Fox

1915		Dr.	*			1915		Cr.	*		
Jan.	4	Flour	1	15	40	Jan.	4	Cash	1	10	
	7	Oats	4	8	70		12	Carting	8	12	50
	7	Cash	4	5			19	"	15	10	
	15	Corn Meal	11	17	50		24	"	21	17	
	24	Hay	21	24			31	Balance		32	47
	29	Oats	34	11	87						
				81	97					81	97
Feb.	1	Balance		32	47						

1. What are the debits in this account ? Find their sum.

2. Name the credits. Find their sum.

3. How much is the balance ? How is it found ? See whether it has been correctly computed.

4. Is the balance in favor of or against George Fox ?

On which side of the account would it have been entered if it had been in his favor ?

On which side would it have been brought down ?

5. 'What is the *footing* of the debit side of the account ? of the credit side ? How do the footings compare ?

Test. — When an account is properly balanced, the footing of one side is equal to the footing of the other.

* These columns contain numbers of the daybook pages upon which the several transactions were entered.

Prepare a ledger form for each of the following accounts, supply the year, enter the items, close, and test the account:

1. George Griffin.

Debits. — Jan. 8, Groceries, $10.37; Jan. 22, Groceries, $18.91; Feb. 10, Groceries, $7.89; Feb. 24, Groceries, 48¢; Mar. 6, Groceries, $15.75; Mar. 20, Groceries, 66¢; April 4, Groceries, $9.72; April 23, Groceries, $13.96.

Credits. — Jan. 16, Cash, $5; Feb. 15, Cash, $14.50; Mar. 8, Labor, $6.30; Mar. 14, Cash, $17; April 15, Cash, $20; May 3, Labor, $8.75; May 17, Cash, $5.

2. Thomas Hinds & Co.

Debits. — July 12, Mdse., $185.50; July 19, Cash, $210; July 25, Mdse., $119.63; July 30, Mdse., $223.29; Aug. 11, Mdse., $88.72; Aug. 18, Cash, $335; Sept. 10, Mdse., $47.54.

Credits. — July 3, Mdse., $237.45; July 19, Mdse., $149.80; July 28, Cash, $125; Aug. 6, Mdse., $92.24; Aug. 14, Mdse., $422.18; Aug. 29, Mdse., $116.40; Sept. 4, Cash, $25; Sept. 8, Mdse., $173.91; Sept. 24, Mdse., $275.16.

3. Johnson & Mason.

Debits. — May 6, 7 carriages, $875; May 14, 5 double wagons, $450; May 20, 4 runabouts, $210; May 25, 3 surreys, $525; June 1, 3 single wagons, $225; June 8, 1 phaëton, $200; and 6 open buggies, $480; June 15, 2 buckboards, $100; and 2 road carts, $70.

Credits. — Jan. 4, White ash lumber, $449.25; Jan. 9, Hub timber (elm), $375.40; Feb. 4, Spoke timber (hickory), $274.65; April 8, Basswood lumber, $124.80; May 10, Cash, $400; May 18, Cash, $325; June 4, Cash, $560; June 12, Cash, $350.

A **cash account** is similar in form to a ledger account with " Cash." The *receipts* (money received) are recorded as debits on the left and the *expenditures* (money paid out), as credits on the right. .

The following form illustrates a balanced *cash account* :

1914 Sept.					1914 Sept.				
	1	Cash on hand	98	75		1	Rent	25	00
		Cash sales	89	16			Clerk hire	12	00
		H. Swan on Acc't.	29	37			Balance	180	28
			217	28				217	28
Sept.	2	Cash on hand	180	28					

Cash accounts are usually kept in a separate book from other accounts, the left page being used for the debits and the right page for the credits.

A cash account may be balanced at any convenient interval, as each day, or week, or month. The first debit item in the above illustration, "cash on hand," shows the last previous balance.

NOTE. — The "Cash Register" is so generally used by merchants that it has largely done away with the necessity of keeping a cash account.

The "Account Register" has further simplified bookkeeping by taking the place of the daybook. By its use each individual's account is kept on slips (one for each time he makes purchases), the amount being carried from one slip to the next so that the last slip shows at once his total indebtedness. The groups of slips for the several purchasers are arranged alphabetically.

It is suggested that the teacher take the class to visit a store that has such registers and have their use explained.

Written Exercises

Supply dates, arrange, and balance these cash accounts :

1. Albert Fenner's cash account for the month of January.

Receipts. — Caring for horses, 30 days @ 15 ¢ ; cleaning sidewalks, 26 hr. @ 7½ ¢ ; sweeping out store, 36 hr. @ 7½ ¢.

Expenditures. — Shoes, $2.25 ; mittens, 49 ¢ ; tablet and pencil, 15 ¢ ; sweater, $2.89 ; pocketknife, 50 ¢.

2. Grace Williams' cash account for one week.

Receipts. — Allowance, 75 ¢; picking 78 qt. of strawberries @ 1½ ¢; helping a neighbor one day, 75 ¢.

Expenditures. — 2 car fares @ 5 ¢; 1 ice-cream soda, 5 ¢; 2 pencils @ 5 ¢; a book, 65 ¢ ; ½ lb. candy @ 40 ¢.

3. John Allen's farm account Nov. 1 to 15.

Receipts. — Nov. 1, 2 calves @ $7.50; Nov. 8, 12 tons hay @ $18.75; Nov. 12, 20 bu. potatoes @ 62½ ¢; Nov. 15, 3650 lb. milk @ 2½ ¢.

Expenditures. — Nov. 3, baling hay, $18.25 ; Nov. 7, 2 tons mixed feed @ $28.50; Nov. 12, thrashing, $30.75.

4. Cash account of a livery business for week ending June 14.

Receipts. — Cash on hand, $78.75; rent of carriages, $65.50; rec'd. on acc't., $35.75; rent of saddle horses, 48 hr. @ 75 ¢.

Expenditures. — 2 tons hay @ $19.50 ; 50 bu. oats @ 48½ ¢ ; painting carriages, $45.85; barn man, $12.75; driver, $10.25 ; horseshoeing, $7.75; rent of barn, $14.50.

5. The cash business of a grocer for a week.

Receipts. — Cash on hand, $35.60 ; cash sales for the several days, $52.99, $43.86, $51.74, $47.32, $45.28, $68.43 ; rec'd. on acc't., J. Lusk $15.81, F. Lee $20.25, G. C. Cross $40.57.

Expenditures. — Empire Grocery Co., $136.55; clerk hire, $21; rent, $9.50; express, $2.75; cartage, $3.30; Lenox Produce Co., $17.25; vegetables, $84.62.

6. A carpenter's cash account for one month.

Receipts. — G. Black, 48 hr. @ 40 ¢ ; E. Stone, 56½ hr. @ 40 ¢ ; R. Crandall, 39½ hr. @ 40 ¢ ; Imperial Novelty Co., 54¾ hr. @ 40 ¢ ; L. Wilson, 8¼ hr. @ 40 ¢.

Expenditures. — House rent, $18; household expenses, $28; fuel and light, $4.50; personal, $4.25; saw, $2.15; square, $2.25; doctor's bill, $3.50; overalls, 75 ¢; shoes, $4; cap, 50 ¢.

BILLS

A detailed written statement of indebtedness for goods sold or for services rendered is a **bill**.

A bill should be dated and should state the names of the parties concerned (debtor and creditor); the terms of credit; the name, quantity, and price of a unit of each item; the whole value of each item, or **extension**; and the entire amount, or **footing**.

When a bill is paid, the creditor writes "Received Payment," or "Paid" near the bottom and signs his name. The bill is then **receipted**.

The creditor may authorize some person to act as his agent in receipting bills. In this case the agent signs the creditor's name and below that his own surname or initial preceded by "Per" or "By."

The following form illustrates a bill receipted by an authorized agent of the creditor.

Buffalo, N.Y. *April 1 1914*

M. *Chas R Davis 36 West Main St.*

Bought of **HENRY MAXWELL**
214 SWAN STREET
HARDWARE, IRON, TOOLS, IMPLEMENTS, WAGONS, ETC.

Terms: *Cash*

Mar.	15	10 kegs 20-penny nails	2 40	24			131 20
	15	25 kegs 5-penny nails	2 50	62 50			
	21	12 Diston saws	1 25	15			
	21	27 Stanley planes	1 10	29 70			
		Rec'd pay't					
		Henry Maxwell					
		per. B.					

1. Name the debtor in the above bill; the creditor.

2. What are the terms of credit? the extensions? Give the footing.

Make out a bill for each of the following, supplying the dates, names, and place of business of both debtor and creditor when lacking. Find the footing, and receipt in due form :

NOTE. — Extend items to the nearest cent.

1. 18 lb. rice @ 8 ¢
12 lb. soda @ 7 ¢
15 lb. starch @ 5 ¢

2. 12 lb. tea @ 60 ¢
15 lb. figs @ 22 ¢
18 lb. coffee @ 35 ¢

3. 24 shirts @ $1.15
36 neckties @ 75 ¢
72 collars @ 12½ ¢

4. 36 pairs hose @ 25 ¢
34 pairs shoes @ $2.25
57 pairs rubbers @ 60 ¢

5. 34 pineapples @ 18 ¢
60 muskmelons @ 10 ¢
21 doz. lemons @ 30 ¢
82 grapefruits @ 12½ ¢

6. 24 cans peas 14 ¢
18 cans corn @ 12½ ¢
16 cans beans @ 15 ¢
48 cans spinach @ 18 ¢

7. 36 yd. muslin @ 9 ¢
15 yd. denim @ 18 ¢
32 yd. percale @ 13 ¢
45 yd. gingham @ 12 ¢

8. 24 lb. ham @ 20 ¢
36 lb. pork chops @ 22 ¢
48 lb. round steak @ 25 ¢
54 lb. smoked beef @ 35 ¢

9. 27 chisels @ 48 ¢
19 hammers @ 55 ¢
13 jointers @ $3.34
36 rip saws @ $1.60
49 back saws @ $1.56

10. 18 try squares @ 38 ¢
30 jack planes @ $2.17
45 block planes @ 77 ¢
54 screw drivers @ 40 ¢
48 drawing knives @ 95 ¢

11. 75 bu. rye @ 87½ ¢
250 bu. oats @ 42 ¢
120 bu. corn @ 62½ ¢
80 bu. barley @ 60 ¢
50 bu. millet @ $2.50

12. 28 yd. carpet @ 85 ¢
30 rolls paper @ 27 ¢
32 yd. matting @ 35 ¢
4 door mats @ $1.74
25 window shades @ 35 ¢

13. H. H. Moore bought of Lackawanna Coal Co. 1343 tons of coal @ $ 3.40. Terms: cash, 10 %.

14. H. H. Moore sold to the President of Board of Contract and Supply, Albany, N. Y., 1260 tons of coal @ $ 4.25.

15. Mrs. M. Hodge bought of Jones & Company 12 yd. voile @ $ 1.25, 6 yd. batiste @ 75 ¢, 9 yd. dimity @ 25 ¢, and 7 yd. madras @ 85 ¢.

16. J. Borst sold to Dow Bros. 75 lb. butter @ 28 ¢, 120 lb. lard @ 14½ ¢, 25 doz. eggs @ 24 ¢, and 45 bu. potatoes @ 62½ ¢.

17. Hayes & Reynolds sold to Empire Milling Co. 1000 bu. No. 1 hard Manitoba wheat @ 96 ¢, 1200 bu. No. 2 red wheat @ $ 1.02, and 5000 bu. No. 2 corn @ 62½ ¢. Terms : 2/30.

18. George Saxe bought of John Stillman 2 brass bedsteads @ $ 36.50, 2 bureaus @ $ 28, 1 mahogany chiffonier @ $ 75, and 1 golden oak toilet table @ $ 42.50.

19. The Champlain Canning Co. is debtor to Stephen A. Dole, for carting and advancing freight charges, as follows :

Jan. 6, 2 loads @ $ 1.50 ; freight charges $ 8.50.
Jan. 7, 3 loads @ $ 1.50 ; freight charges $ 12.00.
Jan. 8, 1 load @ $ 1.50 ; freight charges $ 7.50.
Jan. 9, 5 loads @ $ 1.50 ; freight charges $ 16.25.

20. Albert Perkins bought of the Continental Hardware Co. 15 Excelsior ranges @ $ 38, 12 Alps ranges @ $ 32.25, 16 Jewel gas ranges @ $ 13.50, 42 cook stoves @ $ 23, 18 Safety oil stoves @ $ 7.75, 2 parlor stoves @ $ 35, 6 parlor stoves @ $ 31.50, 27 double boilers @ $ .89, and 4 doz. razors @ $ 11.50.

21. Allen Bros. bought of Storrs & Stoneman 6 doz. blank books @ $ 1.40, 4 doz. blank books @ $ 2.25, 72 pt. ink @ 35 ¢, 64 steel rulers @ 39 ¢, 15 cameras @ $ 4.95, 3 doz. trays @ $ 1.50, 5 M clips @ $ 1.10, 16 doz. blotting pads @ 49 ¢, and 48 bottles library paste @ 19 ¢.

STATEMENTS

It is customary for some dealers to send each customer at certain intervals, usually the first of each month, a *statement* showing the charges against him and the payments he has made.

The following is a **statement of account**:

1915		-Dr.-			
Jan.	4	Flour	15	40	
	7	Oats	5	70	
	7	Cash	5		
	15	Corn meal	17	50	
	24	Hay	24		
	29	Oats	11	37	51 97
		-Cr.-			
Jan.	4	Cash	10		
	12	Carting	12	50	
	19	"	10		
	24	"	17		49 50
		Balance due			32 47

Auburn, N.Y. Feb. 1, 1915

Mr. George Fox

To **MC LEAN BROTHERS,** Dr.
DEALERS IN FLOUR, GRAIN, AND HAY

Telephone 946-A   275 WENDEL ST.

Observe that this is a statement of the ledger account on page 380.

A statement of account is often used instead of a bill and it may be receipted like a bill.

The above is an itemized statement. If a subsequent statement is sent, it would simply say "account rendered, $32.47."

1. What is the sum, or footing, of the debits in the above statement? the footing of the credits?

2. Is the footing of the debits correctly given? of the credits?

3. How is the balance found? Is it correct?

Make out the monthly statement for each of these accounts, dating it the first of the following month, and find the balance :

1. V. E. Champlin in account with B. S. Sheffield.

Debits. — Balance on account, $14.85 ; May 6, 1 hat, $5.00 ; May 16, 2 umbrellas @ $2.50 ; 1 sweater, $6.50.

Credits. — Returned 1 umbrella, $2.50 ; May 16, cash, $15.00.

2. Clayton Rudiger in account with B. A. Vernon & Co.

Debits. — Mar. 14, 1 bed, $9.25 ; 2 mattresses @ $7.50 ; 1 set bed springs, $4.75 ; Mar. 26, 6 dining chairs @ $2.25.

Credits. — Mar. 26, cash, $24.00.

3. George Robinson in account with Wilson & Sons.

Debits. — Balance of previous account, $15.45 ; April 10, 1 pair shoes, $3.50 ; 7⅜ yd. chiffon @ 75¢ ; 11 yd. satin @ $1.50 ; April 25, 1 pair gloves, $2.50 ; 2 shirts @ $1.25.

Credits. — Returned 7⅜ yd. chiffon @ 75¢ ; April 15, cash, $25.50.

4. Daniel Marsh in account with Charles Miller & Co.

Debits. — Balance from previous account, $10 ; Nov. 8, 1 pair gloves, $1.50 ; Nov. 26, 1 card tray, $1.80 ; 3 books @ 75¢ ; Nov. 27, 2 vases @ $2.45 ; Nov. 28, 1 vest, $5.25.

Credits. — Nov. 20, cash, $5 ; Nov. 29, returned 1 vase, $2.45 ; Nov. 29, cash, $10.

5. Peter Cole in account with Mason Bros.

Debits. — Balance on account, $15.25 ; Sept. 3, table linen set, $6.75 ; 4 doz. buttons @ $.15 ; Sept. 5, 6 collars @ $.12½ ; 2 shirt waists @ $1.98 ; Sept. 14, 1¼ doz. collars @ $1.50 ; Sept. 15, 1 overcoat, $30.

Credits. — Returned 6 collars @ $.12½ ; Sept. 8, cash, $20.

RECEIPTS

Sometimes a **receipt** like the following is given instead of a receipted bill or statement :

```
No. 45        Elmira, N.Y. July 26, 1914
Received from Charles Reid.
Twenty-one and 75/100 ———————— Dollars
For services rendered.
$21.75              Roger Caldwell
```

A receipt should contain·: the date when the money is paid ; the name of the person who pays and of the person who receives the money ; and the amount of money paid, both in figures and in words.

It is desirable to indicate the nature of the debt as illustrated above.

When a receipt is given for the full amount of an account, the words "in full of account" are written instead of "for services rendered" and if for only *part* of the amount, the words "on account" are used.

Written Exercises

Supply necessary data and make out receipts as follows :

1. $20.00, rent
2. $28.75, board·
3. $5.50, class dues

4. $70.16, grain purchased
5. $15.75, professional advice
6. $50.50, professional services

Write a receipt "in full" for :

7. $21.45   8. $30.70   9. $58.60   10. $100.50

Write a receipt "on account" for :

11. $27.20   12. $44.28   13. $65.84   14. $115.18

ACCOUNT SALES

A statement in detail rendered by a commission merchant to his principal showing the goods sold, the charges connected therewith, and the net proceeds of the sale is an **account sales.** The following illustrates an *account sales :*

			Albany, N.Y. *Oct.6,.1914*						
Sold for the Account of									
...*John D. Hammond, Chateaugay, N. Y.*.........									
By **WILLIAM D. BANKS & SON,** **740 BROADWAY.**									
COMMISSION MERCHANTS									
Oct.	*5*	*620 bu. potatoes*	*@ 85*	*527*					
	6	*200 bu. onions*	*@ 95*	*190*			*717*		
		Charges							
		Freight and cartage		*29*	*50*				
		Commission, 5%		*35*	*85*	*65*	*35*		
		Net proceeds remitted				*651*	*65*		

Written Exercises

Supplying necessary data, make out an account sales for :

1. 250 bbl. red apples @ $2.50 ; 175 bbl. green apples @ $2.25 ; sorting, $32.50 ; storage, $24.80 ; commission, 7 %.

2. 80 bbl. flour @ $5.20 ; 75 bbl. flour @ $5.80 ; 60 bbl. flour @ $5.50 ; freight, $120.25 ; commission, 8 ¢ per barrel.

3. 1000 baskets cherries @ 45 ¢ ; 1200 baskets peaches @ 75 ¢ ; transportation charges, $48.50 ; commission, 2½ %.

4. 60 tons hay @ $18.60 ; 40 tons hay @ $20 ; 58 tons hay @ $19.50 ; storage and freight, $40.20 ; commission, 4 %.

5. 680 lb. butter @ 28½ ¢ ; ·1200 lb. cheese @ 15 ¢ ; 840 doz. eggs @ 24 ¢ ; freight and cartage, $57.20 ; commission, 5 %.

6. 10 doz. dining chairs @ $24 ; 15 dining tables @ $22 ; 24 buffets @ $24 ; freight, $63.75 ; commission, 4½ %.

ACCOUNT PURCHASES

A statement in detail rendered by a commission merchant to his principal showing the cost of goods bought, the charges connected therewith, and the amount charged is an **account purchase**, illustrated as follows:

	Bought for the Account of	Syracuse, N.Y. .... *Dec. 12, 1914.*			
	Clark Wilber, 249 East 23d St, New York				
	By **THE W. C. SCOTT COMMISSION CO.,**				
	476 SALINA ST.				
	50 tons timothy hay ● *$17.50*	*875*			
	40 tons rye straw ● *$13.75*	*550*		*1425*	
	Charges				
	Loading		*9*	*50*	
	Commission, 50¢ a ton.		*45*	*00*	*54 50*
	Amount chg'd your acct				*1479 50*

Written Exercises

Supplying necessary data, make out an account purchase for:

1. 120 boxes oranges @ $2.80; 80 boxes grapefruits @ $4.50; freight and refrigeration charges, $140.50 ; commission, 3 %.

2. 200 yd. Brussels carpet @ $1.25 ; 250 yd. ingrain carpet @ 75 ¢ ; freight, $15.40 ; cartage, $8 ; commission, 2 %.

3. 250 bbl. potatoes @ $1.50 ; 420 bbl. cabbages @ $1.25 ; cartage and freight, $40.60 ; storage, $12 ; commission, 5 %.

4. 12,250 ft. pine flooring at $45 per M ; 24,600 ft. oak flooring at $80 per M ; freight, $210.20 ; commission, 2¼ %.

5. 8000 bricks at $7.50 per M ; 16,750 bricks at $6.50 per M ; transportation charges, $100.90 ; commission, 50 ¢ per M.

6. 720 bu. wheat @ 87½ ¢ ; 1250 bu. corn @ 75 ¢ ; 800 bu. rye @ 70 ¢ ; storage, $75.50 ; commission, ½ ¢ per bushel.

Miscellaneous Exercises

1. Make out a receipt for $75.25, supplying necessary data and indicating the nature of the indebtedness.

2. Explain what is meant by a receipt " in full "; a receipt " on account." Illustrate each.

3. Define : account, bill, creditor, balance, account sales.

Make out the following business forms, supplying names and such necessary data as are not given :

4. Bill, receipting it as the creditor.   225 tons hay @ $16.25 ; 165 tons straw @ $14.50; 850 bu. oats @ 38 ¢.   Terms : 2/30.

5. Bill, receipting it as the agent of the creditor.

150 sap buckets @ 20 ¢ ; 4 milk pails @ 60 ¢ ; 50 # nails @ 2½ ¢ ; 5 spools barbed wire @ $2.65 ; 20 # staples @ 4 ¢ ; 25½ gal. paint @ $1.60.

6. Ledger account of James Gardiner against Oliver Shaw.

Debits. — June 3, library table, $18.50 ; rocking-chair, $7.75 ; June 14, 4 jardinieres @ 75 ¢ ; 2½ doz. tumblers @ 60 ¢ ; June 30, cedar chest, $17.75 ; desk, $11.

Credits. — June 3, check, $25.35 ; June 16, returned 2 jardinieres @ 75 ¢ ; June 30, cash, $18.

Balance the account.

7. Monthly statement of the ledger account in exercise **6**.

8. Cash account of B. F. Kent.

Receipts. — Cash on hand, $47.75 ; cash sales, $226.54 ; received on account, H. S. Bloss $19.85, A. L. Clay $16.60, G. N. Potter $12.38.

Expenditures. — Imperial Fruit & Vegetable Co., $156.65 ; Meade Bros., $52.75 ; rent, $22.50 ; labor, $15 ; cartage, $2.35 ; personal, $20.

Balance the account.

STOCKS AND BONDS

Corporations. — A group of persons authorized by law to transact business as an individual is a **corporation**.

The authority conferred upon a corporation is contained in a *charter* or *certificate of incorporation* secured from the state or national government. This defines the powers and limitations of the corporation.

Corporations are growing in number and in importance. One advantage in them is that by the combination of the capital of many individuals they are able to undertake vast enterprises involving the expenditure of sums too large for a single individual or a partnership firm to undertake.

Capital Stock. — When a number of persons unite to form a corporation, or *stock company* as it is often called, it is customary for them to make pledges of money to an amount that shall become the **capital stock** of the corporation.

Organization. — With the capital stock provided for, the members proceed to organize, usually by electing from their number a board of directors to represent them in the management of the business. The directors in turn elect the officers of the corporation, including a president, a secretary, and a treasurer. Application is then made for a charter.

Sometimes the full membership of a corporation vote to elect officers.

Shares. — The stock of a corporation is divided into equal parts, called **shares**.

A share is usually $100, but it may be more or less than that sum. Unless stated to the contrary, the face value of a share, in this book, is $100.

If the capital of a corporation is $400,000, it may be divided into 4000 shares of $100 each.

Stockholders. — A person who is the owner of one or more of the shares of a corporation is a **shareholder**, or a **stockholder.**

Certificates of Stock. — Every stockholder receives a *certificate* showing the number of shares to which he is entitled and the *original*, or **par value**, of each.

Dividends. — When the business has been successful, the profits, after all expenses are paid and a working surplus is laid aside, are divided among the stockholders according to the number of shares each holds. Such a payment to a stockholder is called a **dividend.** The company is said to **declare a dividend.**

A dividend is usually a certain per cent of the par value of the stock.

Thus, if the capital stock is \$100,000 and the net earnings are \$5000, a 5% dividend may be declared, and a man who owns stock to the par value of \$1000 will receive \$50 as his part of the profits.

Dividends are usually declared quarterly, semiannually, or annually.

Assessments. — If the business has been unsuccessful, the dividend may be omitted, or *passed ;* and sometimes the stockholders may be required to make up a deficiency coming from losses by an **assessment.**

Kinds of Stock. — There are two general kinds of stock issued by corporations, namely, **preferred** and **common.**

The holders of preferred stock are generally entitled to a fixed rate of dividend that must be paid before the holders of common stock are entitled to participate in the profits. Preferred stock is therefore more secure than common stock. The common stock, however, often pays a higher rate of dividend than the preferred stock.

Market Value. — The price at which a stock is selling at a particular time is called its **market value.** Stock is said to be

at par, when it is worth its face value in the market ; **above par,** or **at a premium,** when it is worth more than its face value ; and **below par,** or **at a discount,** when it is worth less than its face value.

When the dividends are large enough to pay stockholders good interest on their investment, the stock will usually be at or above par; otherwise below par.

A **corporate bond** is a corporation's formal written promise under seal to pay a certain sum of money to the purchaser, on or before a specified time, with interest at regular intervals at a fixed rate.

Corporate bonds must be secured by a **mortgage,** which is an agreement that the holder of the bond may sell the property of the corporation, if the bond is not paid.

Government Bonds. — Bonds are issued by the United States, or other countries, by states, cities, counties, and other political divisions of a country, to raise money for various purposes of government.

Government bonds are not secured by mortgage.

When bonds are recorded by number and in the name of the person owning them, they are **registered bonds.**

Registered bonds cannot be transferred without indicating the transfer in the records of the bonds.

Bonds to which interest certificates, called **coupons,** are attached, are **coupon bonds.**

Coupon bonds are payable to the bearer. When interest is due, a coupon is cut off and presented for payment at a bank or elsewhere.

Stocks and Bonds Compared. — Stockholders of a corporation are owners of the property ; bondholders are lenders to the corporation. Bonds bear interest at a fixed rate ; the income from stocks depends upon the prosperity of the corporation and the rate of dividend declared.

How Stocks and Bonds are Quoted. — The following quotations were found in a newspaper :

Stocks		Bonds	
Un. Pac.	156¼	U. S. 4s coup.	111
Un. Pac. pf.	84¹	N. Y. City 4½s, 1960	102
Nat'l. Bis.	132¼	Va. Ry. 1st 5s	98¼

These quotations in order mean : Union Pacific common stock, at 156¼ % of its par value; Union Pacific preferred stock, at 84¼ % of par; National Biscuit Company common stock, at 132¼ %; United States coupon bonds, paying 4 % interest, at 111 %; New York City bonds, paying 4½ % interest and due in 1960, at 102 %; and Virginia Railway first mortgage bonds, paying 5 % interest, at 98¼ %.

Bonds are commonly issued for $1000, though denominations of $500 or even $100 are sometimes issued to interest small investors.

Unless otherwise stated, a bond in this book means a bond for $1000.

A person whose business it is to buy and sell stocks and bonds for others is a **stock broker** ; his compensation is **brokerage.**

The customary brokerage, for 100-dollar shares, is ⅛ % of the *par value* for buying and the same for selling.

In the exercises in this book, unless stated to the contrary, stocks and bonds are assumed to be bought and sold through a broker, and *the customary brokerage should be considered.*

Oral Exercises

1. Which of the stocks and bonds quoted at the top of the page are above par ? below par ?

From the quotations, find the market value of a share (or a bond) of :

2. Un. Pac.   **4.** Un. Pac. pf.   **6.** U. S. 4s coup.

3. Nat'l. Bis.   **5.** Va. Ry. 1st 5s   **7.** N. Y. City 4½s

8. What is the brokerage on 1 share of stock ? on 1 bond ?

9. A bank fails and the stockholders are assessed 25 %. How much must I pay, if I own 10 shares ?

10. If on a capital stock of $100,000 there is a net gain of $4000, what per cent of dividend may be declared ?

Cost of a given number of shares at a given market value.

1. Find the cost of 500 shares of Baltimore & Ohio railroad stock at 92⅜.

SOLUTION

Brokerage per share, ⅛% of $100 = $⅛.
Total cost per share, $92⅜ + $⅛ = $92½.
Then, 500 shares cost 500 × $92½, or $46,250.

Find the cost of:

2. 40 shares of Atchison at 98⅞.

3. 25 shares of Atchison pf. at 100⅞.

4. 30 shares of Texas Co. at 146½.

5. 50 shares of Pullman Co. at 155.

6. 70 shares of Montana Power at 52⅝.

7. 60 shares of Montana Power pf. at 103¼.

8. 48 shares of Boston & Albany at 177¼.

9. 56 shares of American Sugar at 109½.

10. 72 shares of American Sugar pf. at 113.

11. 125 shares of Chesapeake & Ohio at 52⅜.

12. 175 shares of Amalgamated Copper at 71⅞.

13. 320 shares of American Beet Sugar at 28¾.

14. 750 shares of United States Rubber at 59½.

15. 875 shares of United States Rubber pf. at 109⅞.

16. 460 shares of Woolworth (F. W.) Co. at 96½.

17. 600 shares of Woolworth (F. W.) Co. pf. at 117.

18. How much must be paid for 1050 shares of Western Union Telegraph at 61¾?

19. If Great Northern preferred is worth 124⅝, how much will 1250 shares cost?

20. A man sold 80 shares of United States Steel preferred at 109⅛. How much did he receive for the stock?

<div align="center">SOLUTION</div>

Brokerage per share, ⅛% of $100 = $⅛.
Sum received per share, $109⅛ − $⅛ = $109.
He received for 80 shares 80 × $109¾, or $8780.

Find the amount received for:

21. 40 shares of Erie 1st pf. at 44½.

22. 36 shares of American Can at 28⅝.

23. 60 shares of American Can pf. at 93¼.

24. 80 shares of General Electric at 148¼.

25. 54 shares of New York Central at 92⅝.

26. 75 shares of American Woolen pf. at 75⅝.

27. 93 shares of Atlantic Coast Line at 123⅝.

28. 120 shares of Central Leather Co. at 86¾.

29. 150 shares of Central Leather Co. pf. at 102½.

30. 180 shares of Brooklyn Rapid Transit at 91¾.

31. 175 shares of Atchison, Topeka & Santa Fe at 99½.

32. 210 shares of Atchison, Topeka & Santa Fe pf. at 100¾.

Find the gain or loss, brokerage for both buying and selling:

	SHARES	BOUGHT AT	SOLD AT		SHARES	BOUGHT AT	SOLD AT
33.	20	71½	75¼	**41.**	112	111	105¼
34.	25	64¾	62½	**42.**	130	97⅜	104½
35.	40	87½	92⅛	**43.**	125	121½	115¼
36.	36	41⅛	56¾	**44.**	140	107⅝	100½
37.	50	102	99½	**45.**	160	101¼	106⅜
38.	64	96¼	89⅜	**46.**	185	124⅞	135⅝
39.	75	98½	112	**47.**	215	114½	103¼
40.	96	114¼	118	**48.**	250	125⅛	137½

49. How much must be paid for 50 shares of Delaware & Hudson R. R. stock at 148⅛?

50. When United States registered bonds, paying 3 % interest, are selling at 101¼, what is the cost of 5 such bonds?

51. How much must be paid for 75 shares of Canadian Pacific at 195⅛?

52. A man sold 100 shares of Wells Fargo Express stock at 101. How much did he receive for them?

53. Mr. Case sold 15 United States Rubber 6s at 102⅞. How much did he receive from the sale?

54. Find the cost of 500 shares of an industrial stock at 127¾, brokerage ¼ %, the par value of a share being $50.

55. How much will it cost to buy 40 Armour and Company 4½s, at 7¼ % discount?

56. A man bought 10 American Wrapping Paper 5 % bonds at 65¼ and sold them at 64¼. How much did he lose?

57. Mr. Newton purchased 150 shares of Pittsburgh Coal pf. at 87½ and sold them the same day at 89⅞. Find his gain.

58. How much must be paid for 25 Indiana Steel 5s, if they are selling at 1⅝ % premium?

59. Find the gain on 25 Argentine 5s bought at 94⅝ and sold at 97½.

60. If a man purchased 350 shares of Norfolk & Western common stock at 104½ and sold them the same day at 106¼, how much did he gain?

61. A speculator bought 500 shares of railroad stock at 67½ and sold them the same day at 68¼, in each case through a certain broker. How much did the broker receive? How much did the speculator gain?

Number of shares or bonds that can be bought for a fixed sum.

Written Exercises

1. When United States Rubber common is selling at $58\frac{3}{8}$, how many shares can be bought for \$4914 ?

SOLUTION

Total cost per share, \$58$\frac{3}{8}$ + \$$\frac{1}{8}$ = \$58.50.
Number of shares, \$4914 ÷ \$58.50 = 84.

Find the number of shares that can be bought :

	INVESTMENT	MARKET PRICE		INVESTMENT	MARKET PRICE
2.	\$ 357	$25\frac{3}{8}$	**13.**	\$ 7350	$91\frac{1}{4}$
3.	\$ 495	$61\frac{3}{4}$	**14.**	\$ 8671	$94\frac{1}{8}$
4.	\$1077	$44\frac{3}{4}$	**15.**	\$10,737	149
5.	\$1228	$38\frac{1}{4}$	**16.**	\$15,852	165
6.	\$1755	$64\frac{7}{8}$	**17.**	\$11,158	$99\frac{1}{2}$
7.	\$2478	$51\frac{1}{2}$	**18.**	\$10,710	$101\frac{7}{8}$
8.	\$3231	$89\frac{5}{8}$	**19.**	\$13,035	$108\frac{1}{2}$
9.	\$5976	$93\frac{1}{4}$	**20.**	\$14,320	$111\frac{3}{4}$
10.	\$3731	$66\frac{1}{2}$	**21.**	\$13,365	$123\frac{5}{8}$
11.	\$7281	101	**22.**	\$20,700	$129\frac{1}{4}$
12.	\$6743	$76\frac{1}{2}$	**23.**	\$19,674	$136\frac{1}{2}$

24. How many bonds, New York City 4s registered, quoted at $98\frac{3}{4}$, can be bought for \$19,775 ?

25. A broker was given \$5000 to invest in American Locomotive preferred stock at $98\frac{1}{4}$. How many shares did he buy ? How much money had he left ?

26. When General Electric stock is selling at $148\frac{1}{2}$, how many shares can be bought for \$12,000 ? How much money will be left ?

27. How many shares of National Lead Co. stock at 107½ can be purchased for $8610 ?

28. How many bonds, New York Telephone 4½s, at 98⅛ can be bought for $10,000, and how much money will be left ?

29. How many shares of Pressed Steel Car stock pf. at 102¾ can be bought for $45,265?

30. How many bonds, Texas Co. 6s, selling at 2⅝ % premium can be bought for $30,825 ?

31. Mr. Haskins received $6210 from the sale of Consolidated Gas stock at 129¼. How many shares did he sell ? How much was the brokerage ?

32. A man received $15,412.50 from the sale of Chicago, Milwaukee, & St. Paul 4½s at 102⅞. How many of these bonds did he sell ?

33. How many shares of Bank of Commerce stock at 70¼ % premium can be bought for $8000, and how much will be left ?

34. When Illinois Steel 4½s are selling at 87¼, how many bonds can be bought for $20,000, and how much will be left ?

35. How many shares of Republic Iron and Steel preferred stock at 11¼ % discount must a man sell to realize $2485?

36. How many bonds, New York City 4½s, selling at 7⅜ % premium, can be bought for $80,625 ?

37. How many shares of Brown Shoe Co. preferred stock at 10¾ % discount must I sell to realize $10,695 ?

38. How many shares of People's Gas, Chicago, stock selling at 21⅛ % premium, can be bought for $15,000, and how much money will be left over ?

39. Mr. Wood received $5915 from the sale of International Harvester Corporation stock, at 5¾ % premium. How many shares did he sell?

Rate of income on an investment in stocks or bonds.

Written Exercises

1. Find the rate of income on an investment in 6 % stock, when the market price is 74⅞.

SOLUTION

Cost per share, $74⅞ + $⅛ = $75.

Income per share, 6 % of $100 = $6.

Rate of income, $6 ÷ $75 = 8 %.

Find the rate of income on the investment, to the nearest .01 % :

RATE OF DIVIDEND	MARKET PRICE		RATE OF DIVIDEND	MARKET PRICE		RATE OF DIVIDEND	MARKET PRICE
2. 3 %	59⅞	**9.** 7 %	75¾		**16.** 8 %	190⅞
3. 5 %	93½	**10.** 2 %	52⅜		**17.** 10 %	178
4. 7 %	90	**11.** 5 %	129½		**18.** 12 %	159⅝
5. 8 %	138	**12.** 8 %	148⅜		**19.** 7 %	114⅜
6. 4 %	61⅜	**13.** 6 %	75⅞		**20.** 20 %	456
7. 7 %	85½	**14.** 7 %	117¼		**21.** 7¼ %	149½
8. 6 %	96½	**15.** 4 %	57		**22.** 15½ %	203¼

23. What per cent shall I make on my money, if I buy 4 % bonds at 79⅞ ?

SUGGESTION. $40 is what per cent of $800?

24. What is the rate of income, to the nearest hundredth, on an investment in N. Y. Telephone 4½ % bonds bought at 98⅛ ?

25. Find, to the nearest hundredth, the rate of income on an investment in National Biscuit Co. stock at 132½, paying a 7 % dividend.

26. If I buy Western Electric 5 % bonds at 101⅝, what per cent, to the nearest hundredth, do I make on my investment?

27. How much better is the rate of income on an investment in 4½ % stock quoted at 56⅛ than in 6 % stock at 119⅛ ?

28. Find, to the nearest hundredth, the rate of income on an investment in U. S. Steel stock at $38\frac{1}{2}\%$ discount, paying a 3% dividend.

29. Which yields the better rate of income on the investment, 7% stock at 35% premium, or 5% stock at 10% discount?

30. How much must I pay for 4% stock in order to realize 5% on my investment? What is the market price?

SUGGESTION. $4 is 5% of the amount that must be paid for a share, including the brokerage.

Find the cost and market price of a share:

RATE OF DIVIDEND	RATE ON INVESTMENT		RATE OF DIVIDEND	RATE ON INVESTMENT		RATE OF DIVIDEND	RATE ON INVESTMENT
31. 6%	5%	**36.**	8%	$6\frac{1}{4}\%$	**41.**	$4\frac{1}{2}\%$	3%
32. 7%	4%	**37.**	6%	4.8%	**42.**	$3\frac{3}{4}\%$	6%
33. 5%	8%	**38.**	9%	6.4%	**43.**	11%	8%
34. 8%	5%	**39.**	7%	5.6%	**44.**	$2\frac{1}{2}\%$	4%
35. 7%	8%	**40.**	6%	7.5%	**45.**	$4\frac{1}{2}\%$	8%

46. How much must be invested in Galveston Street Railway 5% bonds at $91\frac{1}{4}$ to secure to the purchaser an annual income of $1000?

SOLUTION

Since the income from each bond is $50, it will take as many bonds to give an income of $1000 as $50 is contained times in $1000, or 20 bonds.

At $91\frac{1}{4}$, each bond, including brokerage, will cost $916.25, and 20 bonds will cost 20 times $916.25, or $18,325. Therefore, $18,325 must be invested to secure an annual income of $1000.

47. What sum must be invested in American Ice Securities 6s at $88\frac{5}{8}$ to give an annual income of $2400?

48. What amount invested in City of Tokio 5s at $86\frac{5}{8}$ will secure an annual income of $3000?

49. What sum must I invest in American Hide and Leather 6s at $102\frac{1}{2}$ to secure an annual income of $7200?

1. Find the market price of 3 % stock when the money invested in it yields 8 %.

2. A man receives a quarterly dividend of $50 on 25 shares of stock. Find the rate of dividend each quarter.

3. A company with a capital of $300,000 pays its stockholders $6000 quarterly. Find the annual income from 40 shares of this stock.

4. How much must be paid for a sufficient number of U. S. 4 % bonds at 111 to obtain an annual income of $1000 ?

5. What is the rate of income on an investment in N. Y. Air Brake 6 % stock that sells at 67⅜ ?

6. A man bought at private sale 50 shares of a bank stock at 191½ and afterwards sold them through a broker at 193. How much did he gain ?

7. What annual income will a man receive who invests $4835.25 in a 6 % stock purchased at 115 ?

8. I purchased 24 shares of Consolidated Gas stock at 128 and, after receiving a quarterly dividend of 1½%, sold the stock at 131¼. How much did I gain ?

9. A man bought 350 shares of North American stock at 75½. He sold 100 shares of it at 73⅝ and the rest at 77¼. Did he gain or lose and how much ?

10. During one year the net earnings of a company whose capital is $360,000 were $65,835. If it retained a surplus of $5835 and distributed the rest in dividends, how much was received by a man owning 60 shares ?

11. I bought 150 shares of Pullman Co. stock at 156, 60 shares of American Express at 103⅜, and 180 shares of Southern Pacific at 91¾. I sold them at 155¼, 101½, and 94⅛, respectively. Did I gain or lose ? How much ?

INSURANCE

Indemnity against loss or damage is **insurance**.

Indemnity against loss of property by fire is **fire insurance**; against loss of property at sea, **marine insurance**; against loss by personal injuries, **accident insurance**; against loss by sickness, **health insurance**; against loss by death, **life insurance**; against loss by injuries to employees, **liability insurance**. There are many other forms of insurance.

The contract or written agreement between the person protected and the insurance company is the **policy**, and the amount of indemnity, or " protection," the **face of the policy**.

The price paid for insurance is the **premium**.

Insurance rates are expressed in per cent, or as a certain sum per $100 or per $1000.

PROPERTY INSURANCE

The principal kinds of **property insurance** are fire insurance, marine insurance, and such special forms as plate glass insurance, tornado insurance, burglar insurance, credit insurance, and so on.

Written Exercises

Find the premium on each policy :

	FACE	RATE		FACE	RATE PER $100		FACE	RATE PER $100
1.	$1500	2 %	7.	$2500	18 ¢	13.	$2200	$1.15
2.	$3600	3 %	8.	$4000	16 ¢	14.	$3850	$1.30
3.	$4500	1½ %	9.	$5600	12 ¢	15.	$5300	$1.25
4.	$4200	2¼ %	10.	$8300	22 ¢	16.	$4650	$1.50
5.	$5000	4½ %	11.	$7500	19 ¢	17.	$6280	$2.10
6.	$6400	2½ %	12.	$9600	15 ¢	18.	$8700	$2.40

Find the missing term, giving the rate per $100:

	Face	Rate	Premium		Face	Rate	Premium
19.	$2400	30¢	——	**27.**	$10,200	60¢	——
20.	$2850	——	$ 7.98	**28.**	$12,500	——	$ 68.75
21.	——	45¢	$15.75	**29.**	——	65¢	$100.10
22.	$4100	52¢	——	**30.**	$14,800	74¢	——
23.	$5450	——	$19.62	**31.**	$18,500	——	$222.00
24.	——	24¢	$17.28	**32.**	——	$1.25	$208.75
25.	$6350	32¢	——	**33.**	$17,850	$1.40	——
26.	$8400	——	$40.32	**34.**	$20,300	——	$274.05

35. A merchant insured his stock for $50,000 against fire, at $2.25 per $100. Find his yearly premium.

36. A frame house with a shingle roof was insured for $3500 at the 3-year rate of 60¢ per $100. Find the premium.

37. A frame house with a tin roof was insured for $4800 at the 3-year rate of 45¢ per $100. Find the premium.

38. The contents of a frame house were insured for $1750 at the 3-year rate of 50¢ per $100. What was the premium?

39. A new automobile costing $7000 was insured for full value at $1\frac{5}{8}\%$. Find the annual premium.

40. The annual premium on a building insured at 24¢ per $100 is $13.56. For what sum is the building insured?

41. If the premium is $19.50 and the rate is $\frac{1}{2}\%$, what is the face of the policy?

42. If the premium on a policy for $3800 is $15.20, what is the rate in cents per $100?

43. A man had 3 plate glass windows insured for $100 each. If he paid a premium of $21.72, what was the rate per $100?

44. For how much is a merchant's stock insured, if he pays a premium of $210 at the rate of $1.40 per $100?

45. A merchant had goods in storage insured for $1500 against fire, at $1.90 per $100. Find the annual premium.

46. An automobile was insured for 80 % of its value at 1¾ %. What was the value of the automobile, if the premium was $49?

47. A store insured for $5000 in the Phœnix, $3000 in the Firemen's, and $2000 in the Protective, was damaged by fire to the extent of $4000. Find the loss of each company.

48. What is the average annual cost of insurance on a brick dwelling valued at $7500, if it is insured for ¾ of its value for 5 yr. at the 5-year rate of 48¢ per $100 ?

49. A man insured his house for $8000 for 3 yr. at the 3-year rate of 30¢ per $100. How much more would it have cost him to insure it for 3 yr. at the annual rate of 12¢ per $100 ?

50. A merchant insured his stock of silk for $15,000 for fifteen days, and paid 14 % of the annual rate, which was $2 per $100. Find the amount of premium paid.

51. A barge contains 8000 bu. of corn worth 42¢ a bushel. The corn is insured for ⅞ of its value at 85¢ per $100. What will be the owner's net loss, if the cargo is destroyed?

52. A frame house is insured for $4200. If the rate for 5 yr. is only 4 times the annual rate, what is the saving in 10 yr. by insuring in 5-year periods, the annual rate being 18¢ per $100?

53. A jeweler paid $7.10 a year to insure each of 2 plate glass windows, and he insured his stock for $25,000 against burglary at $18 per $1000. Find his yearly premium for both.

54. A factory worth $64,800 was insured for ¾ of its value at 75¢ per $100. After 8 premiums had been paid, the factory was damaged by fire to the extent of $50,000. What was the loss to the owners, including the premiums paid ?

PERSONAL INSURANCE

The principal kinds of insurance of persons, or **personal insurance**, are life insurance, health insurance, accident insurance, and liability insurance.

The most important kinds of life insurance policies are :

1. Ordinary Life Policies. — The holder of an ordinary life policy pays a certain premium at the beginning of each year from the time he secures the policy until his death; at his death the company pays the face of the policy 'to his estate or to the persons named in the policy as his beneficiaries.

2. Limited-payment Life Policies. — These are paid for in a limited number of years, after which they are said to be *paid up*. Thus, a "20-payment life" policy is fully paid up in 20 years.

3. Endowment Policies. — These run for a specified number of years. The face of the policy is paid to the insured if he lives to the end of the term, or to his estate or beneficiaries if he dies before that time. Premiums are usually payable annually, for the whole term.

The average number of years a healthy person of a given age has yet to live is his **expectation of life**.

Life insurance premiums are given in sums per $1000. The rate depends upon the kind of policy and upon the age of the insured.

BRIEF TABLE OF ANNUAL PREMIUMS FOR INSURANCE OF $1000 WITH PARTICIPATION IN PROFITS ANNUALLY

AGE	ORDINARY LIFE	10-PAYMENT LIFE	15-PAYMENT LIFE	20-PAYMENT LIFE	20-YEAR ENDOWMENT	25-YEAR ENDOWMENT	30-YEAR ENDOWMENT	EXPECTATION OF LIFE (YEARS)
20	$ 17.52	$ 41.38	$ 30.94	$ 25.85	$ 46.76	$ 36.41	$ 29.83	42.20
25	19.63	44.97	33.68	28.18	47.28	37.01	30.53	38.81
30	22.35	49.28	36.98	31.00	48.02	37.87	31.57	35.33
35	25.88	54.49	41.00	34.50	49.13	39.22	33.25	31.78
40	30.55	60.78	45.95	38.89	50.89	41.40	35.94	28.18
45	36.8C	68.44	52.16	44.58	53.79	44.99	40.31	24.54
50	45.49	77.93	60.19	52.19	58.68	50.94	47.31	20.91

Written Exercises

Find the annual premium on each policy:

In exercises 1–15, use the rates given in the table on page 408.

	Age	Kind of Policy	Face		Age	Kind of Policy	Face
1.	25	10-Payment Life	$2000	8.	50	Ordinary Life	$ 8000
2.	20	Ordinary Life	$1500	9.	25	10-Payment Life	$ 7000
3.	40	20-Payment Life	$3000	10.	40	20-Year Endowment	$ 9000.
4.	30	15-Payment Life	$2500	11.	20	15-Payment Life	$10,000
5.	35	20-Year Endowment	$4000	12.	35	25-Year Endowment	$12,000
6.	45	30-Year Endowment	$6000	13.	30	20-Payment Life	$20,000
7.	30	25-Year Endowment	$5000	14.	45	30-Year Endowment	$25,000

15. At the age of 35, Mr. Adams took out a 20-year endowment policy for $5000. How much less would it have cost him for the term, if he had taken it out at the age of 20?

16. A man having a 20-year endowment policy for $3000 at $48.02 per $1000 lived the full term. Find the total premium.

17. A man took out an ordinary life policy for $2000 at $30.55 per $1000. If he lived 32 yr., how much more or less than the face of the policy did he pay in premiums?

18. Mr. Ide took out an ordinary life policy for $4500 at $36.86 per $1000. If he died after making 12 payments, how much more did his beneficiary receive than the total premiums?

19. A man obtained insurance for $4000 against accident for nine months and was charged 85% of the annual premium of $7.50 for each $1000. Find the cost of his protection.

20. Mr. Roberts obtained insurance for $1800 against accident. If he paid an annual premium of $13.50, what was the rate per $1000?

21. A man secured a health policy that would pay him $30 per week indemnity in case of illness. For each $5 of weekly indemnity he paid $7 per year. Find the cost of carrying the policy for 12 years.

22. Under the Workmen's Compensation Law an automobile manufacturer insured in the state fund, paying $1.19 per $100 on a total payroll of $195,400. What premium did he pay?

23. To insure his bakers an employer paid the state fund $30.48 on a payroll of $2400. Find the premium per $100.

24. The members of a State League baseball team received a combined salary of $15,600 during a period of six months. On this sum the owners of the club paid to the state fund under the Workmen's Compensation Law a premium of $4.01 per $100 as a protection to the players. Find the amount of the premium.

25. A road-making contractor, in conformance with New York state law, paid a stock company $1566.58 to secure compensation to his men in case of injury. If this premium was based on a total salary payment of $53,650, what was the rate per $100?

26. In compliance with the Workmen's Compensation Law a large company decided to give bonds for security and carry its own insurance. If its drivers and helpers were paid $370,000 annually, and its chauffeurs, $35,000, how much was thus saved in premiums, the state fund rates being, respectively, $2.02 and $2.23 per $100?

27. A workman whose average weekly wages were $16.50 lost one hand by an accident in his work. What compensation did he receive, if the law allowed him $66\frac{2}{3}\%$ of his average weekly wages, for a period of 244 weeks?

28. Injury causes the death of a mill employee. The Workmen's Compensation Law entitles his wife to 30% of his weekly wages of $18 during widowhood and each of his two children to 10% of his wages until 18 yr. of age. What will be their total compensation, if his widow is paid for 35 yr., one child for 12 yr., and one for 9 yr.? (Count a year as 52 weeks.)

GOVERNMENT REVENUE

The expenses of government are paid, for the most part, by
taxes, which are sums of money levied on persons, on property,
or on business; by duties or customs, which are charges on goods
imported from other countries; and by taxes on the incomes of
individuals.

The expenses of our national government consist of outlays for the army
and navy, for pensions, for the salaries of officers, etc. The state expends
money for salaries of state officers, for educational purposes, and for the care
of criminals and defective persons; while local governments need money for
maintaining their various departments, such as Fire Department, Police
Department, Department of Education, etc.

TAXES

Fixed property, such as land and structures built upon it, is
real estate.

Movable property, such as money, bonds, mortgages, cattle,
lumber, etc., is personal property.

State and local taxes are usually a certain *per cent* of the *esti-
mated value* of real estate and of personal property.

In addition to the property tax, there is sometimes a *poll tax* (head tax),
varying from $1 to $3, from each male citizen over 21 years of age.

Besides the taxes specified, there are special taxes for *privilege*, as for
instance a license to sell liquors, to hunt game during a prescribed season of
the year, to own a dog, to run an automobile, etc.

The officers who estimate, or *assess*, the value of the property
subject to taxation are assessors.

Property is not usually assessed at its full value.

411

The officer who collects the taxes, when there is a special officer for the purpose, is a collector. He receives either a salary or a per cent of the tax collected.

The collector's fee is sometimes paid by the person who pays the tax. In such cases it is not included in the amount of tax to be raised.

The rate of taxation is usually expressed as a *number of mills* on each *dollar*, a *certain sum* on each *hundred dollars*, or a *certain per cent* of the assessed valuation.

A tax rate of 1.5 mills means a tax of 1.5 mills on each dollar of the assessed valuation, and is therefore a tax of $1.50 on each hundred dollars, or 1.5% of the assessed valuation.

Written Exercises

1. What is my tax on property assessed at $2000, if the tax rate is 10 mills on a dollar?

Find the tax on the following property:

2. $1500 @ 3 mills
3. $6000 @ 4 mills
4. $7500 @ 2 mills

5. $3600 @ 1¼ mills
6. $10,000 @ 5½ mills
7. $20,000 @ 4¾ mills

Find the tax rate expressed as mills on a dollar:

VALUATION	TAX	VALUATION	TAX	VALUATION	TAX
8. $1500	$4.50	11. $10,000	$40	14. $100,000	$100
9. $2500	$5.00	12. $15,000	$45	15. $250,000	$625
10. $4000	$6.00	13. $75,000	$300	16. $300,000	$900

Find the assessed valuation:

TAX	RATE	TAX	RATE	TAX	RATE
17. $10	1 mill	20. $75	5 mills	23. $5000	5 mills
18. $40	2 mills	21. $600	3 mills	24. $6000	6 mills
19. $60	3 mills	22. $800	4 mills	25. $8000	4 mills

26. A village levied a tax of $3420 on an assessed valuation of $456,000. Find the tax rate; also the tax paid by a citizen whose real property was assessed at $9000 and personal property at $3000.

SOLUTION

Tax rate, $3420 ÷ $456,000 = .0075, or 7¼ mills on a dollar.
Tax on $9000 + $3000, or $12,000, is .0075 of $12,000, or $90.

Find the tax rate:

	REAL ESTATE	PERSONAL PROPERTY	TAX
27.	$450,000	$18,000	$2808
28.	$587,650	$123,350	$5688
29.	$474,300	$121,700	$4321
30.	$648,670	$287,330	$11,700
31.	$834,560	$325,440	$17,690

32. If the tax rate is 2.4%, how much is the tax on property assessed at $75,250?

33. A town whose assessed valuation is $23,462,500 is to raise $206,200 by taxation. It contains 9115 persons subject to a poll tax of $1 each. What will be the rate of property tax?

SUGGESTION. — First deduct the poll tax from the amount to be raised.

34. Mr. Richmond paid a tax of $81 of which $1 was poll tax. If the tax rate was 12½ mills, what was the assessed valuation of his property?

35. If the assessed valuation of a town is $12,346,000, what will be the rate of tax to raise $82,965.12?

36. A man bequeathed $65,000 to his wife, $18,000 to his son, $18,000 to his daughter, and $2000 to his nephew. On these bequests the state of New York levied an inheritance tax varying with the amount and the relationship of the person. What was the total tax, if the rate was 2% for the wife, 1% for the son, 1% for the daughter, and 5% for the nephew?

To facilitate calculation, **tax tables** are often used.

TAX TABLE.   RATE, 1.74 MILLS ON $1

PROP.	TAX	PROP.	TAX	PROP.	TAX	PROP.	TAX
$1	$.002	$10	$.017	$100	$ .174	$1000	$ 1.74
2	.003	20	.035	200	.348	2000	3.48
3	.005	30	.052	300	.522	3000	5.22
4	.007	40	.070	400	.696	4000	6.96
5	.009	50	.087	500	.870	5000	8.70
6	.010	60	.104	600	1.044	6000	10.44
7	.012	70	.122	700	1.218	7000	12.18
8	.014	80	.139	800	1.392	8000	13.92
9	.016	90	.157	900	1.566	9000	15.66

Use the table in solving the exercises on this page.

37. Find Mr. Mark's tax on property assessed at $9750.

SOLUTION

Tax on $9000 = $15.66
Tax on   700 =   1.218
Tax on   50 =   .087
Tax on $9750 = $16.965, or $16.97.

38. Mr. Bell's property is valued at $2504, and his poll tax is $1.   Find his tax.

Find the tax on property valued at:

39. $3500	**42.** $7300	**45.** $43,200	**48.** $50,000
40. $4200	**43.** $8650	**46.** $21,000	**49.** $41,000
41. $5800	**44.** $6475	**47.** $92,400	**50.** $27,950

51. A man's property is assessed at $25,000.   How much tax must he pay, including the collector's fee of 2%?

52. How much tax must Mr. Morton pay, if he has real property assessed at $35,000 and personal property at $15,000, the collector's fee being 1%?

FEDERAL INCOME TAX

The tax on the incomes of individuals levied under United States law is known as the **federal income tax**.

The law requires that every unmarried person shall pay an annual tax of 1% on his or her net income, from whatever source, in excess of $3000, and married persons living together an annual tax of 1% on their joint income in excess of $4000.

When the net income exceeds $20,000 an additional tax, or **surtax**, is collected according to the following rates:

1% on the amount over $20,000 and not exceeding $50,000.
2% on the amount over $50,000 and not exceeding $75,000.
3% on the amount over $75,000 and not exceeding $100,000.
4% on the amount over $100,000 and not exceeding $250,000.
5% on the amount over $250,000 and not exceeding $500,000.
6% on the amount over $500,000.

Written Exercises

1. Find the federal income tax of an unmarried man whose net income is $55,000.

SOLUTION

Since an unmarried man is exempt from paying the tax on $3000 of his income, the taxable income is $55,000 − $3000, or $52,000.

Tax of 1% on $52,000 = $520
Surtax of 1% on $50,000 − $20,000, or on $30,000 = 300
Surtax of 2% on $52,000 − $50,000, or on $2000 = 40
Total federal income tax = $860

Find the income tax of a single man whose net income is:

2. $7400 4. $19,000 6. $65,000 8. $125,000 10. $350,000

3. $8500 5. $48,000 7. $84,500 9. $150,650 11. $480,500

12-21. Find the income tax of a married man whose net income is that given in each of the exercises 2-11.

22. What is the income tax of an unmarried woman who has an income of $4000 from stocks, $1800 from rent, and a salary of $3000?

DUTIES OR CUSTOMS

The income for the support of our national government is derived from **internal revenue**, which consists chiefly of taxes on spirits, tobacco, etc. ; from a **tax** on **incomes** ; and from **duties** or **customs.**

Duties are collected at **customhouses**, which are maintained at *ports of entry.*

Imported goods may be divided into four classes : first, goods admitted free of duty; second, goods subject to an **ad valorem duty**, that is, a certain per cent of the cost of the goods ; third, goods subject to a **specific duty**, a certain amount per yard, pound, etc., without regard to value; fourth, goods subject to both specific and ad valorem duties.

For example, coffee is admitted free ; cotton clothing pays an ad valorem duty of 30 %; the specific duty on hay is $ 2 per ton; and the duty on perfume containing alcohol is 40 ¢ per pound and 60 % ad valorem.

Before computing specific duties, allowance is often made for *tare*, or the weight of the box, bag, etc.; also for *leakage* and *breakage*.

A schedule of duties on merchandise is known as a **tariff.**

Written Exercises

Find the ad valorem duty on merchandise as follows:

	Cost	Rate		Cost	Rate		Cost	Rate
1.	$250	10 %	6.	$3000	20 %	11.	$6214	40 %
2.	$325	15 %	7.	$4375	15 %	12.	$4680	35 %
3.	$429	20 %	8.	$2640	30 %	13.	$7356	50 %
4.	$670	25 %	9.	$1228	40 %	14.	$8521	45 %
5.	$865	35 %	10.	$5250	25 %	15.	$9250	60 %

16. The tariff rate on uncleaned rice is ⅚ ¢ per pound. Find the duty on 7200 lb. of uncleaned rice, deducting 5 % for tare.

Find the duty:

Imports	Value or Quantity	Tariff
17. Almonds	2150 lb.	8 ¢ per lb.
18. Bay rum	840 gal.	$ 1.75 per gal.
19. Blankets	$ 2650	25 %
20. Clocks	$ 4325	30 %
21. Grindstones	325 tons	$ 1.50 per ton
22. Lace	$ 3725	60 %
23. Oats	9420 bu.	6 ¢ per bu.
24. Pineapples	6450	$ 5 per 1000
25. Silk	$ 8250	45 %
26. Straw	3560 tons	50 ¢ per ton
27. Umbrellas	$ 7500	35 %

28. The ad valorem duty on jewelry is 60 %. Find the duty on a bracelet appraised at $ 550.

29. If the tariff rate on barley is 15 ¢ per bushel, what is the duty on 2180 bu. of barley ?

30. What is the duty on 740 lb. of cheese worth $15\frac{1}{4}$ ¢ a pound, if the tariff rate is 20 % ad valorem ?

31. Find the duty on 12,250 hyacinth bulbs, the tariff rate being $ 2.50 per M.

32. What is the duty on 400 boxes of raisins, each box containing 36 lb., if the tariff rate is 2 ¢ a pound ?

33. A merchant imported 80 doz. razors valued at $ 6 a dozen. Transportation cost $ 4.00. If the tariff rate was 55 % ad valorem, how much, to the nearest cent, did each razor cost ?

34. What is the duty on 25,000 cigars, invoiced at $ 62.50 per M, weighing on an average $10\frac{1}{2}$ oz. per box of 50 cigars, if the tariff rate is $ 4.50 per pound and 25 % ad valorem ?

35. Compute the dutiable value and the duty on this invoice:

MANIFEST No. *751.* INVOICED AT *Bristol, Eng., June 21, 1914.*

INWARD FOREIGN ENTRY OF MERCHANDISE

IMPORTED BY *Bates & Holman* IN THE STEAMER *Holloway*

Henry Maxwell, Master. From *Liverpool, Eng.*

Arrived *July 5, '14.*

MARKS	NUM-BERS	PACKAGES AND CONTENTS	QUANTITY	FREE LIST	VALUE	AD VAL-OREM DUTY	SPECIFIC DUTY	TOTAL DUTY
A.A.	1624	8 cases woolen yarn	2400 lb.		£252.10.0	18%	£...	$ ..
⊕	833	12 cases cotton hose	384 dos.		£149.12.0	50%	£....	$ ..
					£		£....	$ ..
					$		$....	$....

36. Find the duty on 600 doz. linen handkerchiefs costing 7s. 6d. per dozen at 40 % ad valorem.

37. A shipment of skein silk costing 2754 francs in Lyons was taxed 35 % ad valorem. How much duty was paid on it?

38. Find the entire cost in this country of 2000 lb. of macaroni bought in Italy for 500 lire, the duty being 1¢ per pound and the freight charges $10.60.

39. Paintings valued at 125,400 francs were imported from Paris. Freight and insurance charges were $84.50. If the tariff rate was 15 % ad valorem, what was the total cost?

40. A merchant imported cut and stained glass invoiced at 12,440 crowns (20 crowns = $4.052). If the tariff rate was 45 % ad valorem, how much duty did he pay?

41. What was the duty on 8250 yd. of English wool suiting, costing 7s. 2¼d. per yard, at 35 % ad valorem?

SIMPLE FORMULAS

A **formula** expresses a principle, a rule, or a law in brief form by means of letters and signs.

Thus, if an automobile goes 20 feet per second, in 25 seconds it will go 25 times 20 feet.

In general, *the space passed over by anything moving with uniform velocity in a given time equals the velocity times the time.*

Briefly,   *space = velocity × time.*

More briefly, with only the initial letters of these terms,

$$s = vt,$$

which is the **algebraic formula** for this **physical law.**

It must be understood that in formulas the letters stand simply for *numerical values,* that is, the *number* of units in the space equals the *number* of units in the velocity times the *number* of units in the time.

Written Exercises

1. Find the space passed óver by a train in 30 sec., its average velocity being 48 ft. per second.

SOLUTION. — Substituting the values of t and v in the formula, we have

$$s = 30 \times 48$$
$$= 1440.$$

Hence, the space passed over is 1440 ft.

Find the space passed over by:

2. A launch in 11 hr., average velocity 8.2 mi. per hr.

3. A torpedo in 75 sec., average velocity 44 ft. per sec.

4. The earth in 1 min., uniform velocity 18.64 mi. per sec.

5. A steamship in 2 da., average velocity 21.17 mi. per hr.

6. An aeroplane in $6\frac{1}{2}$ hr., average velocity 44.46 mi. per hr.

419

MEASUREMENT FORMULAS

PLANE FIGURES

On page 305 you learned that :

The area of a rectangle is equal to the product of its base and altitude (height).

Briefly,   *area = base × height.*

More briefly, $A = bh$,

. which is the **formula** for the **area** of a **rectangle**.

By reference to the principle on page 307, it is seen that the *formula* for the *area* of a *parallelogram* is also $A = bh$.

Written Exercises

Find the area of a rectangle or a parallelogram when :

1. $b = 14$; $h = 6$ 5. $b = 24$; $h = 8\frac{1}{2}$ 9. $b = 12\frac{1}{2}$; $h = 11\frac{1}{5}$

2. $b = 18$; $h = 9$ 6. $b = 36$; $h = 9\frac{3}{4}$ 10. $b = 9.75$; $h = 12\frac{1}{4}$

3. $b = 4.6$; $h = 7$ 7. $b = 5.4$; $h = 3.5$ 11. $b = 32.4$; $h = 9.25$

4. $b = 6.5$; $h = 8$ 8. $b = 8.5$; $h = 6.7$ 12. $b = 58\frac{7}{8}$; $h = 47.76$

Since a formula is an algebraic equation (pages 168–172), it may be solved for any letter in it.

Thus, from $A = bh$, $bh = A$. This divided by h gives $b = \dfrac{A}{h}$, the **formula** for the **base** when the *area* and *altitude* are given.

Also, dividing $bh = A$ by b gives $h = \dfrac{A}{b}$, the **formula** for the **altitude** when the *area* and *base* are given.

Find b when :   Find h when :

13. $A = 168$; $h = 12$ 17. $A = 652\frac{1}{3}$; $b = 25\frac{3}{4}$

14. $A = 432$; $h = 16$ 18. $A = 303.6$; $b = 18.4$

15. $A = 79.9$; $h = 8.5$ 19. $A = 369.46$; $b = 22.75$

16. $A = 1155$; $h = 27\frac{1}{2}$ 20. $A = 11,049$; $b = 105\frac{1}{4}$

Referring to page 308, we have :

The area of a triangle is equal to one half the product of its base and altitude (height).

That is, $\quad\quad\quad area = \frac{1}{2}$ of *base* × *height*.

Or, $\quad\quad\quad\quad\quad A = \frac{1}{2}\,bh,$

which is the **formula** for the **area** of a **triangle**.

Written Exercises

Find the area of a triangle when :

1. $b = 18$; $\quad h = 12$
2. $b = 35$; $\quad h = 26$
3. $b = 7.8$; $\quad h = 5.5$
4. $b = 16\frac{2}{3}$; $\quad h = 13\frac{1}{2}$

5. $b = 24.8$; $\quad h = 18.5$
6. $b = 37\frac{1}{2}$; $\quad h = 21\frac{1}{3}$
7. $b = 58.4$; $\quad h = 47.9$
8. $b = 41.32$; $\quad h = 35.75$

Solving $A = \frac{1}{2}\,bh$ for b, gives $b = \dfrac{2A}{h}$, the **formula** for the **base** of a triangle when the *area* and *altitude* are given.

Similarly, solving for h, gives $h = \dfrac{2A}{b}$, the **formula** for the **altitude** when the *area* and *base* are given.

How do these formulas agree with the principle for either dimension of a triangle given on page 308?

Find b when :

9. $A = 120$; $h = 16$
10. $A = 875$; $h = 25$
11. $A = 8.64$; $h = 4.8$

Find h when:

12. $A = 151.7$; $\quad b = 18\frac{1}{2}$
13. $A = 425\frac{3}{4}$; $\quad b = 32.5$
14. $A = 454.26$; $\quad b = 28\frac{1}{4}$

15. Find the area of a triangular flower bed whose base is 6 ft. and whose altitude is $3\frac{1}{2}$ ft.

16. How many square yards of canvas are needed for a triangular sail whose base is 18 ft. and whose altitude is 27 ft. ?

17. Find, in square inches, the area of a triangular weather flag the base of which is 2 ft. 4 in. and altitude 1 ft. 3 in.

The principle (page 295) that *the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides,* may be stated:

$$H^2 = a^2 + b^2, \qquad (1)$$

in which H stands for the hypotenuse and a and b for the legs. Then, by taking the square root of each member, we have

$H = \sqrt{a^2 + b^2}$, the **formula** for the **hypotenuse.**

Solving (1) for a and for b, we have

$a = \sqrt{H^2 - b^2}$, the **formula** for the **leg a,**

and $b = \sqrt{H^2 - a^2}$, the **formula** for the **leg b.**

Written Exercises

1. Find H when $a = 3$ and $b = 4$.

SOLUTION. — Substituting in the formula $H = \sqrt{a^2 + b^2}$, we have

$$H = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$

Given two of the terms H, a, and b, find the other term :

2. $a = 6$; $b = 8$ **4.** $H = 5$; $b = 3$,   **6.** $H = 13$; $a = 12$

3. $a = 21$; $b = 20$ **5.** $H = 10$; $b = 8$ **7.** $H = 25$; $a = 15$

According to the principle given on page 309 :

The area of a trapezoid is equal to one half the product of its altitude and the sum of its bases.

Then, if we let h stand for the altitude, and a and b for the bases, the **formula** for the area of a **trapezoid** is

$$A = \tfrac{1}{2} h (a + b).$$

Written Exercises

Find the area of a trapezoid when :

1. $a = 12$; $b = 14$; $h = 10$ **5.** $a = 45$; $b = 38$; $h = 36$

2. $a = 18$; $b = 25$; $h = 16$ **6.** $a = 12\frac{1}{2}$; $b = 15\frac{1}{2}$; $h = 13$

3. $a = 6\frac{1}{2}$; $b = 3\frac{1}{2}$; $h = 12$ **7.** $a = 25\frac{1}{4}$; $b = 28\frac{3}{4}$; $h = 20$

4. $a = 7.4$; $b = 8.6$; $h = 55$ **8.** $a = 8.85$; $b = 9.45$; $h = 11$

On page 310 is given the principle that :

The circumference of a circle is equal to π times the diameter, or 2π times the radius.

That is, $C = \pi d$ or $C = 2\pi r$,

which are **formulas** for the **circumference** (C) of a circle in terms of the *radius* (r) and in terms of the *diameter* (d), respectively.

Solve the first formula for d and the second for r, and thus express the formulas for the **diameter** and the **radius**, when the *circumference* is given.

Use $\pi = 3\frac{1}{7}$ unless it is specified to use $\pi = 3.1416$.

Written Exercises

Using $\pi = 3.1416$, find the circumference of a circle when:

1. $d = 5$	**6.** $r = 3$	**11.** $d = 1.5$	**16.** $r = 4.5$
2. $d = 8$	**7.** $r = 7$	**12.** $d = .25$	**17.** $r = 7\frac{1}{2}$
3. $d = 12$	**8.** $r = 9$	**13.** $d = 1\frac{1}{8}$	**18.** $r = 8.5$
4. $d = 16$	**9.** $r = 15$	**14.** $d = 2\frac{3}{4}$	**19.** $r = 12\frac{1}{2}$
5. $d = 25$	**10.** $r = 17$	**15.** $d = 6\frac{3}{8}$	**20.** $r = 9.75$

Find d when: Find r when :

21. $C = 22$	**25.** $C = 4.4$	**29.** $C = 3\frac{2}{3}$	**33.** $C = 121$
22. $C = 33$	**26.** $C = 110$	**30.** $C = 5\frac{1}{2}$	**34.** $C = 143$
23. $C = 2\frac{1}{6}$	**27.** $C = 13\frac{1}{6}$	**31.** $C = 66$	**35.** $C = 17\frac{2}{3}$
24. $C = 7\frac{1}{8}$	**28.** $C = 27.5$	**32.** $C = 8.8$	**36.** $C = 73\frac{1}{8}$

37. The diameter of a circular fountain basin is 14 ft. Find its circumference.

38. If the minute hand of a watch is $\frac{7}{8}$ in. long, what is the distance covered by its point in an hour ?

39. If the circumference of a circular tablecloth is 11 ft., what is the diameter of the tablecloth in yards and inches ?

On page 312 we learned that:

The area of a circle is π times the square of the radius.

That is, $A = \pi r^2$.

Solve this formula for r, and thus express the formula for finding the **radius** when the *area* is given.

Written Exercises

1. Find the area of a circle whose radius is $3\frac{1}{2}$ ft.

SOLUTION. — Substituting $3\frac{1}{2}$ for r and $3\frac{1}{7}$ for π in the formula for the area of a circle, we have

$$A = 3\frac{1}{7} \times 3\frac{1}{2} \times 3\frac{1}{2} = \frac{\overset{11}{\cancel{22}}}{7} \times \frac{\overset{7}{\cancel{7}}}{\underset{2}{\cancel{2}}} \times \frac{7}{2} = 38\frac{1}{2}.$$

Hence, the area is $38\frac{1}{2}$ sq. ft.

Find the area of a circle when:

2. $r = 7$	**5.** $r = 2.1$	**8.** $r = 9\frac{1}{8}$	**11.** $r = 84$
3. $r = 2\frac{1}{8}$	**6.** $r = 35$	**9.** $r = 6.3$	**12.** $r = 9.1$
4. $r = 14$	**7.** $r = 56$	**10.** $r = 7.7$	**13.** $r = 10\frac{1}{2}$

Using $\pi = 3.1416$, find the area of a circle when:

14. $r = 5$	**16.** $r = 2\frac{1}{2}$	**18.** $r = 3.5$	**20.** $r = 20$
15. $r = 8$	**17.** $r = 15$	**19.** $r = 6\frac{1}{2}$	**21.** $r = 25$

22. Find the radius of a circle whose area is 154 sq. ft.

SOLUTION. — Substituting 154 for A and $3\frac{1}{7}$ for π in the formula for the radius of a circle, we have

$$r = \sqrt{\frac{154}{3\frac{1}{7}}} = \sqrt{49} = 7.$$

Hence, the radius is 7 ft.

Find the radius of a circle when:

23. $A = 9\frac{5}{8}$	**25.** $A = 86\frac{5}{8}$	**27.** $A = 346\frac{1}{2}$	**29.** $A = 1386$
24. $A = 38\frac{1}{2}$	**26.** $A = 616$	**28.** $A = 962\frac{1}{2}$	**30.** $A = 2464$

31. What is the area of a circular window 24 in. in diameter?

32. A circular park has an area of 15,400 sq. ft. Find the length of a path from the center to the hedge that bounds it.

SOLIDS

On page 333 we learned that :

The volume of a prism or a cylinder is equal to the product of its altitude (height) and the area of its base.

Then, the **formula** for the **volume** of a **prism** or a **cylinder** is

$$V = bh,$$

in which V stands for volume, b for area of base, and h for altitude.

In the case of the *cylinder*, since the base is a circle, the area of which is πr^2, this may be substituted for b. Express the formula.

Written Exercises

Find the volume of a prism or a cylinder when :

1. $b = 32$; $h = 6$ 4. $b = 84\frac{1}{2}$; $h = 16$

2. $b = 56$; $h = 9$ 5. $b = 116$; $h = 33$

3. $b = 75$; $h = 12$ 6. $b = 95.4$; $h = 25$

Find the volume of a cylinder when :

7. $r = 7$; $h = 9$ 10. $r = 10$; $h = 14$ 13. $r = 4.2$; $h = 18$

8. $r = 5$; $h = 7$ 11. $r = 21$; $h = 26$ 14. $r = 5.5$; $h = 4.9$

9. $r = 3\frac{1}{2}$; $h = 8$ 12. $r = 32$; $h = 35$ 15. $r = 70$; $h = 105$

16. How many cubic yards of earth will be removed in digging a cellar 6 ft. deep, if the area of the bottom is 810 sq. ft. ?

17. Find the volume of a cylindrical tank whose base has a radius of 7 ft. and whose altitude is 15 ft.

18. A sand box has for its base a rectangle 4 ft. long and 18 in. wide and is 6 in. high. Find its volume.

19. The depth of water in a well $3\frac{1}{2}$ ft. in diameter is 32 ft. How many cubic feet of water does the well contain ?

20. Referring to the principle on page 331, write a formula for the lateral surface of a cylinder, in which S shall stand for the lateral surface, r for the radius, and h for the height.

On page 336 the following principle is developed:

The volume of a pyramid or a cone is equal to one third the product of its altitude and the area of its base.

Using the same notation as on the preceding page, this principle may be expressed by the formula,

$$V = \tfrac{1}{3} bh,$$

which is the formula for the volume of a pyramid or a cone.

Substituting πr^2 for b, express the formula for finding the volume of a cone in terms of r and h.

Written Exercises

Find the volume of a pyramid or a cone when:

1. $b = 36$; $h = 6$	**5.** $b = 25\tfrac{1}{2}$; $h = 15$
2. $b = 42$; $h = 9$	**6.** $b = 22.5$; $h = 30$
3. $b = 57$; $h = 8$	**7.** $b = 144$; $h = 48$
4. $b = 74$; $h = 12$	**8.** $b = 272$; $h = 53$

Find the volume of a cone when:

9. $r = 3$; $h = 7$	**13.** $r = 12$; $h = 21$
10. $r = 7$; $h = 9$	**14.** $r = 14$; $h = 24$
11. $r = 6$; $h = 14$	**15.** $r = 10\tfrac{1}{2}$; $h = 32$
12. $r = 3\tfrac{1}{2}$; $h = 15$	**16.** $r = 17.5$; $h = 36$

17. Find the number of cubic feet in a conical spire, the radius of its base being 7 ft. and its altitude 60 ft.

18. Find the volume of the top of a fountain in the form of a pyramid, the base being $2\tfrac{1}{4}$ sq. ft. and the height 3 ft.

19. The receptacle of a vase is in the form of an inverted cone with an altitude of 6 in. and a diameter, at the top, of $3\tfrac{1}{2}$ in. Find its capacity in cubic inches.

From the principles on page 389, with the notation given there, letting S stand for the surface, we have

$$S = 4\,\pi r^2 \text{ and } S = \pi d^2,$$

which are **formulas** for the **surface** of a **sphere**, the first in terms of r and the second in terms of d.

Express the formula for finding the **volume** (V) of a **sphere**, as given in the principle on page 340, in terms of π and r.

Written Exercises

Find the surface of a sphere when :

1. $r = 7$	**5.** $d = 21$	**9.** $r = 56$	**13.** $d = 70$
2. $r = 3\frac{1}{2}$	**6.** $d = 35$	**10.** $r = 49$	**14.** $d = 84$
3. $r = 14$	**7.** $d = 2.8$	**11.** $r = 24\frac{1}{2}$	**15.** $d = 91$
4. $r = 5\frac{1}{4}$	**8.** $d = 4.2$	**12.** $r = 31.5$	**16.** $d = 105$

Find the volume of a sphere when :

17. $r = 14$	**19.** $r = 35$	**21.** $r = 45$	**23.** $r = 12.6$
18. $r = 21$	**20.** $r = 49$	**22.** $r = 63$	**24.** $r = 16.8$

Using $\pi = 3.1416$, find the volume of a sphere when :

25. $r = 5$	**27.** $r = 15$	**29.** $r = 45$	**31.** $r = 25\frac{1}{2}$
26. $r = 9$	**28.** $r = 25$	**30.** $r = 7\frac{1}{2}$	**32.** $r = 32\frac{1}{4}$

33. Find, to the nearest square inch, the surface of a basket ball 10 in. in diameter.

34. A bowl in the form of a hemisphere is 14 in. in diameter. How many cubic inches of water will it hold ?

35. If steel weighs 490 lb. per cubic foot, what is the weight, in pounds and ounces, of a steel ball 9 in. in diameter ?

36. If $\frac{1}{4}$ of the surface of the earth is land, how many square miles of land are there on the earth's surface, taking 4000 mi. as the earth's radius ?

PERCENTAGE FORMULAS

The principle on page 179 may be written

percentage = base × rate.

Or, $p = br$,

the **formula** for the **percentage** when the *base* and *rate* are given.
The solution of this formula for r and for b gives

$$r = \frac{p}{b} \text{ and } b = \frac{p}{r},$$

which are the **formulas** for the **rate** and **base**, respectively.

How does the formula for *rate* agree with the principle on page 183?
the formula for *base* with the principle on page 187?

Written Exercises

Find the percentage when:

1. $b = 32; \ r = 5\%$ 6. $b = 680; \ r = \frac{1}{2}\%$

2. $b = 75; \ r = 8\%$ 7. $b = 927; \ r = 6\%$

3. $b = 140; \ r = 12\%$ 8. $b = 1250; \ r = 40\%$

4. $b = 175; \ r = 20\%$ 9. $b = 2325; \ r = 35\%$

5. $b = 384; \ r = 25\%$ 10. $b = 5718; \ r = 33\frac{1}{3}\%$

Find r when: Find b when:

11. $p = 6.6; \ b = 165$ 16. $p = 18.5; \ r = 10\%$

12. $p = 27.3; \ b = 390$ 17. $p = .825; \ r = 30\%$

13. $p = 323; \ b = 2584$ 18. $p = 77.76; \ r = 18\%$

14. $p = 170.7; \ b = 1138$ 19. $p = 4125; \ r = 50\%$

15. $p = 719.2; \ b = 4495$ 20. $p = 4660; \ r = 16\frac{2}{3}\%$

21. If a boy solved correctly $87\frac{1}{2}\%$ of the 24 exercises in his lesson, how many exercises did he solve correctly?

22. If a man has $12,500, of which $5625 is invested in real estate, what per cent of his money is invested in real estate?

INTEREST FORMULAS

From the principle on page 224 we have the **formula,**

$$i = prt,$$

in which i denotes the **interest** on a *principal* of p dollars on simple interest at $r\%$ for t years.

The solution of this formula for p, for r, and for t gives

$$p = \frac{i}{rt}, \quad r = \frac{i}{pt}, \quad \text{and } t = \frac{i}{pr}.$$

Written Exercises

Find the interest when:

1. $p = \$150$; $r = 4\%$; $t = 2$ 3. $p = \$460$; $r = 3\%$; $t = 1\frac{1}{2}$

2. $p = \$325$; $r = 6\%$; $t = 3$ 4. $p = \$684$; $r = 4\%$; $t = 3\frac{1}{4}$

Find the principal when:

5. $i = \$7.50$; $r = 3\%$; $t = 1$ 7. $i = \$66.56$; $r = 4\%$; $t = 2$

6. $i = \$42.75$; $r = 5\%$; $t = 3$ 8. $i = \$197.50$; $r = 6\%$; $t = 3\frac{1}{3}$

Find the rate when:

9. $i = \$38.50$; $p = \$550$; $t = 2$ 10. $i = \$140$; $p = \$1000$; $t = 3\frac{1}{2}$

Find the time when:

11. $i = \$18$; $p = \$450$; $r = 2\%$ 12. $i = \$81$; $p = \$675$; $r = 5\%$

SUMMARY OF FORMULAS

Translate each formula into the principle for which it stands:

$s = vt$	$C = \pi d$	$V = \pi r^2 h$	$V = \frac{4}{3}\pi r^3$
$A = bh$	$d = \dfrac{C}{\pi}$	$V = \frac{1}{3} bh$	$p = br$
$A = \frac{1}{2} bh$	$r = \dfrac{C}{2\pi}$	$V = \frac{1}{3}\pi r^2 h$	$i = prt$
$H^2 = a^2 + b^2$	$V = bh$	$S = 4\pi r^2$	$p = \dfrac{i}{rt}$

Using the proper formula, solve each problem :

1. Find the interest on $1845 for $2\frac{1}{2}$ yr. at 4 %.

2. If a man walks at an average rate of $\frac{1}{15}$ mi. per minute, how far will he walk in 40 min.?

3. How many square feet of land are there in a rectangular lot that is 75 ft. long and 55 ft. wide?

4. A boy's yearly allowance in college was $750. If he spent $13\frac{1}{3}$ % of this for tuition, how much was his tuition each year?

5. What is the area of the top of a stand, if it is in the form of a triangle whose base is 18 in. and altitude 14 in.?

6. Find the circumference of a circular platform 21 ft. in . diameter.

7. Find the capacity of a cylindrical drinking glass that is 3 in. in diameter and $3\frac{1}{2}$ in. high.

8. Find the number of cubic feet of stone in a stone pyramid whose base is a 6-foot square and whose altitude is 15 ft.

9. How many cubic feet of space does a conical tent contain whose base has a diameter of 7 ft. and whose altitude is 9 ft.?

10. A field in the form of a rectangle is 40 ft. long and 30 ft. wide. Find the length of the line dividing it into two equal triangles.

11. Find the number of square yards of canvas in a sail in the form of a trapezoid whose bases are 12 ft. and 18 ft., respectively, and whose altitude is 15 ft.

12. If the radius of the planet Mercury is approximately 1500 mi., what is the approximate number of square miles on the surface of that planet?

GENERAL REVIEW

The exercises given in this review are selected from examinations given by the New York State Education Department, based on the syllabus of 1910.

Ten minutes allowed for this question.

Twelve minutes allowed for this question.

1.

2.

(*a*) Add :

6 2 1 9 5
9 5 4 6 2
4 8 7 3 3
7 4 3 7 9
5 7 6 4 0
2 9 9 2 7
3 6 5 5 6
6 9 8 2 8
8 7 6 9 5
6 8 7 3 9
2 1 0 7 6
9 0 9 1 3
7 4 3 5 7
5 6 2 6 4
6 3 5 8 1

(*b*) Subtract :

3 5 0 7 5
1 5 1 6 9

(*a*) Add :

2 9 7 6 4
8 3 2 1 5
6 4 2 9 1
3 7 8 4 9
8 2 1 5 6
7 4 6 3 9
8 4 6 3 7
6 9 1 8 2
3 5 7 4 9
2 1 8 5 4
7 9 3 2 7
6 8 2 9 8
6 1 5 4 3
2 9 8 7 6

(*b*) Multiply :

6 9 8
7 5 4

(*c*) Divide :

487)3267283

(*c*) Divide :

679)5798660

3. Find the least common multiple of 26, 104, 156, 78.

4. Write in words each of the following : (*a*) 90,909, (*b*) 125.00036, (*c*) $149\frac{24}{15}$, (*d*) 500,000,000, (*e*) XCIV.

5. Solve (*a*) $6\frac{2}{3} + 9\frac{1}{8} + 17\frac{5}{12}$ (*c*) $6\frac{3}{4} \times 4\frac{1}{2}$

     (*b*) $19\frac{1}{3} - 8\frac{5}{6}$ (*d*) $5\frac{1}{3} \div 2\frac{3}{4}$

6. Write in words (*a*) $\frac{18}{25}$, (*b*) 153926, (*c*) 201.293. Subtract $\frac{1}{10}$ from .1.   Multiply $\frac{3}{10}$ by .3.

7. What fractional part of a dollar is each of the following: $12\frac{1}{2}\cancel{c}$, $16\frac{2}{3}\cancel{c}$, $5\cancel{c}$, $33\frac{1}{3}\cancel{c}$, $75\cancel{c}$, $10\cancel{c}$, $37\frac{1}{2}\cancel{c}$, $20\cancel{c}$, $25\cancel{c}$, $66\frac{2}{3}\cancel{c}$?

8. A family uses $2\frac{1}{4}$ lb. of butter a week for a month of 30 days.   At $35\cancel{c}$ a pound, how much will the butter cost?

9. A merchant bought potatoes at $1 a bushel and sold them at $30\cancel{c}$ a peck.   How much did he gain on $48\frac{1}{2}$ bu.?

10. A farmer raises 27.5 acres of sugar beets that average 9.81 tons to the acre.   If he sells his beets for $4.40 a ton, how much money should he receive?

11. At $32 per ton, find the cost of fertilizer for 600 young peach trees, allowing $1\frac{1}{4}$ lb. to a tree.

12. Write the following tables of measures: linear, liquid, avoirdupois, dry.

13. I handed the grocer $5 to pay for 4 lb. tea at $62\frac{1}{2}\cancel{c}$ a pound and $5\frac{1}{4}$ lb. crackers at $8\cancel{c}$ a pound.   What change should I receive?

14. If .125 of an acre of land is worth 15.87\frac{1}{2}$, how much are 25.43 acres worth?

15. A baseball team won 30 games and lost 50 during the season.   What per cent of the games played did the team win?

16. A merchant has three pieces of silk containing $13\frac{1}{4}$ yd., $21\frac{1}{4}$ yd., and $16\frac{1}{2}$ yd., respectively.   At $80\cancel{c}$ a yard, what must be paid for these three pieces?

17. The Empire State Express leaves Buffalo at 1 P.M. and arrives in Albany at 6.57 P.M.   The distance is 296.53 miles. What is the average number of miles per hour made by the train?

18. Reduce the decimal fractions .875, .3125, and .06¼ to common fractions in their lowest terms.

19. Find the total cost of these loads of coal at $6.25 per ton: 3849 lb., 2935 lb., 4832 lb., 2364 lb., 2837 lb., 4378 lb., 1352 lb.

20. (a) Express in figures, one million one thousand fifty-six and four hundred twenty-five hundred-thousandths.

(b) Express 1911 in Roman notation.

21. The standing of a seventh grade pupil in arithmetic was as follows: Sept. 89 %, Oct. 85%, Nov. 91 %, Dec. 85%, Jan. 96 %, Feb. 87 %, Mar. 89 %, April 97 %, May 96 %, June 88%. What was his average standing for the year?

22. Extract the square root of 44,521.

23. Make out a bill for the following articles which you have sold to-day to E. C. Gray : 4 lb. sugar @ 5½ ¢; 1½ lb. meat @ 18¢; 2¾ lb. butter @ 32 ¢; 1½ doz. eggs @ 38 ¢.

24. A family purchases 47⅝ lb. of meat per month at an average price of $.167 per pound. If other foods are substituted for the meat at an average cost of $5.23 per month, how much money will be saved thereby in a year?

25. The distance around a rectangular field is 192 rd. If the field is 56 rd. long, how many acres does it contain?

26. Charles Jones bought of the George N. Johnson Company an automobile for $4000. He paid $2500 in cash and for the balance gave a promissory note for 3 mo., with interest at 5 %. Write the note, dating it Jan. 15, 1912.

27. How much will it cost to excavate a ditch 180 ft. long, 4 ft. wide, and 6 ft. deep at 69¢ a cubic yard?

28. A farmer sold 200 bu. of potatoes at 75¢ a bushel, 10 doz. eggs at 24¢ a dozen, and 40 lb. of butter at 30¢ a pound. He invested ⅔ of the proceeds in coal and deposited the balance in the bank. How much did he deposit?

29. Define each of the following: improper fraction, commission, discount, square root of a number, radius.

30. (*a*) Express in decimal form: $\frac{1}{2}\%$, $\frac{1}{8}\%$, 50%, $87\frac{3}{4}\%$, 105%.

(*b*) Express as per cents: .98375; 2.50; .0025; 1.00; .625.

31. One side of a square field is 40 rd. long. How many acres are there in this field? How much will it cost to fence it at 70¢ a rod?

32. Find the interest on $450 for 1 yr. 3 mo. 20 da. at 6%.

33. Find the area of a circle whose circumference is 50.2656 ft. (Use $\pi = 3.1416$.)

34. What is the commission on 87 tubs of butter, each containing 50 lb., if sold at 27¢ a pound, commission $3\frac{1}{2}\%$?

35. A grocer bought 625 doz. pound packages of a certain food for $900, less $16\frac{2}{3}\%$ discount. He sold each package for 16¢. How much did he gain in all?

36. The cost of food purchased for twelve hens for the 6 mo. from April 1 to Oct. 1 was $1.48 per month. During that entire period the owner collected on an average 7 eggs a day. These eggs were sold at $24\frac{1}{2}$¢ per dozen. Allowing 30 days to the month, how much money was gained by keeping the hens?

37. Find the compound interest on $420 for 2 yr. at 4%, the interest being compounded semiannually.

38. A street half a mile long and 60 ft. wide is paved with granite blocks. If 36 blocks are required to pave 1 sq. yd., how many will be required to pave the street?

39. A man purchased a house for $5000; the first year the expense for repairs was $45, for taxes $75, for insurance $10. If the house was rented for $40 a month, what per cent did he gain on his investment that year?

40. Change these per cents to equivalent common fractions in their simplest forms, showing all the work : $6\frac{1}{4}$ %, $3\frac{1}{8}$ %, 120 %, $\frac{1}{2}$ %, .2 %.

41. A boy bought apples at the rate of 12 for 3 ¢ and sold them at the rate of 6 for 3 ¢.   How many must he sell to gain $ 6 ?

42. A square lot has an area of 6889 sq. ft.   How many feet long is one side ?

43. A man bought a horse for $ 175 and sold it for $ 210. - What was his per cent of gain in this transaction ?

44. Write a promissory note for $ 900, dated Jan. 1, 1911, payable without interest to John Smith 3 mo. after date.

45. A dealer buys 320 yd. of cloth at $87\frac{1}{2}$ ¢ a yard.   At what price per yard must he sell the cloth to make a profit of $ 40 ?

46. Find the simple interest at 5 % on a note for $ 5375 dated Sept. 15, 1905 and payable June 12, 1911.

47. Soap that a druggist bought at $1.20 a dozen cakes was so damaged that he had to sell it for 8 ¢ a cake.   What was his per cent of loss and what was his loss on 165 doz. ?

48. Richard, James, and Henry formed a partnership to sell fruit.   Richard invested $ 48, James $ 60, and Henry $ 72.   If they gained $ 60 the first week, what was each partner's share of the profit ?

49. A wholesale dealer offers two successive discounts of · 20 % and 5 % on a bill of goods.   What single discount would be equivalent to these two successive discounts ?

50. A dealer bought 300 tons of ice for $ 750.   He sold the ice at the rate of 25 ¢ per hundred pounds.   Find (*a*) how much he gained, (*b*) what per cent he gained.

51. A farmer desired to grind together oats and barley at the ratio of 3 bu. of oats to 2 bu. of barley.   He had 64 bu. of barley. What quantity of oats was required ?   (Solve by proportion.)

52. What is the interest on $825 from May 16, 1910 to Feb. 21, 1912, at $5\frac{1}{2}\%$?

53. A tank measuring 24 ft. by 8 ft. by 10 ft. is half full of water. What is the weight of the water in tons? (A cubic foot of water weighs $62\frac{1}{2}$ lb.)

54. In a certain quantity of milk the ratio of the cream to the rest of the milk is as 2 to 9. How many pounds of cream are there in 253 lb. of the milk?

55. On Jan. 1, 1913, William Jones bought of Henry Abel a bill of goods amounting to $500, for which he paid by giving his note due Mar. 15, 1913, with interest at 5%. Write the note. Compute the interest on the note when due.

56. Compute the cost of fencing a field 58 rd. $12\frac{1}{2}$ ft. long and 39 rd. wide, if the fence cost $3.25 per rod.

57. If 3 men can do a piece of work in $9\frac{1}{3}$ days, how long will it take 10 men to do the same work?

58. A ton of coal requires 36 cu. ft. of space. A coal bin is 9 ft. long and 8 ft. wide. How deep must the bin be to hold 10 tons of coal?

59. Find the cost, at $25 per thousand, of 100 boards, each 18 ft. long, 8 in. wide, and $\frac{1}{2}$ in. thick.

60. I sold a consignment of 2000 bbl. of flour at $5.60 a barrel. I paid $73 for storage and $27 for carting. How much ought I to remit after deducting a commission of $\frac{1}{2}\%$?

61. Make a receipted bill for the following: C. A. Bryant bought of Harris, Smith & Co., Troy, N. Y., 37 bu. of oats @ $.40; 50 bu. of corn @ $.67$\frac{1}{2}$; 76 bu. of wheat @ 1.04\frac{1}{4}$; 75 bu. of rye @ $1.04; 95 bu. of beans @ $4.00; 16 bu. of potatoes @ $.95. (Date the bill to-day.)

62. Define five of the following : factor, prime factor, denominate number, proper fraction, trapezoid, bank discount.

63. Find the amount of a note for $640.50 dated April 9, 1908, and paid June 7, 1909, with interest at 5 %.

64. The triangular gable of a house has a base of 13 ft. and an altitude 10½ ft. Find the area of the triangle.

65. A note for $500, without interest, is payable in 90 days. On the day the note is made it is discounted at a bank at 6 %. Find the proceeds.

66. How many yards of carpet 27 in. wide will be required for a hall 11 ft. 3 in. wide and 64 ft. long, if the strips run lengthwise and there is no waste in matching ? How much will the carpet cost at $1.50 per yard ?

67. At 21 ¢ a square yard, what will be the cost of painting the walls of a kitchen 15 ft. long by 12 ft. wide by 9 ft. 4 in. high ?

68. A merchant buys a bill of goods amounting to $850, with discounts of 20 % and 5 %. Find the net amount of his bill.

69. The assessed valuation of a certain district is $3,250,000. The amount to be raised by tax is $8125. What is the rate of taxation and what is the tax of a man whose property is assessed at $50,000?

70. The inside dimensions of a car are 36 ft. by 8 ft. by 4 ft. Find the cost of a carload of oats, at 31 ¢ per bushel.

71. A commercial traveler receives a salary of $30 per week and in addition a 3% commission on his sales. If his sales amount to $6000 per month, how much is his income per month? (Assume that there are 4 weeks in one month.)

72. At 11 ¢ per square foot, find the cost of laying a concrete walk 5 ft. wide, along the front and along one side of a corner lot 80 ft. long by 60 ft. wide. Represent the lot and the walk by a diagram.

73. A cylindric tank is 7 ft. in diameter and 10 ft. high. How many gallons does it hold? (1 gal. = 231 cu. in.)

74. The shorter side of a rectangular field is 30 rd., and the diagonal is 50 rd. Find in acres the area of the field.

75. Find the bank discount on a note for $900, dated Jan. 1, 1911 and payable in 3 mo. without interest, if discounted Feb. 20, 1911 at 5 %.

76. A bicycle wheel is 35 in. in diameter. How many times will the wheel turn in going 20 rd.?

77. A quart of floor varnish will cover a space of 150 sq. ft. How much will it cost, at $2.80 a gallon, to varnish two floors whose dimensions are one 18 ft. by 15 ft. and the other 15 ft. by 12 ft.?

78. What per cent is realized from an investment in 6 % stock at 119⅞, brokerage ⅛ %?

79. A man bought a house and lot for $6835. He repaired the house at a cost of $1250. The house was burned and he received $3575 insurance. He then sold the lot for $4516. Did he gain or lose and how much?

80. Mr. Brown sent to a commission firm 480 bu. of potatoes and 500 bu. of beans. The potatoes were sold at 72 ¢ a bushel and the beans at 98 ¢ a bushel. The firm charged 2½ % commission. What amount was sent to Mr. Brown?

81. A wire attached to the top of a pole 30 ft. high reaches the ground 40 ft. from the bottom of the pole. How long is the wire, if the pole stands in a level field?

82. A 60-day note for $2450, dated Dec. 10, 1912, with interest at 6 %, was discounted Jan. 9, 1913, at 6 %. Find the discount.

INDUSTRIAL PROBLEMS

Work as many as possible of these problems mentally. In other cases make a mental estimate, before finding the exact result by written work.

HOUSEHOLD ARTS

Household Accounts. The following illustrates a convenient form of account for the home:

WEEK COMMENCING *July 6*, 1914.

	MON.	TUES.	WED.	THURS	FRI.	SAT.	SUN.	TOTALS	DAILY NOTES
Baker	.10		.25	.10	.10	.40			Monday
Books, papers, etc.	.02	.02	.17	.02	.02	.02			
Butcher	.35	.40	.87	.20	.48	1.15			Tuesday
Car fare . . .	.20		1.08		.25		.10		
Clothing		1.25			2.75				Wednesday
Fuel			6.75						
Furnishings . .		1.50		.25					Thursday
Grocer	.50	.18	.81	.47	.60	1.92			
Hired help . . .	.75	.75			.75				Friday
Ice						.60			
Laundry		.24							Saturday
Lighting . . .			4.20						
Sundries		.50	.25	1.05	.45	.20	.45		Sunday
Totals . . .									

NOTE. — From such an account the total daily and weekly expenses are readily obtained; also the total for any class of expenses as those at the baker's, the grocer's, or the butcher's.

In the " Daily Notes " column may be entered any desired details, as quantities, prices, etc., similar to the entries in a Day Book.

1. Find the total expenditure for each day in the week.
2. Find the total for each class of expense for the week.
3. What was the total expenditure for the week ?

4. Mr. Day's expenses for a year as shown by the yearly balance sheet were : rent, $360; food, $536.41; fuel and light, $89.75; house furnishings, $75.50; service, $91; clothing, $220.38; dentist's and doctor's bills, $21.50; sundries, $175.60. If his income was $1800, what was the amount saved?

5. Mrs. White's receipts for a month were $95. Her expenditures were as follows : rent, $25; fuel and lighting, $4.70; groceries, $18.16; meat, $10.73; ice, $1.60; milk and butter, $5.54; laundry service and supplies, $6.95; clothing, $8.25; sundries, $7.20. Supply dates, arrange, and balance the cash account for the month.

Economy in Buying. Find how much less each of these items costs when bought in large quantities than in small quantities :

		PRICE IN LARGE QUANTITIES	PRICE IN SMALL QUANTITIES
6.	10 lb. tea	$7 per 10 lb.	$.80 per lb.
7.	300 lb. sugar	$4.50 per cwt.	$.05 per lb.
8.	2 doz. cans beans	$1.65 per doz.	$.15 per can
9.	3 doz. cans peas	$2.25 per doz.	$.20 per can
10.	60 lb. coffee	$3.25 per 10 lb.	$.37 per lb.
11.	3 gal. maple sirup	$1.25 per gal.	$.25 per pt.
12.	12 bu. potatoes	$.95 per bu.	$.30 per pk.
13.	2 bbl. flour	$6.50 per bbl.	$.85 per ⅛-bbl.

14. How much is saved by buying 1 lb. of baking powder at 35¢ a pound can instead of buying it at 10¢ a ¼-pound can ?

15. Mrs. Deyo bought 12 200-yard spools of thread at 5¢ each. How much would she have saved by buying a 2400-yard spool of the thread at 39¢ ?

16. If soap that costs 5¢ per cake can be bought at $3.75 per box of 100 cakes, how much is saved per 100 cakes by buying it by the box ?

Household Measurements. 17. In making chocolate, Clara used 6 cupfuls of milk. Find the cost of the milk at 6 ¢ per quart. (1 cupful = $\frac{1}{2}$ pt.)

18. How many square yards of oilcloth are required to cover the top of a table 51 in. by 27 in., if 1$\frac{1}{2}$ in. are allowed on each side for turning under?

19. Ruth made some candy for a sale. If she used 8 $\frac{1}{2}$-pint cupfuls of molasses, what part of a gallon did she use?

20. How many pairs of pillowcases can be cut from 10 yd. of tubing, if each case is to be 42 in. long and 3 in. are allowed on each for the seam and hem?

21. Mrs. Wilson made 12 sheets each 92 in. long finished. If 4 in. was allowed per sheet for hems, how many yards of sheeting did she use?

22. One month Mrs. Rudd used 1$\frac{1}{4}$ bu. of potatoes costing 35 ¢ per peck. Find the total cost of the potatoes.

23. If a ton of coal occupies 35 cu. ft. of space, how high must a bin 8 ft. by 3$\frac{1}{2}$ ft. be made to hold 4 tons of coal?

24. Mrs. Bixby made 24 pint cans of strawberry preserves. She used 1 pt. of berries and $\frac{1}{2}$ lb. of sugar per can. Find the total cost when berries are 12 ¢ a quart and sugar 5 ¢ a pound.

25. The floor of Mr. Hale's living room is 18 ft. by 15 ft. How many strips of 27-inch carpet are required to cover it, strips running lengthwise? how many yards of plain carpet?

26. There are 48 scallops on the bottom of a dress flounce that are to be edged evenly with lace. If 3 in. of lace are required for each scallop, how many yards are required for the flounce?

27. Allowing 5 in. for fullness, find, to the nearest tenth of a yard, the amount of lace required to put around a centerpiece 27 in. in diameter.

House Furnishing and Renovating. 28. One spring Mrs. Butler bought an ingrain carpet for a bedroom floor. If the room was 12 ft. square, what was the cost of the carpet required at 80 ¢ a yard, allowing ¾ yd. for matching ?

29. Mrs. Swift bought the following utensils for the kitchen: a food cutter, $1.25; a bread mixer, $2; 2 aluminum kettles @ $2.50; 3 stone crocks @ 75¢. Find the total cost.

30. The walls and ceiling of a laundry 10 ft. by 12 ft. and 8¾ ft. high were calcimined at a cost of 6¢ per square yard. Find the total cost, making no allowance for openings.

31. Make out, foot, and receipt the bill for the following, bought of West Brothers by Mrs. Day : 3 chairs @ $4.75; desk, $14.50; chiffonier, $17.25; 6 pairs curtains @ $2.75; 6 curtain rods @ 30¢.

32. Mrs. Moore's kitchen was 13 ft. by 15 ft. and 8 ft. 9 in. high. Find the cost of painting the walls and ceiling at 15¢ a square yard, allowing 28 sq. ft. for openings.

33. A dining room floor 10 ft. by 15 ft. and a living room floor 14 ft. by 15 ft. were varnished two coats. If 1 gal. of varnish covered 240 sq. ft. two coats, what was the cost of the varnish used at $2.50 per gallon ?

34. The side walls of a living room 15 ft. by 15 ft. were covered with paper costing 60¢ per double roll. If the strips were cut 8¾ ft. long, and 8 strips were allowed for openings, what was the cost of the paper required ?

35. Mrs. W. Lambert bought these furnishings for a bedroom of Ward & Co. on Sept. 12 : rug, $6.75; bed, $21.50; mattress, $10; dresser, $24.25; dressing table, $14.75; 2 chairs @ $4.50; 2 pairs curtains @ $2.25. Mrs. Lambert paid $25 on account the day she bought the furnishings. Make out a statement of account such as Ward & Co. would send Mrs. Lambert on Oct. 1.

Heating and Lighting. The dials of a gas meter and other kinds of meters give readings in the decimal scale.

Each division of dial *a* represents 100 cu. ft. of gas, and each complete revolution of the hand records the passage of 1000 cu. ft. of gas through the meter, as indicated over the top of the dial. With each complete revolution of the hand of *a* the hand of *b* moves from one division to the next, and 10 revolutions of the hand of any dial produce 1 revolution of the hand of the dial next on the left.
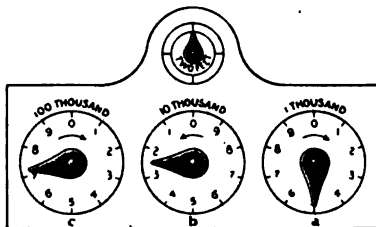
Tell how much gas passes through the meter while the hand of :

36. *a* passes from 0 to 3.

38. *b* passes from 0 to 4.

37. *a* passes from 3 to 5.

39. *c* passes from 0 to 6.

40. Read *c*, *b*, and *a* together. How much gas is the meter now recording ?

41. On June 1, my gas meter recorded 25,600 cu. ft. ; on July 1, 27,400 cu. ft. ; and on Aug. 1, 29,300 cu. ft. How many cubic feet of gas were used in June ? in July ? At $1.10 per 1000 cu. ft., how much was my gas bill for each month ?

SUGGESTION. — Amount of gas used in June = 27,400 cu. ft. − 25,600 cu. ft.

42. If 25,000 cu. ft. of natural gas is equivalent to 1 ton of anthracite coal, which is cheaper, gas at $.30 per 1000 cubic feet or coal at $7.10 per ton ? how much cheaper ?

43. At $.90 per 1000 cu. ft., what is the cost of the gas required to burn 4 gas jets, 3 hr. every day during April, if each gas jet consumes 5 cu. ft. of gas per hour ?

44. Find the cost, at 10 ¢ per 1000 watts per hour (electrical units), of burning 4 electric lights 3 hr. every day during June. Each light consumes approximately 40 watts.

House Maintenance.

A man bought a two-family house for $6000, paying $2000 cash and giving a 5% mortgage for the balance. The assessed valuation of the property was $4200 and the tax rate was $2.20 per $100. The house was insured for $4800 at a 3-year rate of $.37½ per $100. Repairs and upkeep amounted to 1½% of the cost of the property. The water rent was $12 per year. Half of the house was rented at $25 per month.

45. How much would the cash payment of $2000 earn per year, if invested at 4½%?

46. Find the interest on the mortgage for one year.

47. Find the amount of taxes for one year.

48. How much did the insurance cost per year?

49. How much was paid per year for repairs and upkeep?

50. Find the amount received each year in rent.

51. The total expenses per year for maintaining this house were $490.40, and the income from rent was $300. How much did it cost the owner per month to live in one half the house?

52. What per cent of the investment were the taxes, insurance, repairs, and water rent, if they amounted to $200.40?

53. A farmer rented 200 acres of land at $2.25 per acre per year. He also agreed to pay the taxes, which were $1.85 per $100 on a valuation of $4000. Find the total cost of the use of the farm.

54. Mr. Ellis leased a house for $35 per month. The amount of the rent for a year was just 10% of the cost of the property. Find the cost of the property.

55. A man rented a house at $492 per year. After living in it 8 mo. he sublet it for the rest of the year at $35 per month. How much rent did he pay per month for the first 8 mo.? How much per month did he lose during the rest of the year?

MANUAL ARTS

Extend the items in this **bill of lumber** for the **school supply** :

"Thickness" and "Number of Pieces" simply describe the lumber and do not enter into the computation. "Feet" means the total number of board feet.

The thickness is often given in fourths of an inch, $\frac{4}{4}$ meaning 1″ thick, etc.

	THICKNESS	DESCRIPTION	NUMBER OF PIECES	FEET	PRICE PER M	COST
1.	$\frac{4}{4}$	White Oak	40	309	$65	—
2.	$\frac{5}{4}$	White Oak	12	102	68	—
3.	$\frac{8}{4}$	White Oak	13	202	68	—
4.	$\frac{12}{4}$	White Oak	5	97	75	—
5.	$\frac{12}{4}$	Maple	7	161	55	—
6.	$\frac{5}{8}$	White Wood	8	162	65	—
7.	$\frac{4}{4}$	White Wood	18	209	72	—
8.	$\frac{8}{4}$	White Wood	6	119	75	—
9.	$\frac{4}{4}$	Good Pine	30	312	90	—
10.	$\frac{5}{4}$	Good Pine	11	160	90	—
11.	$\frac{8}{4}$	Good Pine	4	115	92	—
12.	$\frac{4}{4}$	No. 2 Pine	30	316	45	—

Over-all Dimensions. From the dimensions on this drawing find in feet :
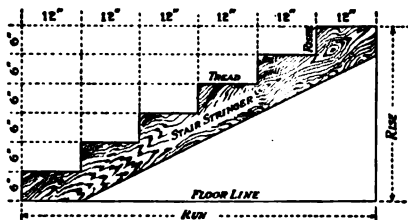
13. The rise of the stairs.

14. The run of the stairs.

15. The length (nearest foot) of the plank required for the stair stringer.

16. Find the rise and run of stairs that have 16 risers, each $7\frac{1}{4}''$, and 16 treads, each $10\frac{3}{4}''$.

17. How many yards of carpet will be needed for a flight of stairs that has 16 risers, each $6\frac{3}{4}''$, and 16 treads, each $11\frac{1}{4}''$?

From the detailed dimensions given on this hall mirror find:

18. The length of each end piece.

19. The length of the top and bottom pieces.

20. How many linear feet of lumber are needed for this mirror, if $1'$ is allowed for joints and waste?
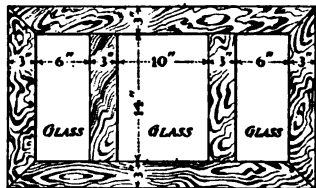
21. This cut shows the end view of a library table. The

top is $1\frac{1}{8}''$ thick, the rail is $4''$ wide, the slats are $14\frac{13}{16}''$ long, the stretcher is $1\frac{1}{2}''$ wide, and the distance from the bottom of the stretcher to the bottom of the leg is $8\frac{13}{16}''$. Find the total height of the table.

22. The top projects $1\frac{1}{16}''$ beyond the legs which are $2\frac{1}{8}''$ square, the slats are $3''$ wide and $1''$ apart, and the distance from the outside slats to the leg is $3\frac{1}{2}''$. Find the length of the rail; the total width of the table.

NOTE. — In schools where there is a shop, the students should add the detailed dimensions on the drawings used in manual training.

Cost of Materials. This chair is made of oak. All joints are mortise and tenon. The cushion is made of leather sewed with leather thongs.

23. From the following estimate, find the total cost of the materials for the chair :

Legs, 13 linear feet, $2\frac{1}{2}'' \times 2\frac{1}{2}''$, @ 8¢.

Rails, 9 linear feet, $3\frac{1}{2}'' \times \frac{7}{8}''$, @ 5¢.

Arms, 7 linear feet, $2\frac{1}{2}'' \times \frac{7}{8}''$, @ 4¢.

Filler, stain, and varnish, 40¢.

Leather for cushion, $8\frac{1}{3}$ sq. ft. @ 24¢; filling for cushion, 25¢.

24. The following is a bill of materials used in the construction of a library table. Find the total cost.

2 pieces quartered oak, $8' \times 9'' \times \frac{7}{8}''$, at 12¢ per board foot.

2 pieces quartered oak, $8' \times 3'' \times \frac{7}{8}''$, at 12¢ per board foot.

1 piece quartered oak, $9' \times 2'' \times 2''$, at 12¢ per board foot.

1 piece quartered oak, $6' \times 2'' \times \frac{1}{2}''$, at 8¢ per board foot.

1 piece white wood, $6' \times 12'' \times \frac{1}{2}''$, at 10¢ per board foot.

Screws, stain, filler, and varnish, 60¢.

Cost Slip. 25. From the given slip, find the total cost of this bench, the lumber costing $50 per M.

In each item, consider any part of a board foot as a whole board foot, and add $\frac{1}{3}$ of the total for waste.

Bench top, $42'' \times 15'' \times 2\frac{1}{2}''$, 1 piece.

Top board, $42'' \times 8'' \times 1''$, 1 piece.

Tool back, $42'' \times 10'' \times 1''$, 1 piece.

Tool rack, $42'' \times 3'' \times 1''$, 1 piece.

Stretcher, $24'' \times 4'' \times 2''$, 3 pieces.

Leg, $24'' \times 4'' \times 2''$, 4 pieces.

Brace, $30'' \times 4'' \times 2''$, 2 pieces.

Sill, $24'' \times 4'' \times 2''$, 2 pieces.
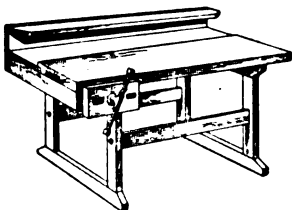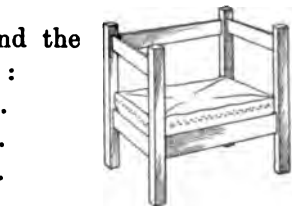
The other cost items were: vise, bench stop, screws, and bolts, $4.75; power and use of shop, 75¢; and 48 hr. of boy's time at $7\frac{1}{2}$¢ per hour.

Geometrical Forms. The drawing represents a board 13″ square from which is to be cut a hexagon, 6″ on a side, for the top of a taboret.
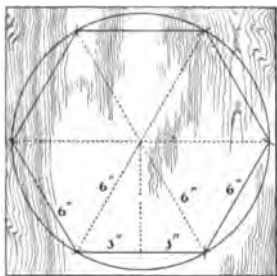
26. Draw a circle having a radius of 6″. Lay off on the circle the radius as shown in the cut. If you have made no mistake you can lay off the radius exactly 6 times.

27. How do you lay out a hexagon having a side of any desired length?

28. Lay out a hexagon whose sides are each 4″.

29. Draw lines connecting the opposite corners of a hexagon as shown above. How many equal angles are formed at the center? Knowing the number of degrees in all the *central angles*, how many degrees does each central angle contain?

30. By referring to the above figure compute, to the nearest square inch, the area of a hexagon whose sides are each 6″.
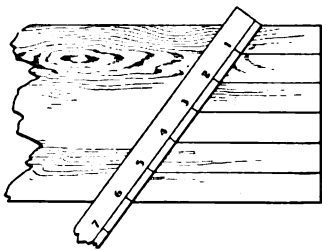
31. Here is a board $4\frac{7}{8}''$ wide which I wish to divide into 6 strips of equal width. I place my ruler on it diagonally as shown in the diagram, so that I measure 6″. Then I draw a line to each inch division parallel to the edge of the board. Measure the distance between these marks and you will find that they are equal. The same method will apply to a board of any width.

32. Divide a board $6\frac{1}{2}''$ wide into 8 strips of equal width.

33. Tell how you would divide a board 9″ wide into 5 strips of equal width.

Lathes. Following is a simple relation for pulleys belted together, in which R. P. M. means the number of *revolutions* made *per minute:*

The R. P. M. of the driving pulley is to the R. P. M. of the driven pulley as the diameter of the driven pulley is to the diameter of the driving pulley.

34. A driving pulley is 8″ in diameter and its speed is 400 R. P. M. What is the diameter of the driven pulley, if its speed is 800 R. P. M. ?

SUGGESTION. $400 : 800 = x : 8.$

35. What is the R. P. M. of a driving pulley 10″ in diameter, if the R. P. M. of the driven pulley is 500 and its diameter is 12″ ?

36. What is the diameter of a driven pulley whose speed is 900 R. P. M., if the speed of the motor which drives it is 2700 R. P. M. and its pulley is 4½″ in diameter?

37. The pulley on a turning lathe is connected with a pulley on a motor. The motor makes 2000 R. P. M. and its pulley is 6″ in diameter. The diameter of the pulley on the lathe is 8″. Required the R. P. M. of the lathe spindle.

SUGGESTION. — In this case the pulley on the motor is the driving pulley and the pulley on the lathe the driven pulley.

38. A turning lathe is driven by a motor with a speed of 1800 R. P. M. The pulley on the motor is 6″ in diameter. What is the diameter of the pulley on the lathe, if the lathe spindle makes 1200 R. P. M. ?

39. The R. P. M. of a lathe is 1200 and the diameter of the object being turned is 7″. How many feet per minute pass the point of the cutting tool ?

SUGGESTION. — Find the circumference of the object in feet and multiply it by 1200.

AGRICULTURE

Farm Measurements. 1. Mr. Black's corn field is 80 rd. long and 60 rd. wide. How many acres does it contain?

2. Lay out a rectangular flower garden 30 ft. long and 18 ft. wide. Find its area in square yards.

3. A concrete feeding floor for cattle was 72 ft. long by 60 ft. wide. Find the cost of paving it at $1.15 per square yard.

4. Two boys planted a piece of ground 132 ft. long by 66 ft. wide to potatoes. The potatoes when dug averaged 180 bu. to the acre. How many bushels did their plot of land yield?

5. Measure some garden and find its area.

6. Measure your school yard and find the cost of sodding it at 25 ¢ per square yard.

7. Measure your cellar at home and find the cost of paving it with concrete at 12½ ¢ per square foot.

8. What part of a 150-foot bale of poultry netting is required to inclose a chicken yard 25 ft. by 12½ ft. ?

9. At $2.10 per 80-rod spool, find the cost of enough wire for a fence 5 wires high around a field 56 rd. square.

10. At 32 ¢ per rod for woven wire and 25 ¢ per rod for posts and labor, find the cost of inclosing a farm that contains 100 acres and is 100 rd. wide, with a woven wire fence.

11. Find the cost, at 2½ ¢ per rod, of enough barbed wire for a fence 5 wires high around a field 64 rd. by 54 rd.

12. A school yard 200 ft. square has a tight board fence 6 ft. high on 3 sides. How many feet of inch boards were used?

13. Compute the cost, at $6.25 per cubic yard, of a retaining wall 27 yd. long, 6 ft. high, and 2 ft. thick.

14. Find the number of perch of stone in a wall 60 rd. long, 4 ft. high, and 2 ft. thick.

Capacity Measurements. 15. Find the depth of a bin, 15 ft. by 12 ft., that will hold 2304 bu. of grain when level full.

16. A vat 8 ft. long and 3 ft. wide contains $2\frac{1}{2}$ ft. of milk. About how many gallons of milk does the vat contain?

17. An underground cylindrical tank for storing gasoline is 7 ft. in diameter and 12 ft. deep. How many gallons does it contain when $\frac{3}{4}$ full?

18. A cylindrical silo 14 ft. in diameter contains 20 ft. of silage. Find the approximate weight of the silage in tons. (1 ton = 50 cu. ft.)

19. Find the number of bushels of corn in a box car 36 ft. long by 8 ft. 6 in. wide when filled to a depth of 5 ft.

20. A watering trough is 8 ft. by 3 ft. by 3 ft. on the inside. About how many gallons of water does it contain, when it lacks but 2 in. of being full?

21. At a cucumber salting station, a tank 10 ft. in depth and 16 ft. in diameter is used. Find its capacity in gallons.

22. A cistern 14 ft. in diameter will hold 23,100 gal. of water when full. Find the depth of the cistern.

23. A concrete swimming pool for ducks was 20 ft. by 10 ft. and $2\frac{1}{2}$ ft. deep, on the inside. How many gallons of water are needed to fill it within 3 in. of the top?

24. How many tons of shelled corn will a bin 15 ft. square and 14 ft. deep hold when full?

25. I wish to build a cylindrical silo 14 ft. in diameter, so that it will hold 77 tons of silage. How high must it be, if 1 cu. ft. of silage weighs approximately 40 lb.?

26. A water tank in the attic of Mr. White's house is 8 ft. square and contains $6\frac{1}{2}$ ft. of water. Find the weight of the water in tons.

Dairy Problems. **27.** A dairyman purchased 14 cows at $60 each and 8 cows at $85 each. Find the total cost.

28. One year the milk from a cow furnished 272 lb. of butter. Find the value of the butter at 25¢ a pound.

29. If 16 lb. of hay are fed each of 5 cows per day, how long will a ton of hay last them?

30. A cow ate 7280 lb. of silage in a year. Find the value of the silage eaten at $2.75 per ton.

31. If the average cow requires 80 lb. of water each day, and $\frac{1}{4}$ of this is in her food, about how many gallons of water does she drink per day?

32. The bedding for each of 20 dairy cows is 5 lb. of straw and 10 lb. of sawdust per day. How many tons of each are used per year, if the cows are stabled the year round?

33. A cow gave milk 327 days in the year. The total amount being 5192 lb., find the average yield per day, to the nearest tenth of a pound.

34. During April a cow gave 740 lb. of milk, testing 5.1 % butter fat. Compute the yield of butter, if it is estimated at $\frac{1}{6}$ more than the amount of butter fat in the milk.

35. One cow yielded 5896 lb. of milk, testing 5.14 % butter fat, in a year; another yielded 7563 lb., testing 3.86 %. Find, to the nearest tenth of a pound, the yield of butter fat from each cow.

36. A milkman received from his cows one day these amounts of milk, in pounds: 32, 21, 18, 25, 29, 19, 12, 11, 13, 10, 25. He sold the milk at 8¢ a quart (2.15 lb.). Find his receipts.

37. Mr. Andrews took 8400 lb. of milk to a factory one month and was credited with 308 lb. of butter fat. What per cent of the milk was butter fat? What was the value of the butter fat at $26\frac{1}{2}$ ¢ per pound?

The following is a brief record for a year of the best and the poorest cow in a herd:

	MILK	BUTTER FAT	COST OF FOOD
Bess	7532 lb.	4.85 %	$53.09
Clover	3433 lb.	5.4 %	$51.96

38. How much more milk did Bess produce than Clover?

39. How much butter fat did Bess produce?

40. How much butter fat did Clover produce?

41. Find, to the nearest cent, the cost per 100 lb. of the milk produced by Bess ; by Clover.

42. Find, to the nearest cent, the cost per pound of the butter fat produced by Bess ; by Clover.

43. If the milk was sold at 3¾¢ a quart (2.15 lb.), how much more than the cost of the food were the returns from the two cows?

44. A dairyman sold 360 lb. of milk at a cheese factory for $1.10 per 100 lb.; 85 % of the weight of the milk was returned to him as whey which he disposed of at $.15 per 100 lb. Find his total receipts, to the nearest cent.

45. The daily ration of green forage for some dairy cows was 110 lb. per 1000 lb. of their weight. What was the amount of forage fed all the cows per day, if they weighed 987 lb., 1037 lb., 1186 lb., 826 lb., and 814 lb., respectively?

46. The composition of a grain mixture was as follows : wheat bran, 400 lb.; gluten feed, 200 lb.; dried distiller's grains, 100 lb.; corn meal, 100 lb. The price per ton of each of these ingredients was $23.50, $31, $31, and $28, respectively. Find the cost of the mixture per 100 lb.

47. On a dairy farm two men are regularly employed at $25 a month each, one at $6 a week, one at $23 a month, and one at $18 a month. Find their total wages for a year.

FILLING SILOS

Number of Farm	Total Quantity of Silage Cut	Area Cut	Number of Days Cutting	Cost per Ton of Silage				
				Labor	Teams	Twine	Fuel	Engine Hire
1	70 tons	9 acres	1.5	$.42	$.18	$.04	$.06	$.10
2	325 tons	19 acres	7	.28	.14	.04	.06	.10
3	810 tons	85 acres	14	.34	.16	.00	.05	.08
4	242 tons	20 acres	5.5	.22	.13	.03	.03	.10
5	630 tons	57 acres	13.2	.26	.15	.03	.03	.09
6	205 tons	21 acres	6	.24	.14	.04	.04	.13

From the table above, find:

48. To the nearest tenth, the yield of silage per acre for each farm.

49. To the nearest tenth, the quantity of silage cut per day on each farm.

50. The total cost per ton of silage on each farm; the average cost per ton on all the farms.

51. The total cost of filling each silo; to the nearest cent, the average cost per acre on each farm.

52. To the nearest tenth of a ton, the average yield of silage per acre.

53. One year a man found that it cost him 79¾ ¢ per ton of silage to fill his silo. The next year he hired fewer men and teams so that no time was lost in waiting to load or unload the corn and the cost was 64½ ¢ a ton. If the man had 296 tons of silage each year, how much did he save the second year?

Business Forms. The following is a *farm inventory.*

PROPERTY	APRIL 1, 1914		
	Number	Rate	Valuation
REAL ESTATE			
Farm	*60 acres*	*$ 83¼*	——
Buildings			*$ 4000*
Fences and drains			*2500* ——
LIVE STOCK			
Cows	*5*	*60.00*	——
Calves	*4*	*12.00*	——
Horses	*2*	*160.00*	——
Horses	*2*	*200.00*	——
Pigs	*4*	*4.00*	——
Hens	*75*	*.60*	—— ——
MACHINERY AND TOOLS			
Sulky plows	*2*	*45.00*	——
Disk harrow	*1*	*27.50*	——
Spring-tooth harrow	*1*	*14.75*	——
Potato planter	*1*	*35.00*	——
Potato hiller	*1*	*8.00*	——
Potato digger	*1*	*35.00*	——
Cultivators	*2*	*6.50*	——
Mowing machine	*1*	*25.00*	——
Hay rake	*1*	*20.00*	——
Spraying outfit	*1*	*50.00*	——
Wagons and small tools			*325.00* ——
FEED AND SUPPLIES			
Seed potatoes	*396 bu.*	*.45*	——
Oats	*60 bu.*	*.48*	——
Hay, clover	*10 tons*	*16.50*	——
Seed wheat	*25 bu.*	*1.00*	——
Other feed			*40.50* ——
Total Investment			——

54. Fill the blanks in the first column under valuation.

55. Find the total investment in each class of property.

56. Find the total investment.

This is an account of one class of live stock, the *dairy herd*.

DATA		ITEM	SUBTOTAL	TOTAL
Inventory, 1913 :	*Live stock — 45 head*		*$2575.00*	
	Equipment		*409.30*	
	Total			———
Inventory, 1914 :	*Live stock — 38 head*		*2395.00*	
	Equipment		*373.60*	
	Total			———
	Difference (decrease)			═══
Receipts :	*Milk and cream*	*$3630.29*		
	Cows and calves sold	*1749.20*		
	Total receipts	═══		———
Expenses :	*Feed*		*2124.70*	
	Labor		*1202.69*	
	Equipment		*6.54*	
	Decrease in Inventory		*215.70*	
	Miscellaneous		*142.38*	
	Total expenses			———
	Difference			═══
Interest at 5 % on average investment			*143.82*	
Building charge			*255.00*	
	Balance or profit			———

57. Fill the blanks and find the balance or profit.

58. Make out a stock account, similar to the one above, for the horses on some farm you know of.

59. James Stone paid Robert Levitt $12 for veterinary services. Make out a receipt in full for James Stone.

60. George Becker's expenses one week were: ¾ ton of coal @ $6; 40 bu. seed potatoes @ 50¢; household expenses, $3.89; personal expenses, $2.25. His receipts were: cash on hand, $20.75; 20 doz. eggs @ 28¢; 2 tons hay @ $16.50; calf, $8. Supply necessary data, make out his cash account and balance it.

This is a sheet from the farm *time book* of Henry Cox.

The time is given in hours and the rate in cents per hour.

Week ending July 13, 1914.

NAMES OF EMPLOYEES	S.	M.	T.	W.	T.	F.	S.	TOTAL TIME	RATE	TOTAL WAGES
James Brown	4	10	11½	6	12	10	5½	——	25	——
Frank Newtown		12	12	8½	11	12	9½	——	15	——
John Stevens	4	6		8½	10	10		——	20	——
Wm. Buell, with team			10	5			10	——	45	——

61. Find the number of hours each man worked that week.

62. How much were the wages of each man for the week?

63. Make out a time sheet, similar to the one above.

64. James Ross borrowed $100 of Henry Case on Aug. 12, 1914, and gave him a non-negotiable note for the amount, payable on demand. Write the note.

65. Mr. Parks sold to Mrs. James on July 7: 2 qt. milk @ 7¢; 1 qt. cream, 40¢; 4 doz. eggs @ 24½¢; 12 qt. raspberries @ 15¢. Make out and receipt the bill.

66. Supply necessary data and make out a statement of Mrs. Rand's account with George Fiske, the debits being 8 lb. butter @ 34¼¢, 6 doz. eggs @ 22½¢, and the credits, cash $3.

67. On June 12, John Cristy bought a cow of J. Smith for $75. In payment he gave a negotiable note due in 4 mo. with interest at 6%. Write the note and find the amount of the note when due.

68. One year Mr. Atwood's cash receipts were: potatoes, $1467.93; wheat, $715; hay, $91; butter and eggs, $90.16; miscellaneous, $335.77. His cash expenditures were: labor, $600; seed, $75; feed, $200; fertilizer, $500; taxes and insurance, $40; spraying material, $110; repairs, $150; miscellaneous, $100. Supply necessary data, make out, and balance the expense account for the year.

Applications of Percentage, Ratio, and Proportion to Farm Problems. **69.** When milk tests 5.35% butter fat, how many pounds of butter fat are there in 480 lb. of milk?

70. A wash for spraying trees contained 96 lb. of lime and 72 lb. of sulphur. Find the ratio of lime to sulphur.

71. Clay loam contains .148% nitrogen, .156% phosphoric acid, and 1.415% potash. How many pounds of each *plant food* are there in 20 tons of clay loam?

72. A bushel of stone lime weighs 90 lb. If it contains 84.6 lb. of pure lime, what per cent of the stone lime is pure lime?

73. A 10% solution of water glass was used to preserve eggs. At 25¢ per quart, find the cost of the water glass required for 15 gal. of the solution.

74. A kerosene emulsion used in winter spraying for scale is composed of 1 part of sour milk to 2 parts of kerosene. What per cent of the emulsion is kerosene?

75. If the weight of the butter fat in a gallon of cream testing 25% fat was 2.1 lb., what was the weight of a gallon of such cream?

76. If sandy loam is .125% nitrogen, .165% phosphoric acid, and 1.965% potash, how many pounds of each are there in 100 tons of the loam?

77. Mr. Dunn mixed 2016 lb. of feed consisting of 1 part cottonseed meal, 2 parts ground corn, 2 parts ground oats, and 4 parts wheat bran. How much of each kind did he use?

78. To spray an orchard a farmer used some Bordeaux mixture, containing 6 lb. of copper sulphate and 4 lb. of lime to 50 gal. of water. This mixture is known as the 6-4-50 formula. Find the amount of each ingredient in 625 gal. of the mixture.

SUGGESTION. — The amount of the mixture is considered to be the same as the quantity of water used.

79. Mr. Jones used a 5-5-50 formula for Bordeaux mixture to spray his apple orchard of 125 trees. He used 6 gal. of the mixture per tree. Find the amount of the mixture required; the amount of each ingredient.

80. If an acre of soil three inches deep weighs 500 tons and is 2.295 % nitrogen, .415 % phosphoric acid, and .395 % potash, how many tons of each does the soil contain?

81. The dry feed of a certain dairy herd is mixed in the following proportions: bran, 50 %; crushed oats, 25 %; corn meal, 25 %. If a cow eats, on the average, 7 lb. of this grain per day, how much of each substance does she consume in a year?

82. A lime-sulphur mixture used to spray an apple orchard for San Jose scale contained 10 lb. of sulphur and 10 lb. of lime to 50 gal. of water. Find the amount of each substance in the 820 gal. of the mixture required.

83. To protect some carrots from the carrot rust fly, ½ pt. of kerosene mixed with 3 gal. of land plaster was sprinkled along the rows. Find the ratio of kerosene to land plaster.

84. The ratio of the copper sulphate to the lime in a Bordeaux mixture used for spraying a peach orchard was 1:8. If 18 lb. of lime were used to 100 gal. of water, how much copper sulphate was used? Find the amount of each substance in 450 gal. of the mixture.

85. A sample of soil from a farm where cauliflower was grown was 5.58 % organic matter, .146 % nitrogen, .144 % phosphoric acid, and .181 % potash. How many pounds of each were there in a ton of the soil?

86. In order to kill the wild mustard in an 8-acre field of oats, it was sprayed with a copper sulphate solution. 8 lb. of copper sulphate dissolved in 50 gal. of water were used per acre. Find the amount of copper sulphate required for the field; its cost at 6 ¢ a pound.

Fertilizers are used for the nitrogen, phosphoric acid, and potash that they contain. Only a small percentage of any fertilizer is used by the plant, the rest being a container of the plant food. A fertilizer labeled 6-8-5 contains 6% of nitrogen, 8% of phosphoric acid, and 5% of potash. In these exercises the ingredients are named in this order.

The following table shows some of the principal sources of the plant foods and the per cent of each available from these sources.

NITROGEN	%	PHOSPHORIC ACID	%	POTASH	%
Nitrate of soda . .	16	Acid phosphate	16	Muriate of potash .	50
Sulphate of ammonia	19	Florida superphosphate .	16	Sulphate of potash	52
Dried blood . . .	12	Boneblack superphosphate	17	Kainit	12½
Tankage	11	Thomas slag	18	Carbonate of potash	18

87. In a ton of 6-8-5 fertilizer, how many pounds of nitrogen are there? of phosphoric acid? of potash?

88. On an acre of celery, 2 tons of a 6-5-10 fertilizer were used. How many pounds of each plant food did it contain?

89. As a source of potash, which is cheaper, muriate of potash at $44 a ton or kainit at $12 a ton? How much cheaper per pound of potash?

90. How many pounds each of nitrate of soda, boneblack superphosphate, and sulphate of potash are required to mix a ton of 4-9-10 fertilizer?

SUGGESTION. 4% of 2000 lb., or 80 lb., of nitrogen are required. From the table we find that nitrate of soda is 16% nitrogen. Hence, 80 ÷ .16, or 500 lb., of nitrate of soda are used.

The total weight of the ingredients used is sometimes a little less than a ton. This difference is supplied by mixing in dry dirt or cinders, called filler.

Compute final results to nearest pound in these exercises.

91. Find the number of pounds each of dried blood, Thomas slag, and muriate of potash required to make a ton of 3-6-12 fertilizer. How much filler is needed?

92. Find the number of pounds each of sulphate of ammonia, Thomas slag, and carbonate of potash required to make 500 lb. of 4-10-2 fertilizer. Find the amount of filler required.

93. As a fertilizer for potatoes, a New York farmer used a 3-8-10 fertilizer, made of nitrate of soda, acid phosphate, and muriate of potash. How much of each did it contain per 100 pounds?

94. A farmer mixed a 2-8-5 fertilizer for his tomatoes, using tankage, acid phosphate, and carbonate of potash. How much of each was required for a ton of fertilizer? how much filler?

95. A ton of fertilizer used for barley contained 300 lb. of tankage, 100 lb. of nitrate of soda, 1400 lb. of acid phosphate, and 200 lb. of muriate of potash. Calculate the analysis of the mixture.

SUGGESTION. — Find how many pounds of each plant food the ingredients of the fertilizer furnish. Each amount of plant food is what per cent of 2000 lb.?

Compute the per cent to the nearest tenth in these exercises.

96. A farmer mixed 50 lb. of nitrate of soda, 50 lb. of sulphate of ammonia, 300 lb. of acid phosphate, and 100 lb. of sulphate of potash as a fertilizer for sugar beets. Calculate the analysis of the fertilizer.

97. A ton of fertilizer used on an asparagus bed contained 500 lb. of nitrate of soda, 1100 lb. of acid phosphate, and 400 lb. of muriate of potash. Calculate the analysis.

98. As a fertilizer for onions, a farmer mixed 150 lb. of nitrate of soda, 250 lb. of dried blood, 400 lb. of acid phosphate, and 200 lb. of muriate of potash. What is the analysis of the mixture?

99. What is the analysis of a fertilizer for timothy hay that contains 200 lb. of nitrate of soda, 100 lb. of Florida superphosphate, and 50 lb. of muriate of potash?

Farm Buildings. Estimated cost of a two-story *farmhouse.*

EXCAVATION AND MASONRY

100. A cellar 38′ × 31′ 6″ by 6′ deep was excavated for a two-story farmhouse. Find the cost at 35¢ per cubic yard.

101. The concrete foundation is 38′ long and 31′ 6″ wide, the walls being 7′ high and 16″ thick. Find the cost of the foundation at $6.50 per cubic yard. (Count corners only once.)

102. What is the cost of the chimney and fireplace for this house, if 2000 bricks are required at a cost of $21 per M laid?

103. There are 450 sq. yd. in this house to be covered with lath and plaster. Find the cost at 30¢ per square yard.

FRAMING LUMBER

At $30 per thousand, find the cost of:

104. 93 joists, 2″ × 10″ × 16′ **108.** 4 rafters, 2″ × 10″ × 24′
105. 58 joists, 2″ × 6″ × 16′ **109.** 50 rafters, 2″ × 6″ × 18′
106. 70 studs, 2″ × 4″ × 18′ **110.** 24 rafters, 2″ × 6″ × 14′
107. 141 studs, 2″ × 4″ × 16′ **111.** 4000 ft. sheathing

EXTERIOR MATERIAL

Find the cost of each item and the total cost of:

112. 1838 board feet siding ($\frac{1}{2}″ × 5\frac{1}{2}″$) at $30 per M.

113. 2350 board feet flooring ($\frac{7}{8}″ × 2\frac{1}{4}″$) at $40 per M.

114. 240 board feet ceiling ($\frac{5}{8}″ × 3\frac{1}{2}″$) at $28.50 per M.

115. 13,500 shingles at $3.80 per M.

116. 180 linear feet cornish material @ 14¢.

117. 90 linear feet corner boards ($\frac{7}{8}″ × 4\frac{1}{2}″$) @ 8¢.

118. 76 linear feet quarter round @ $1\frac{1}{2}$¢.

Columns, rails, lattice, and steps for piazza, $20.

119. Find the cost of 15 windows at $6 each, including frame, sash, weights, and trim; and 6 cellar sash at $.60 each.

120. How much will 15 doors cost at an average of $6 each, including jambs and trim?

121. It will require 810 linear feet of $\frac{7}{8}'' \times 8''$ molded base. Find its cost at 6 ¢ per linear foot.

122. What is the cost of the carpenter work on this house, if 3 men worked 40 days at an average of $3.00 per day?

123. You have found that excavation and masonry for this house cost $569.10; framing lumber, $334.44; exterior material, $256.32; interior trim, doors, and windows, $202.20; and carpenter work, $360. If the painting cost $115 and the hardware, tin work, and plumbing $255, what was the cost of the house?

124. At $8.25 per M, find the cost of enough bricks to build 6 piers for this *corn crib*, each pier being 1' square and 2' high.
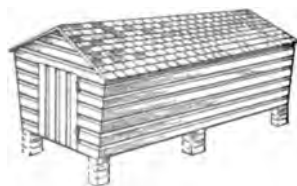
At $28 per M, find, to the nearest cent, the cost of each item of lumber required:

125. Sills and joists, 16 pieces, $2'' \times 8'' \times 16'$.

126. Studs, 30 pieces, $2'' \times 4'' \times 16'$.

127. Rafters, 13 pieces, $2'' \times 6'' \times 14'$.

128. Floor and roof 525 board feet.

129. Sides and ends 3200 board feet.

130. Each span of the roof is $26' \times 7'$. Find the number of bunches of shingles, estimating 1000 shingles per square, required for the roof and their cost at $.95 per bunch.

A builder estimated the cost of this *barn* to be as shown below. Fill the blanks, thus finding the cost of each item to the nearest cent, the total cost of material, the cost of labor, and the total cost of the barn.

131. 26 bbl. cement @ $1.35 ——

132. 22 cu. yd. gravel @ $1.10 ——

133. Studs, 129 pieces, $2'' \times 4'' \times 16'$, at $28.50 per M ——

134. Joists, 72 pieces, $2'' \times 10'' \times 14'$, at $28.50 per M ——

135. Rafters, 62 pieces, $2'' \times 6'' \times 22'$, at $32.00 per M ——

136. Beams, 30 pieces, $2'' \times 6'' \times 8'$, at $26.00 per M ——

137. Posts, 1 piece, $6'' \times 6'' \times 18'$, at $30.00 per M ——

138. Ribbon, 12 pieces, $1'' \times 4'' \times 12'$, at $26.50 per M ——

139. Yellow pine flooring, 1700 ft. at $28.00 per M ——

140. Roof boards, 1600 ft. at $25.00 per M ——

141. Novelty siding, 2540 ft. at $28.50 per M ——

142. Pine, 6 pieces, $1'' \times 5'' \times 16'$, at $40.00 per M ——

143. Shingles, 70 bunches, at $3.75 per M ——

144. 10 4-light sash windows @ $.75 ——

145. 125 lb. 10-penny nails @ 3¢ ——

146. 125 lb. 8-penny nails @ 3¢ ——

147. 100 lb. 20-penny nails @ $2\frac{1}{2}$¢ ——

148. 90 lb. shingle nails @ $3\frac{1}{2}$¢ ——
 Track, rollers, handles, and other hardware, $12.00

149. Paint, $8\frac{1}{2}$ gal. @ $1.25 ——

150. Total cost of material, ——

151. Labor, 54 days, average per day $3.00 ——

152. Total cost of barn, ——

APPENDIX

GREATEST COMMON DIVISOR

When the numbers cannot be factored readily.

Written Exercises

1. Find the greatest common divisor of 221 and 494.

221)494(2
442
—
    52)221(4
    208
    —
      13)52(4
      52
      —

The g. c. d. cannot be greater than the smaller number; therefore, 221 would be the g. c. d., if it were exactly contained in 494; but there is a remainder of 52. Therefore, 221 is *not* the g. c. d. Since 494 and 442, which is 2 times 221, are each divisible by the g. c. d. (p. 72, Ex. 34), their difference, 52, must be divisible by the g. c. d. (p. 72, Ex. 35); therefore, the g. c. d. cannot be greater than 52. 52 would be the g. c. d., if it were exactly contained in 221; for, if it were contained in 221, it would be contained in 442, and in 52 *plus* 442, or 494; but there is a remainder of 13. Therefore, 52 is *not* the g. c. d.

Since 221 and 208, which is 4 times 52, are each divisible by the g. c. d., their difference, 13, must contain the g. c. d.; therefore, the g. c. d. cannot be greater than 13. 13 is the g. c. d., because it is exactly contained in 52; for, since it is contained in itself and 52, it is contained in 208 and in 13 *plus* 208, or 221, also in 494, which is the sum of 52 and 442, the latter being 2 times 221. Hence, **13 is the g. c. d. of 221 and 494.**

To find the greatest common divisor of two numbers:

Divide the greater number by the less, and if there is a re-mainder, divide the less number by it, then the preceding divisor by the last remainder, and so on until there is no remainder. The last divisor will be the greatest common divisor.

NOTE. — If more than two numbers are given, find the greatest common divisor of any two, then of this divisor and another number, and so on.

Find the greatest common divisor of :

2. 286 and 338	**7.** 663 and 1053	**12.** 119, 255, and 357
3. 391 and 460	**8.** 714 and 1683	**13.** 146, 365, and 219
4. 406 and 551	**9.** 836 and 2432	**14.** 430, 602, and 989
5. 496 and 899	**10.** 925 and 5439	**15.** 627, 741, and 817
6. 646 and 714	**11.** 945 and 9114	**16.** 728, 868, and 924

LEAST COMMON MULTIPLE

When the numbers cannot be factored readily.

When the prime factors of the given numbers cannot be discovered by inspection, the factors of the l. c. m. may be found by applying the method of finding the g. c. d. under such circumstances.

Written Exercises

1. Find the least common multiple of 255 and 357.

$$51)\overline{255 \quad 357}$$
$$\overline{5 \quad\;\; 7}$$
$$51 \times 5 \times 7 = 1785$$

By the method on page 465, the g. c. d. of the numbers is found to be 51.

Dividing each of the given numbers by 51, we obtain the quotients 5 and 7, which are prime to each other.

Therefore, 51 × 5 × 7, or 1785, is the l. c. m. of the numbers.

Find the least common multiple of :

2. 282 and 354	**8.** 431 and 573	**14.** 1110 and 777
3. 348 and 638	**9.** 570 and 969	**15.** 1007 and 1855
4. 289 and 425	**10.** 665 and 760	**16.** 2232 and 2352
5. 414 and 529	**11.** 720 and 868	**17.** 3230 and 2550
6. 468 and 923	**12.** 871 and 938	**18.** 4136 and 1504
7. 536 and 737	**13.** 702 and 975	**19.** 5460 and 4725

CUBE ROOT

The cube of an integer or a fraction is a **perfect cube.**

What is the cube root of a number? How may the cube root of a number be found by factoring?

Verify these cube roots by the factoring method :

$$\sqrt[3]{1} = 1 \qquad \sqrt[3]{1000} = 10 \qquad \sqrt[3]{1000000} = 100$$

$$\sqrt[3]{27} = 3 \qquad \sqrt[3]{46656} = 36 \qquad \sqrt[3]{47045881} = 361$$

$$\sqrt[3]{729} = 9 \qquad \sqrt[3]{970299} = 99 \qquad \sqrt[3]{997002999} = 999$$

How many figures are there in the cube root of a perfect cube that is expressed by not more than *three* figures? by *six* figures or by one or two less than six figures? by *nine* figures or by one or two less than nine figures?

The number of figures in the cube root of a perfect cube is the same as the number of periods of three figures each into which the number can be separated, beginning at units.

The left-hand period may contain one, two, or three figures.

Observe this form of the cube of 24, or 20 + 4 :

$$
\begin{array}{ll}
24 = 20 + 4 & \\
24 = 20 + 4 & \\
\hline
480 = 20^2 + 20 \times 4 & \text{(Product by 20)} \\
96 = + 20 \times 4 + 4^2 & \text{(Product by 4)} \\
\hline
576 = 20^2 + 2(20 \times 4) + 4^2 & \\
24 = 20 + 4 & \\
\hline
11520 = 20^3 + 2(20^2 \times 4) + (20 \times 4^2) & \text{(Product by 20)} \\
2304 = + (20^2 \times 4) + 2(20 \times 4^2) + 4^3 & \text{(Product by 4)} \\
\hline
13824 = 20^3 + 3(20^2 \times 4) + 3(20 \times 4^2) + 4^3 &
\end{array}
$$

Hence, if the tens of a number are represented by t and the units by u, the cube of a number consisting of tens and units will be the cube of $(t + u)$, or $t^3 + 3\,t^2u + 3\,tu^2 + u^3$.

Written Exercises

1. What is the cube root of 13,824, or what is the edge of a cube whose volume is 13,824 cubic units?

$$13'824(20 + 4 = 24$$

$$20^3 = \quad 8\ 000$$

$$3 \times 20^2 = 1200 \overline{\smash{)}\ 5\ 824}$$

$$3 \times 4 \times 20 = \quad 240$$

$$4^2 = \quad \underline{16}$$

$$1456 \mid 5\ 824$$

Since the number of figures in the cube root of a number may be determined by separating the number into periods of three figures each, beginning at units, the cube root of 13,824 is seen to be composed of *tens* and *units*.

The tens in the cube root of the number cannot be greater than 2, for the cube of 3 tens is 27,000. 2 tens, or 20, cubed is 8000, which, subtracted from 13,824, leaves 5824; therefore, the root, 20, must be increased by a number such that the additions will exhaust the remainder.
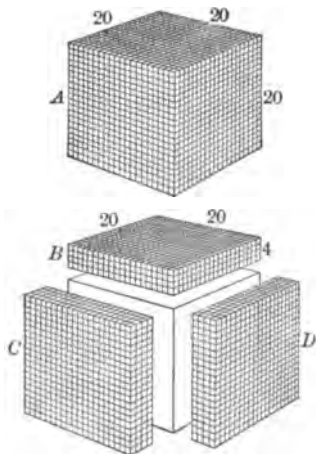
The cube *A* already formed from the 13,824 cubic units is one whose edge is 20 units in length. The additions to be made, keeping the figure formed a perfect cube, are 3 equal rectangular solids, *B*, *C*, and *D*; 3 other equal rectangular solids, *E*, *F*, and *G*; and a small cube, *H*. Inasmuch as the solids *B*, *C*, and *D* comprise much the greatest part of the additions, their volume will be *nearly* 5824 cubic units, the whole volume to be added.

Since the volume of these three equal solids is nearly equal to 5824 cubic units, and the area of a side of each is 20×20, or 400, square units, if we divide 5824 by 3 times 400, or 1200, we shall obtain the approximate thickness of the additions, which is 4 units. 3×20^2, or 1200, then, is a *trial*, or *partial*, *divisor*.

Since all the additions have the same thickness, if this area, 1200 square units, is multiplied by 4, the result will be the volume of the additions *B*, *C*, and *D*.

Each of the solids E, F, and G is 20 units long and 4 units wide; consequently, the area of one side of each is 4 × 20, or 80, square units, and since there are 3 of these solids, 3 × 80, or 240, square units is the area which multiplied by 4 will give the volume of the additions E, F, and G. The area of one side of the cube H is 4 × 4, or 16, square units, which multiplied by 4 gives the volume added by the cube H.

But, in the process, instead of multiplying each of the areas represented by 1200, 240, and 16 separately by 4, it is more convenient to find their sum, 1456, which multiplied by 4 gives 5824, the total number of cubic units in the volume of the additions. This completes the cube and exhausts the remaining cubic units.

Therefore, the edge of the completed cube is 20 + 4, or 24, units in length, or the cube root of 13,824 is 24.

We may further explain the process (rewritten below) by reference to the form, $t^3 + 3\,t^2u + 3\,tu^2 + u^3$, the cube of a number composed of tens and units.

$$13'824(24$$
$$t^3 = 20^3 = \quad 8\ 000$$
$$3\,t^2 = 3 \times 20^2 = 1200 \overline{5\ 824}$$
$$3\,tu = 3 \times (20 \times 4) \;=\; 240$$
$$u^2 = 4 \times 4 \quad = \quad 16$$
$$3\,t^2 + 3\,tu + u^2 = \overline{1456}$$
$$u \times (3\,t^2 + 3\,tu + u^2) = \quad\;\; 5\ 824$$
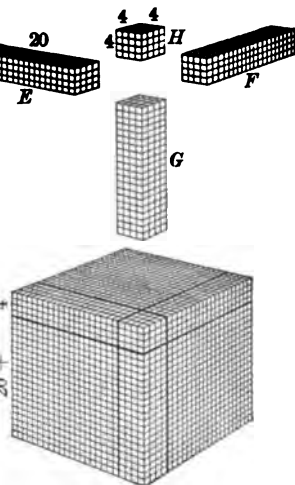
Taking out the cube of the tens ($t^3 = 8000$) there remains 5824 which contains $3\,t^2u + 3\,tu^2 + u^3$, or (3 × the tens^2 × the units) + (3 × the tens × the units^2) + (the units^3). Each of these parts contains the *units* (u) as a factor; hence, 5824 is the product of two factors, one of which is the units (u) and the other is 3 × the tens^2 + 3 × the tens × the units + the units^2 ($3\,t^2 + 3\,tu + u^2$).

Since 3 × the tens^2 is much greater than the rest of the factor, if 5824 is divided by 3 × the tens^2, or 1200, the quotient is about equal to the units or other factor. It is found to be 4. 1200, then, is a *trial*, or *partial*, *divisor*.

The divisor *completed* is therefore $3 \times 20^2 + 3 \times 20 \times 4 + 4^2$, which is equal to 1200 + 240 + 16, or 1456. This multiplied by 4 gives the product 5824. Therefore, the cube root of the number is 24.

Find the cube root of :

2. 1728   **3.** 4096   **4.** 9261   **5.** 15,625

When the number consists of more than two periods of figures, the root may be found in the same manner by considering, each time, the root already found as *tens* and the next figure of the root as *units*.

6. What is the cube root of 48,228,544 ?

$$
\begin{array}{ll}
 & 48'228'544\,\underline{|\,364} \\
3^3 = & 27 \\
\textit{Partial divisor, } 3 \times 30^2 = \quad 2700 & \overline{21228} \\
3 \times 30 \times 6 = \quad 540 \\
6^2 = \quad 36 \\
\textit{Complete divisor,} \qquad \overline{3276} & 19656 \\
 & \overline{1572544} \\
\textit{Partial divisor, } 3 \times 360^2 = 388800 \\
3 \times 360 \times 4 = \quad 4320 \\
4^2 = \quad .16 \\
\textit{Complete divisor,} \qquad \overline{393136} & 1572544
\end{array}
$$

7. What is the cube root of 22.906304 ?

$$
\begin{array}{ll}
 & 22.906'304\,\underline{|\,2.84} \\
2^3 = & 8 \\
3 \times 20^2 = \quad 1200 & \overline{14906} \\
3 \times 20 \times 8 = \quad 480 \\
8^2 = \quad 64 \\
 \qquad \overline{1744} & 13952 \\
 & \overline{954304} \\
3 \times 280^2 = 235200 \\
3 \times 280 \times 4 = \quad 3360 \\
4^2 = \quad 16 \\
 \qquad \overline{238576} & 954304
\end{array}
$$

In pointing off decimal periods, begin at the decimal point.

Separate the number into periods of three figures each, beginning at units.

Find the greatest cube in the left-hand period, and write its root for the first figure of the required root.

Cube this root, subtract the result from the left-hand period, and annex to the remainder the next period for a dividend.

Take three times the square of the root already found, considered as tens, for a partial divisor, and by it divide the dividend. The quotient or the quotient diminished will be the second part of the root.

To this partial divisor add three times the product of the first part of the root, considered as tens, by the second part, and also the square of the second part. Their sum will be the complete divisor.

Multiply the complete divisor by the second part of the root, and subtract the product from the dividend.

Continue thus until all the figures of the root have been found.

1. When there is a remainder, after subtracting the last product annex periods of decimal ciphers, and continue the process. The figures of the root obtained after the ciphers are annexed will be decimals.

2. Decimals are pointed off into periods of three figures each, beginning at tenths and passing to the right.

3. The cube root of a common fraction is found by extracting the cube root of both numerator and denominator separately, or by reducing the fraction to a decimal and then extracting its root.

Extract the cube root, carrying inexact results to two decimal places:

8. 54,872	**13.** 2	**18.** 3,048,625	**23.** 13,312.053
9. 91,125	**14.** 6	**19.** 4,741,632	**24.** 28.094464
10. 175,616	**15.** $\frac{4}{5}$	**20.** 43,614,208	**25.** .000166375
11. 405,224	**16.** $\frac{5}{7}\frac{1}{2}\frac{3}{5}$	**21.** 75,686,967	**26.** 1,865,409,391
12. 857,375	**17.** $\frac{6859}{19683}$	**22.** 130,323,843	**27.** 4065.356736

28. How deep is a cubical cistern containing 2744 cu. ft. ?

29. Find the number of square inches in one face of a cubical block whose volume is 185,193 cu. in.

30. A cubical box contains 46,656 cu. in. Find its depth.

31. Find, to the nearest tenth of a foot, the depth of a cubical bin that will contain 1200 bu. (Use 1 bu. = 2150.42 cu. in.)

32. A cubical cistern holds 400 bbl. of water. How many feet (nearest .1 ft.) deep is it ? (Use 1 gal. = 231 cu. in.)

33. A bin that is twice as long as it is wide or high holds 200 bu. of grain. Find its length. (Use 1 bu. = 1¼ cu. ft.)

SIMILAR SURFACES

Figures that are of exactly the same shape though they differ in size are **similar figures.**

All circles are similar; also all squares; two maps of the same country drawn to different scales are similar figures.

1. These triangles are similar. The sides of the first two are in the ratio of 1 to 2 and their areas are in the ratio of 1 to 4 (the squares of 1 and 2). The sides of the first and third are in the ratio of 1 to 3 and their areas in the ratio of 1 to 9.
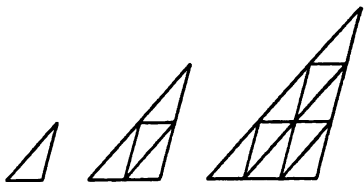
2. Similarly, the sides of these squares are proportional to 1, 2, and 3, and their areas are proportional to 1, 4, and 9, the squares of the sides.
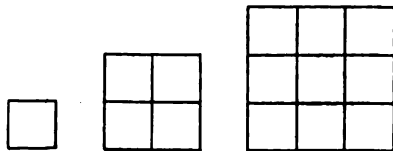
1. *The corresponding sides or like dimensions of similar plane figures are proportional.*

2. *The areas of similar plane figures are proportional to the squares of their corresponding lines.*

3. *The corresponding dimensions of similar plane figures are proportional to the square roots of their areas.*

Written Exercises

1. If a rectangle is 4 in. long and 3 in. wide, what is the width of a similar rectangle that is 8 in. long?

SUGGESTION. $4:8 = 3:x.$

2. The area of a circle is 5 sq. in. Find the area of a circle whose diameter is twice the diameter of the first.

3. The sides of two similar polygons are as 1 to 3. What is the ratio of their areas?

4. A lady has two circular flower beds, one having a radius of 4 ft. and the other a radius of 16 ft. How do they compare in area ?

5. By the principle of similar figures, find the height of a poplar tree that casts a 25-foot shadow when a boy 5 ft. tall casts a shadow $6\frac{1}{4}$ ft. long.

6. When a telephone pole 30 ft. high casts a shadow of 60 ft., what is the height of a church steeple that casts a shadow 300 ft. long ?
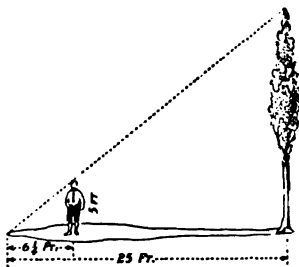
7. Two similar triangles have an area of 4 sq. ft. and 16 sq. ft., respectively. If the base of the first is $1\frac{1}{2}$ ft., what is the base of the second ?

SUGGESTION. $1\frac{1}{2}:x = \sqrt{4}:\sqrt{16}.$

8. The side of a square is 5 in. Find the side of another square that contains 4 times as much area.

9. Two similar rectangles have an area of 144 sq. in. and 1296 sq. in., respectively. If the altitude of the first is 8 in., what is the altitude of the second?

SIMILAR SOLIDS

Solids that have exactly the same shape though they differ in volume are **similar solids.** Cubes are similar solids.
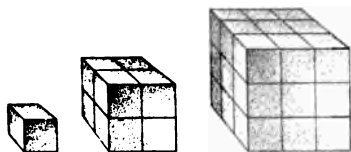
These three cubes have edges proportional to 1, 2, and 3.

How many cubes the size of the first does the second contain? the third?

The volumes of these cubes are proportional to 1, 8, and 27, the cubes of the edges.

1. *The volumes of similar solids are proportional to the cubes of their corresponding lines.*

2. *The corresponding dimensions of similar solids are proportional to the cube roots of their volumes.*

Written Exercises

1. If a prism 5 in. high contains 30 cu. in., how many cubic inches will a similar prism 10 in. high contain?

2. The volume of a sphere is 96 cu. ft. What is the volume of a sphere having a diameter half as long?

3. The altitude of a cone weighing 10 lb. is 2 ft. What is the altitude of a similar cone (same material) weighing 270 lb.?

4. A ball weighs 10 lb. Find the weight of a ball of the same material, if its diameter is 3 times as great.

5. How many more gallons of water can be contained in a tank $10\frac{1}{2}$ ft. in radius and 60 ft. high than in a similar tank 40 ft. high?

6. If a globe 4 in. in diameter weighs 8 lb., what will be the diameter of a similar one that weighs 125 lb.?

SUGGESTION. $4 : x = \sqrt[3]{8} : \sqrt[3]{125}$.

7. If a haystack 18 ft. in diameter contains 27 tons of hay, what is the diameter of a similar stack that contains 64 tons?

FOREIGN EXCHANGE

Foreign exchange does not differ in principle from domestic exchange.

A foreign bill of exchange is similar to a bank draft and is payable in the money of the country on which it is drawn. **Commercial drafts** are also drawn and accepted as in domestic exchange.

Foreign bills of exchange are usually written in duplicate, called a **set of exchange**, illustrated as follows :

EXCHANGE FOR *New York, U.S.A., Dec. 1, 1914.*

£ 200 . 8 . 5

Three days

after sight of this **First** *of Exchange (second unpaid)*

Pay to the order of Hiram Putnam

Two hundred Pounds 8/5 Sterling

Value received and charge the same to account of

To Brown, Shipley & Co.,

London,   *Brown Brothers & Co.*

No. 527   *England.*

In the duplicate, " Second of Exchange (first unpaid) " is substituted for " First of Exchange (second unpaid)," and " 2 " for " 1 " in the left margin. When either one of the set is paid, the other becomes void.

Par of exchange between two countries is the value of the monetary unit of one expressed in that of the other.

The table of equivalents on page 499 gives the par of exchange in the United States, on England, France, and Germany.

How Foreign Exchange is Quoted. — Exchange on :

England, by giving the cost of a bill of exchange for £ 1; thus, 4.87 means that a bill for £ 1 costs $ 4.87.

France (and other countries using the same monetary system), by giving the number of francs of exchange that can be bought for $1 ; thus, 5.18 means that 5.18 fr. can be bought for $ 1.

Germany, by giving the number of cents that 4 marks of exchange cost; thus, 94¼ means that 4 marks cost 94¼ ¢.

Newspapers usually give exchange rates for cable transfers, demand bills, and sixty-day bills, thus:

	CABLES	DEMAND	60 DAYS
Sterling	4.86¼	4.85¼	4.82
Francs	5.16⅞	5.18¼	5.20¼
Marks	95¼	95¼	94¼

Foreign exchange for small amounts is usually effected by means of **international** express and postal **money orders,** at fixed rates.

Letters of Credit. — A person intending to travel abroad may deposit funds with an international banking house to draw against, and receive a **letter of credit** guaranteed by the bank.

The purchaser signs several signature blanks, one of which is sent to each correspondent bank. When he presents the letter at any one of these banks he is asked to sign a draft or check for the amount he wishes to draw. The cashier compares the signature with that on the signature blank, and if they correspond, the money is paid and charged on the letter, which is returned to the owner.

Letters of credit are usually written in sterling money, the holder paying London exchange when he purchases it, and a further exchange when he draws in any other country than England.

Travelers' Checks. — These are guaranteed checks issued in denominations of $10, $20, $50, $100, and $200 by banks and express companies, at a fixed rate of ½ % of the face value.

The purchaser signs them on the face when purchased, and on the face or back when cashed. Identification is by comparison of signatures. These checks are readily received at European hotels, railroads, and business places.

Written Exercises

1. Find the cost of a cable transfer of 265 fr. at 5.16⅞.

SUGGESTION. — The cost in dollars = 265 ÷ 5.16⅞.

2. How much will a sixty-day bill for 250 M. cost at 94½?

SUGGESTION. — One mark costs ¼ of $.94½.

3. Find the cost of a demand bill for £75 6s. 4d. at 4.85¼.

SUGGESTION. — See exercise 13, page 139.

Find the cost, to the nearest cent, of exchange for:

4. £1200 @ 4.8365	**8.** 10,000 fr. @ 5.18
5. £1525 @ 4.8420	**9.** 25,000 M. @ 95¹⁄₁₆
6. £95 12s. @ 4.87¼	**10.** 1224.75 fr. @ 5.19⅞
7. £225 10s. 2d. @ 4.83⅝	**11.** 4520.65 M. @ 94¾

12. A cotton exporter in Mobile drew a sixty-day bill on London for £1075 against a shipment of cotton, and sold the bill at the bank at 4.83⅞. Find the proceeds.

13. An American sent to his family in Milan a bill for 2000 lire, exchange at 5.19. What was the cost of the bill?

14. Find the cost of £5 at 4.87, sent to Dublin at Christmas time by international money order, the fee being 30¢.

15-23. Find the cost of travelers' checks for $4000, at ½%.

The value of these checks in foreign money was according to the following values, printed on a 20-dollar check:

£ s. d.	FRANCS	MARKS	LIRE	CROWNS	FLORINS
4 1 2	102.50	82.50	102.50	73.39	49.02

The purchaser cashed $700 in checks in England, $900 in France, $400 in Germany, $200 in Denmark (crowns), $500 in Belgium (francs), $200 in Holland (florins), and $600 in Italy. Find the amount of money of each kind that he received and the amount refunded to him at the end of the trip.

LONGITUDE AND TIME

An imaginary line passing north and south from one pole of the earth to the other is a **meridian**.

The imaginary line around the earth running east and west halfway between the poles is the **equator**.

Since the equator is the circumference of a circle, distances along it are measured in *degrees*.

Distances east and west are measured from some selected meridian, called the **prime meridian**.
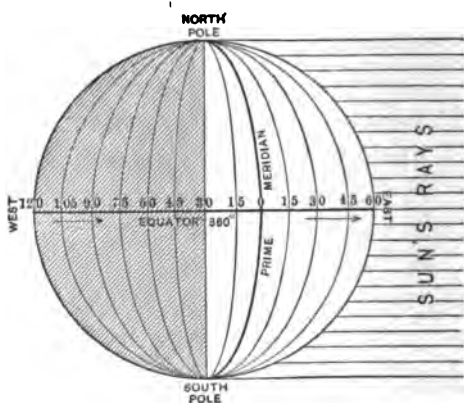
The prime meridian commonly used is that passing through the Royal Observatory at Greenwich, England.

Distance east or west of the prime meridian, measured in degrees along the equator, is **longitude**.

East longitude is the distance *east* of the prime meridian; *west longitude* is the distance *west* of it.

1. Since the earth rotates on its axis once in 24 hr., any meridian passes through 360° in that time.

Then, how many degrees of longitude pass under the sun's rays during 24 hr.? during 1 hr.?

2. Since 15° of longitude pass under the sun's rays in 1 hr., what part of a degree passes in one min.? How many minutes (′) of longitude pass in 1 min. of time?

3. Since 15′ of longitude pass under the sun's rays in 1 min., how many seconds (″) of longitude pass in 1 sec. of time?

As developed in the foregoing, the relation existing between *longitude* and *time* may be expressed briefly as follows:

360° of longitude correspond to 24.hours of time.
15° of longitude correspond to 1 hour of time.
15′ of longitude correspond to 1 minute of time.
15″ of longitude correspond to 1 second of time.

When the sun's rays are vertical at any point on a meridian, it is noon at all places on that meridian.

Since the earth turns from west to east, the sun *appears* to move from east to west. Therefore, when it is noon at any place it is *before* noon, or **earlier**, at all places **west**, because the sun has not yet reached the meridians of those places. It is *after* noon, or **later**, at all places **east**, because the sun has already crossed the meridians of those places.

Oral Exercises

1. What is the difference in longitude between a place 15° west of Greenwich and a place 30° west?

Which place has the earlier time and how much earlier?

2. How far westward must one go to pass from meridian 15° E. to Greenwich? from Greenwich to 15° W.? How many degrees west of 15° E. is 15° W.? Compare their times.

Compare the times of places on the following meridians:

3. 15° W. and 45° W.	**8.** 10° E. and 5° W.
4. 15° W. and 60° W.	**9.** 30° E. and 30° W.
5. 45° E. and 30° E.	**10.** 20° E. and 10° W.
6. 35° E. and 20° E.	**11.** 40° E. and 20° W.
7. 42° E. and 12° E.	**12.** 30° E. and 45° W.

TABLE OF LONGITUDES

WEST LONGITUDES				EAST LONGITUDES				EAST LONGITUDES			
	°	′	″		°	′	″		°	′	″
Boston	71	03	50	Batavia	106	48	37	Bombay	72	48	56
New York	74	00	24	Melbourne	144	58	35	Cape Town	18	28	40
Washington	77	08	06	Tokyo	139	44	80	Berlin	13	23	44
Chicago	87	36	45	Shanghai	121	28	55	Hamburg	9	58	25
Denver	104	58	00	Manila	120	58	06	Amsterdam	4	53	04
San Francisco	122	24	32	Canton	113	16	30	Paris	2	20	14

In the following exercises refer to this table for longitudes and find answers to the *nearest* second.

Written Exercises

1. When it is noon at San Francisco, what is the time at Washington ?

$$122° \ 24′ \ 32″$$
$$77° \ \ 3′ \ \ \ 6″$$
$$15)\overline{45° \ 21′ \ 26″}$$
$$3° \ \ 1′ \ 26″$$

3 hr. 1 min. 26 sec. is the difference in time.

The difference in longitude is found to be 45° 21′ 26″.

Since 15° corresponds to 1 hr., 15′ to 1 min., and 15″ to 1 sec., the difference in time between San Francisco and Washington is as many hours, minutes, and seconds, respectively, as there are degrees, minutes, and seconds in $\frac{1}{15}$ of the difference in longitude.

Therefore, the difference in time is 3 hr. 1 min. 26 sec., and since Washington is *east* of San Francisco, the time is *later* in Washington; that is, when it is noon at San Francisco it is 1 min. 26 sec. past 3 P.M. at Washington.

Find the true time in each of the following cities when the sun is on the meridian of Washington:

2. Boston   **5.** Berlin   **8.** Hamburg

3. New York   **6.** Paris   **9.** Amsterdam

4. Chicago   **7.** London (Greenwich)   **10.** Cape Town

11. What is the longitude of Petrograd, if its time is 6 hr. 57 min. 19 sec. later than that of New York?

hr.	min.	sec.			
6	57	19	104°	19′	45″
		15	74°	00′	24″ W. (N. Y.)
104	19	45	30°	19′	21″ E. (Petro.)

104° 19′ 45″, diff. in longitude.

Since 1 hr. corresponds to 15°, 1 min. to 15′, and 1 sec. to 15″, the difference in longitude between New York and Petrograd is as many degrees, minutes, and seconds, respectively, as there are hours, minutes, and seconds in 15 times the difference in time between the two places.

Therefore, the difference in longitude is 104° 19′ 45″, and since the time of Petrograd is *later* than that of New York, Petrograd is *east* of New York; but New York is only 74° 00′ 24″ west of the prime meridian, and since the difference in longitude is greater than this, Petrograd must be east of the prime meridian. Subtracting, we find the longitude of Petrograd to be 30° 19′ 21″ E.

Find the longitude of places having the following times when the sun is on the meridian of Washington:

12. 2.21 P.M.   **15.** 10.18 A.M.   **18.** 7.32½ A.M.

13. 4.36 P.M.   **16.** 9.54 A.M.   **19.** 1.05¾ P.M.

14. 3.42 P.M.   **17.** 6.09 A.M.   **20.** 15 sec. before 8 A.M.

21. The longitude of Norwich, Eng., is 1° 18′ W.; of Norwich, Conn., 72° 4′ W. How much later does the sun cross the meridian of the American city than that of the English city?

22. An astronomer in Boston observed the moon entering the shadow of an eclipse at 1.13 A.M. At what times did astronomers in Chicago, Denver, and San Francisco observe the same?

23. When the people of Boston were celebrating the passing of the 19th century at midnight Dec. 31, 1900, how long had Parisians been living in the 20th century? How much of the 19th century was left for the people of Galveston, 94° 47′ 26″ W.?

24. At noon a ship's chronometer carrying Greenwich time indicated 1.05 P.M. In what longitude was the ship?

25. A ship's chronometer carrying Greenwich time was 35 min. slow Saturday noon and 13 min. fast the following Tuesday noon. In what longitude was the ship at each observation? How far east or west did she sail?

26. When it is 6 P.M. Jan. 10 at San Francisco, what is the time and date at Tokyo?

First Solution

Since San Francisco is 122° 24′ 32″ W. and Tokyo is 139° 44′ 30″ E., Tokyo is 262° 9′ 2″ *east* of San Francisco, and consequently has 17 hr. 28 min. 36 sec. *later* time. Counting this time on from 6 P.M. Jan. 10, the time at Tokyo is found to be 28 min. 36 sec. after 11 A.M. Jan. 11.

Second Solution

Reckoning in the other direction, Tokyo is 360°–262° 9′ 2″, or 97° 50′ 58″ *west* of San Francisco, and its time is therefore 6 hr. 31 min. 24 sec. *earlier*.

This apparent contradiction of the first solution is explained thus: the time of Tokyo is 17 hr. 28 min. 36 sec. later than that of San Francisco, or a day later *lacking* 6 hr. 31 min. 24 sec. A day later than 6 P.M. Jan. 10 is 6 P.M. Jan. 11, and a day later *less* 6 hr. 31 min. 24 sec. gives the Tokyo time, 28 min. 36 sec. after 11 A.M. Jan. 11.

Note. — When a ship sails *westward* over the 180th meridian the calendar is set *forward* one day; sailing *eastward* its calendar is set back one day.

An irregular line corresponding in general with this meridian marks the place where the calendar changes. It is called the **international date line.**

When the sun is on the meridian of New York, on the first day of May, find the true time and date in:

27. Manila   **29.** Batavia   **31.** Canton

28. Melbourne   **30.** Bombay   **32.** Cape Town

33. At noon, Mar. 10, a ship weighed anchor at Hongkong, 114° 10′ 02″ E., and sailing eastward reached Honolulu, 157° 51′ 34″ W., at noon Mar. 25. How long was the voyage?

STANDARD TIME

In 1883, the railroads of the United States and Canada agreed upon a system of **standard time** that has come into general use because of its convenience. Under this system there are five **time belts**, each approximately 15° of longitude in width, and each having the time of its central meridian.

Each railway has selected the most convenient city on its own road at which to change from the standard time of one belt to that of the next. Since such towns on the various roads are seldom on the same meridian, the line connecting them forms an irregular boundary between the various belts; hence these time belts, shown on the following map, are neither equal in size nor uniform in shape.

STANDARD TIME BELTS
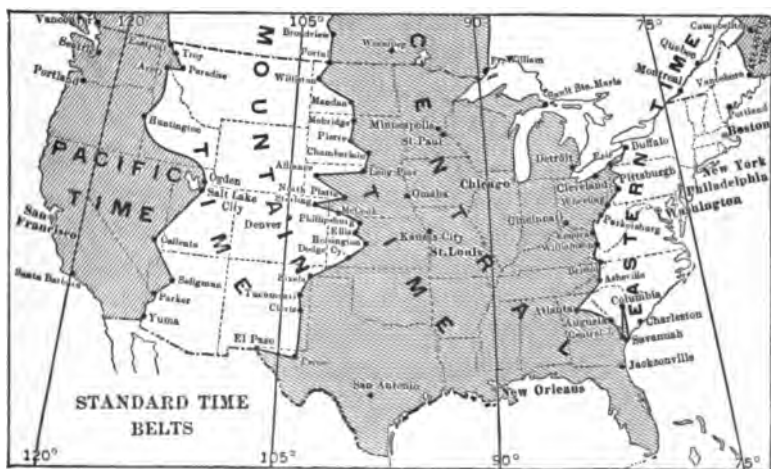
The time belts are called **Atlantic**, with the time of the meridian of 60° W.; **Eastern**, with the time of the meridian of 75° W.; **Central**, with the time of the meridian of 90° W.; **Mountain**, with the time of the meridian of 105° W.; and **Pacific**, with the time of the meridian of 120° W.

It is evident that the time of the various belts differs by *hours*, the minute and second hands of all correct timepieces being the same at any instant. Central time is 1 hr. earlier than Eastern time; Mountain time is 1 hr. earlier than Central time; and Pacific time is 1 hr. earlier than Mountain time.

Correct time is telegraphed each day to all parts of the United States from the Naval Observatory at Washington.

Standard time has been adopted by most civilized governments of the world, the time meridian chosen being, with few exceptions, some multiple of 15° from the prime meridian through Greenwich.

In exercises refer to the map on page 483 for the time meridians of cities in the United States. The standard time meridian for each foreign city will be given in parentheses in the exercises where it is needed.

Written Exercises

1. A certain business transaction was reported by telegraph from Chicago at 10.30 A.M., to New York, New Orleans, San Antonio, and Portland, Ore. Allowing no time for transmission, when did the message reach each city?

2. If the news of the opening of the St. Louis Exposition at 12.15 P.M. was immediately telegraphed all over the world, at what time did Denver receive it? Portland, Me.? Manila (120° E.)? Bombay (75° E.)? Berlin (15° E.)? London (0°)? Tokyo (135° E.)?

3. The news of the *Maine* disaster was cabled to Madrid (0°) from Havana (75° W.) at 2 A.M. Neglecting the time of transmission, find the time when the news reached Madrid.

4. Dewey's flagship, the *Olympia*, opened fire on the defenses of Manila (120° E.) at 5.41 A.M., May 1, and ceased firing at 7.40 A.M. to allow time for breakfast. Between what hours and on what day, Washington time, did the assault occur?

5. The news of the assault reached Madrid (0°) at 6.20 P.M. the same day. How long was this after the *Olympia* opened fire?

6. Cable communication with Manila was severed at 10 A.M., London time, May 2. What was the clock time in Manila (120° E.) when the cable was cut?

7. A London paper received a dispatch from Cairo (30° E.) reporting an explosion in the British barracks. If the dispatch was received in London at 3.15 P.M., at what time was it sent?

8. If 1¼ hr. later the news was cabled to New York (75° W.), what was the time when it reached New York?

9. Germany through her ambassador at Petrograd (30° E.) formally declared war on Russia at 7.30 P.M., Aug. 1, 1914. What time was this by New York time?

10. The first shock of the earthquake at San Francisco was recorded at 5.12 A.M. at the University of California, and at 20 sec. after 8.19 A.M. at Washington. How long did it take the shock to cross the continent?

11. The dispatch regarding the final surrender of Port Arthur (120° E.) was sent from that place at 10 P.M., Jan. 2. At what time was the news received in Tokyo (135° E.)? in Petrograd (30° E.)? in London? in New York?

12. The *Atlantic*, the winner of a yacht race from Sandy Hook (75° W.) to the Lizard, Eng. (0°), started at 12.15 P.M., May 17, and finished at 9.18 P.M., May 29. Find the *Atlantic's* time.

13. Paris uses the time of her meridian, 2° 20′ 14″ E. The Paris stock exchange, or *Bourse*, closes at 3 P.M. At what time will closing quotations reach New York, if cabled immediately?

14. At 12 P.M., Saturday, Dec. 31, the chief of the U.S. Signal Service sent a message from Washington around the world *via* Chicago, Denver, San Francisco, Manila (120° E.), Tokyo (135° E.), Melbourne (150° E.), Bombay (75° E.), Berlin (15° E.), and London. What was the date and the clock time in each city at the instant of sending the message?

METRIC SYSTEM

The **metric system** is a *decimal* system of weights and measures, having for its **principal unit** the **meter**, to which all other units are related.

This system originated in France. Its use is required by law in many countries, and permitted in many others, including the United States. Congress has made it the official system in our Philippine possessions.

Its most general use is in the arts and sciences, where its convenience and accuracy have specially commended it.

MEASURES OF LENGTH

The primary unit of **length** is the **meter**, which is the unit upon which the whole system is based.

100 Millimeters

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

10 Centimeters

$\frac{1}{10}$ Meter, or 1 Decimeter
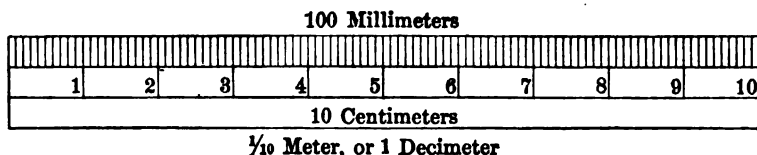
The length of the meter was intended to be a **ten-millionth** part of the distance from the equator to either pole, but subsequent calculations have shown it to differ slightly from that.

The length of the standard meter in the United States is 39.37 inches.

Other metric units of length are **decimal parts** of the meter and **powers of 10** times the meter.

The primary units of surface, volume, capacity, weight, etc., are likewise subdivided and multiplied *decimally*, giving the other units of those measures.

Consequently, in the metric system names are simplified and made to show the size of each unit by giving to each *primary unit* the following *Latin prefixes* to indicate the *decimal parts* and *Greek prefixes* for the *multiples*.

LATIN	GREEK
deci means .1	**deka** means 10
centi means .01	**hekto** means 100
milli means .001	**kilo** means 1000

Thus, decimeter means $\frac{1}{10}$ of a meter; centimeter, $\frac{1}{100}$ of a meter; millimeter, $\frac{1}{1000}$ of a meter; while dekameter means 10 meters; hektometer, 100 meters; kilometer, 1000 meters.

Another prefix sometimes used is **myria**, meaning 10,000.

Table of Measures of Length

10 millimeters (mm.) = 1 centimeter (cm.)
10 centimeters   = 1 decimeter (dm.)
10 decimeters   = 1 **meter** (m.)
10 meters   = 1 dekameter (Dm.)
10 dekameters   = 1 hektometer (Hm.)
10 hektometers   = 1 **kilometer** (Km.)

Abbreviations for *parts* of the primary unit begin with small letters, those for *multiples* of it with capitals.

The tables indicate important units by **heavy type.**

The **kilometer** (about .6 mi.) is used for long distances; the **meter** (about 3¼ ft., a little over 1 yd.), for shorter distances and to measure cloth, etc.; the **millimeter**, in the sciences and to express such small measurements as the thickness of wire.

The decimal relation of the units and their correspondence to the successive orders of units in the decimal notation is illustrated by the following:

Km.	Hm.	Dm.	m.		dm.	cm.	mm.
1	1	1	1	.	1	1	1

Since the meter occupies units' place, this number is read the same as 1111.111 m.

Reduction to any other unit in the table is accomplished by simply moving the decimal point, thus:

1111.111 m. = 1.111111 Km. = 111111.1 cm. = etc.

Express in meters or decimals of a meter :

1. 2 Km.	**7.** 75 cm.	**13.** 5 Km. 300 m.
2. 4 mm.	**8.** 37.5 cm.	**14.** 40 Km. 75 m.
3. 7 Hm.	**9.** 12.5 cm.	**15.** 26 Km. 33.5 m.
4. 9 cm.	**10.** 250 mm.	**16.** 706 m. 82 cm.
5. 5 Dm.	**11.** 981 mm.	**17.** 530 m. 75 cm.
6. 8 dm.	**12.** 3.275 Km.	**18.** 48 cm. 9.65 mm.

19. About how many feet are there in 40 m. ?   meters in 13 ft. ?   miles in 15 Km. ?   kilometers in 60 mi. ?

20. During practice marches, the German soldier walks 30 Km. per day.   About how many miles does he walk per day ?

21. A foot is about 30 cm.   About how many steps must a man take to walk 100 ft., if each step is ⅚ m. long ?

MEASURES OF SURFACE

1. Draw a rectangle 10 cm. long and 1 cm. wide, and divide it into square centimeters.   What is its area ?

one sq. cm.									

10 Square Centimeters

2. What is the area of a rectangle 10 cm. by 2 cm. ?   10 cm. by 3 cm. ?   10 cm. by 5 cm. ?   10 cm. by 10 cm. ?

3. Since 10 cm. = 1 dm., how many square centimeters are there in 1 square decimeter ?

4. How many square decimeters equal 1 sq. m. ?   How many square meters equal 1 sq. Dm. ?

In metric square measure it requires 100 *units of any denomination to make* 1 *unit of the next higher denomination.*

The primary unit of **surface** is the **square meter**.

The official equivalent of the square meter in the United States is 1.196 square yards.

Table of Measures of Surface

100 square millimeters	= 1 square centimeter
100 square centimeters	= 1 square decimeter
100 square decimeters	= 1 square meter
100 square meters	= 1 square dekameter
100 square dekameters	= 1 square hektometer
100 square hektometers	= 1 square kilometer

The **square meter** (about 1.2 sq. yd.) is used for ordinary surfaces, such as floors, walls, etc.; the **square kilometer** (nearly .4 sq. mi.) for such large surfaces as the areas of countries.

From the principle (page 488) and the table, it is seen that the successive metric units of square measure occupy *two* orders of figures, thus:

sq. Km.		sq. Hm.		sq. Dm.		sq. m.		sq. dm.		sq. cm.		sq. mm.
1	0	1	0	1	0	1	.	0	1	0	1	0	1

Sometimes the abbreviation m² is used for sq. m.

The primary unit of **land measures** is the **are**, which is a *square dekameter*.

The official equivalent of the are is 119.6 square yards.

Table of Land Measures

100 centares	= 1 are
100 ares	= 1 hektare

The **hektare** is nearly 2½ acres.

The successive land units occupy two orders of figures:

Ha.		a.		ca.
1	0	1	.	0	1

Written Exercises

Reduce to square meters or decimals of a square meter:

1. 62.5 sq. dm.   3. 122.5 sq. Hm.   5. .125 sq. Dm.
2. 37.5 sq. dm.   4. 1.875 sq. Km.   6. 1000 ares.

7. Reduce to square millimeters : .5 sq. cm.; 1.25 sq. cm.

8. Reduce to square centimeters : 875 sq. mm.; 1500 sq. mm.

9. Find the area of a post card 14 cm. by 8 cm.

10. Find the area, in hektares, of a city block 1 Hm. square.

11. The large sulphur mines at Askhabad, Russia, cover an area of 6000 Ha. About how many acres do they cover?

MEASURES OF VOLUME

1. How many centimeters are there in 1 dm.?

2. Into how many cubic centimeters may a rectangular solid be divided, if it is 1 dm. long, 1 dm. wide, and 1 cm. thick? 2 cm. thick? 5 cm. thick? 10 cm., or 1 dm., thick?

3. Then, how many cubic centimeters are there in a rectangular solid 1 dm. by 1 dm. by 1 dm., that is, in 1 cu. dm.?

4. How many cubic decimeters equal 1 cu. m.? How many cubic meters equal 1 cu. Dm.?

In metric cubic measure it requires 1000 *units of any denomination to make* 1 *unit of the next higher denomination.*

The primary unit of **volume** is the **cubic meter.**

The official equivalent of the cubic meter is 1.308 cubic yards.

Table of Measures of Volume

1000 cubic millimeters = 1 **cubic centimeter**
1000 cubic centimeters = 1 **cubic decimeter**
1000 cubic decimeters = 1 **cubic meter**

and so on. The higher units, however, are little used.

From the principle (page 490) and the table, it is evident that the successive metric units of cubic measure occupy *three* orders:

cu. Km.	cu. Hm.	cu. Dm.	cu. m.	cu. dm.	cu. cm.	cu. mm.
1 0 0	1 0 0	1 0 0	1 . 0 0	1 0 0	1 0 0	1 0 0 1

Sometimes the abbreviation m⁸ is used for cu. m.

The primary unit of **wood measures** is the **stere**, which is a *cubic meter*.

Table of Wood Measures

10 decisteres = 1 stere
10 steres   = 1 dekastere

Written Exercises

1. A book is 19 cm. long, 14½ cm. wide, and 2 cm. thick. Express its volume in cubic centimeters.

2. How many steres of wood can be piled in a shed 6 m. long, 5 m. wide, and 8.2 m. high?

3. How many cubic centimeters of water will fill a cubical box, each inside dimension of which is 1 dm.?

4. A wall along the side of a park 1 Km. long is 1 m. thick and 1.6 m. high. Find the solid contents in cubic meters.

5. How thin must 1 cu. cm. of gold be beaten to cover a rectangle 40 cm. long and 15 cm. wide?

6. A block of white marble quarried at Paros, Greece, was 4 m. long and 1.25 m. square. Find its value at $28 per cubic meter.

7. In a recent year 320 cu. Hm. of gas were produced in New York. How many cubic meters were produced?

8. Timbers of imported Spanish mahogany are 61 cm. square and 8.05 m. long. Find the volume of such a timber.

MEASURES OF CAPACITY

The primary unit of capacity for both liquid and dry measures is the liter (lētēr), which contains 1 *cubic decimeter*, or 1000 *cubic centimeters*.

The official equivalent of the liter is 1.0567 qt. (liquid). The liter is .908 qt. (dry).

Table of Measures of Capacity

10 milliliters = 1 centiliter
10 centiliters = 1 deciliter
10 deciliters = 1 liter
10 liters = 1 dekaliter
10 dekaliters = 1 hektoliter

The liter (about 1 qt., liquid or dry) is used to measure moderate quantities; the hektoliter (nearly 2.84 bu.) is used to measure grain, fruit, vegetables, etc., in large quantities.

The successive units of capacity correspond to the order of units in the decimal notation, thus:

$$\overset{\text{Hl.}}{1} \ \overset{\text{Dl.}}{1} \ \overset{\text{l.}}{1} . \overset{\text{dl.}}{1} \ \overset{\text{cl.}}{1} \ \overset{\text{ml.}}{1}$$

Written Exercises

1. How many liters of rice will it take to fill a box the inside dimensions of which are 60 cm., 50 cm., and 40 cm. ?

2. What is the cost, in Mexico, of 24 Hl. of corn at $12 per 200 l. ?

3. A Manila merchant bought 400 l. of olive oil at 25¢ per liter and retailed it at 30¢ per ¾ l. Find his gain.

4. The French wheat harvest one year was 110½ million hektoliters, grown on 6½ million hektares. Find the yield per hektare.

5. · A Spanish olive orchard of 750 trees yielded 30 l. of olives per tree. How many hektoliters of olives did it yield?

6. An electric fire engine such as is used in France throws 350 l. of water per second. How many cubic meters of water do 4 such engines throw on a fire in an hour?

MEASURES OF WEIGHT

The primary unit of **weight** is the **gram**, which is the weight of 1 *cubic centimeter* of pure water at its greatest density.

The weight of 1000 cubic centimeters (1 cubic decimeter) of water, that is, of 1 *liter* of water, is called a **kilogram**, or a **kilo**.

one cu. cm.

The official equivalent of the kilogram is 2.2046 pounds avoirdupois.

Table of Measures of Weight

10 milligrams = 1 centigram   10 grams   = 1 dekagram
10 centigrams = 1 decigram   10 dekagrams = 1 hektogram
10 decigrams = 1 gram   10 hektograms = 1 kilogram
    Higher units are:
                10 kilograms = 1 myriagram
               100 kilograms = 1 metric quintal
              1000 kilograms = 1 metric ton

The **gram** (about ⅟ the weight of a 5-cent piece) is used in weighing medicines, precious metals, etc.; the **kilogram** (about 2.2 lb.), in weighing groceries; and the **metric ton** (about 2205 lb.), in weighing bulky articles, such as hay, coal, etc.

A 5-cent piece weighs about 5 grams, and is about 2 cm. across and 2 mm. thick.

The successive units of weight and the orders of units in the decimal notation correspond, thus:

M.T.	Q.	Mg.	Kg.	Hg.	Dg.	g.	dg.	cg.	mg.
1	1	1	1	1	1	1 .	1	1	1

Reduce to grams or decimals of a gram :

1. 3.5 Kg.   **3.** 2.5 dg.   **5.** 350 mg.   **7.** 49 Kg. 755 g.

2. 4.2 Hg.   **4.** 7.5 Dg.   **6.** 8400 mg.   **8.** 15 g. 200 mg.

9. Make a table of the units *milligram, gram, kilogram,* and *metric ton.*

10. If a 6-liter jar weighing 2½ Kg. is filled with water, what is the total weight of the jar and the water ?

11. Find the weight of a cubic meter of water.

12. Ice is .92 as heavy as water. Find the weight of a cube of ice 80 cm. on each edge.

13. A boy who weighs 40 kilos weighs about —— pounds.

14. About how many 5-cent pieces weigh 1 kilo?

EQUIVALENTS

Unless stated to the contrary, refer to the table of equivalents given on page 499 when changing from one system to the other, giving final inexact results to the nearest thousandth.

In exercises **1–8** reduce kilometers to miles, meters to feet, centimeters and millimeters to inches :

1. Height of spire of Strassburg Cathedral, 142 m.

2. Height of Pyramid of Cheops, 139.5 m.

3. Height of Leaning Tower of Pisa, 54.5 m.

4. Bore of Mauser rifle, 7 mm.

5. Length of hairspring of a watch, 28 cm.

6. Length of St. Gothard tunnel, Switzerland, 14.9 Km.

7. Length of Panama Railroad, Panama to Colon, 78 Km.

8. Length of French cable, Brest to New York, 5318.9 Km.

9. How many metric tons of lard are there in a shipment of 1000 tubs, each holding 12½ Kg. of lard ?

10. A South American sewer tunnel 1278 m. long, 3.65 m. high, and 3 m. wide was cut through rock. How many cubic meters of rock were removed ?

11. A large dredge removed 4500 cu. m. of mud per hour at a cost of 3 pfennigs per cubic meter. Find the cost in marks of running the dredge 12 hr.

12. Short cotton, grown in Algeria, averages 3 metric tons per hektare. Find the value of 5.5 Ha. of Algerian cotton at $9.25 per metric quintal.

13. A wholesale dealer carefully weighed a sample of cloth 40 mm. by 50 mm. and found its weight to be 62 cg. What was the weight of this cloth per meter, if it was 140 cm. wide?

14. A quart of water is less than a liter, but if frozen will make more than a liter of ice. How many cubic centimeters more, if water expands $\frac{2}{13}$ of its volume in freezing ?

The **specific gravity** of a substance is its weight as compared with that of an equal volume of water.

15. If 2 liters of milk weigh 2.06 Kg., what is the specific gravity of milk, or how many times as heavy as water is it ?

SOLUTION. 1 l. of water weighs 1 Kg.; 1 l. of milk weighs ½ of 2.06 Kg., or 1.03 Kg. Therefore, the specific gravity of milk is 1.03.

Find the specific gravity of the following substances :

16. Olive oil, if 40 cu. cm. weigh 36.6 g.

17. Mercury, if ½ l. weighs 6.799 Kg.

18. Cork, if a piece 10 cm. by 8 cm. by 2 cm. weighs 384 dg.

19. Find the weight of 25 l. of naphtha, specific gravity .848.

20. In a quarry in Saxony an undercut resulted in the fall of 58,000 cu. m. of stone. What was the weight of the stone in metric tons, if its specific gravity was 2.7 ?

TABLES

DENOMINATE NUMBER TABLES

The tables marked with an asterisk (*) are given for reference only.
The other tables are important and should be *memorized*.

Linear Measures

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
5½ yards ⎫	
16½ feet ⎬	= 1 rod (rd.)
320 rods	= 1 mile (statute)

1 mi. = 1760 yd. = 5280 ft. = 63,360 in.

A nautical mile (knot) = 6080.27 ft., or approximately 1.15 mi.

A furlong = ⅛ mi.; a fathom, used in measuring the depth of water, is 6 ft.; a hand, used in measuring the height of horses, is 4 in.

Square Measures

144 square inches	= 1 square foot
9 square feet	= 1 square yard
30¼ square yards	= 1 square rod
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile

1 acre = 43,560 sq. ft.

An acre of land in the form of a square is very nearly 209 ft. on a side.

A tract of land 1 mile square is often called a *section*.

100 sq. ft. of roofing, flooring, or slating is called a *square*.

Origin of Units. In early times a measure of distance was furnished in "a day's journey." Later the *stride* or *pace* came into use as a smaller unit, and the dimensions of the parts of the human body appeared as units.

Inch is supposed by some to be derived from the terminal joint of the thumb, by others to come from *uncia*, the twelfth part. It probably originated only as a convenient subdivision of the foot.

Foot, as its name implies, is probably based on the length of the human foot. It was used by the Romans.

Yard. In England, Henry I established the yard as the distance from the point of his nose to the end of his thumb.

Rod. An old work gives this rule: "Stand at the door of a church on a Sunday and bid sixteen men to stop, tall ones and small ones, as they happen to pass out as service is finished. Then make them put their feet one behind the other and the length thus obtained shall be a right and lawful rood" (rod).

Mile. The statute or "land mile" is from the Latin *mille passuum*, 1000 paces, the Roman pace being a double step about 5 feet in length.

Acre was primarily an open plowed or sowed field.

As civilization advanced the need of a definite and invariable standard arose.

The principal unit of our common system of weights and measures, the unit from which all the others are derived, is the **yard**, which is 36 of the 39.1393 inches in the length of a pendulum beating seconds under certain conditions.

The principal unit of the metric system is the **meter** described on page 486.

Cubic Measures

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

A pile of wood 3 ft. long, 4 ft. wide, and 4 ft. high is called a cord.

*Surveyors' Linear Measures

100 links = 1 chain
80 chains = 1 mile

This chain, called **Gunter's chain**, is 4 rd., or 66 ft. long. 1 link = 7.92 in. Links are written as hundredths of a chain.

*Surveyors' Square Measures

10 square chains = 1 acre
640 acres = 1 square mile

The convenience of this system lies in the easy reduction of square chains to acres, by moving the decimal point one place toward the left.

Liquid Measures

4 gills(gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

1 gal. = 231 cu. in.; 1 cu. ft = $7\frac{1}{2}$ gal., approximately.

A gallon of water weighs about $8\frac{1}{4}$ lb.; a cubic foot of water weighs about $62\frac{1}{4}$ lb.

In measuring the capacity of cisterns, etc. :

$31\frac{1}{2}$ gal. = 1 **barrel.**
63 gal. = 1 **hogshead.**

Dry Measures

2 pints = 1 quart
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

1 bu. (*stricken,* even full) = 2150.42 cu. in., or approximately $1\frac{1}{4}$ cu. ft.

1 bu. (*heaped*) = $1\frac{1}{4}$ cu. ft., approximately.

Our bushel is the **Winchester bushel.** In form it is a cylinder $18\frac{1}{2}$ in. in diameter and 8 in. deep. This has been displaced in England by the **imperial bushel** of 2218.192 cu. in.

Origin of Units. *Gill* is from Low Latin *gilla*, a drinking glass.

Pint. While the origin is uncertain, pint is thought to be from an Anglo-Saxon word *pyndam*, to shut in, to pen, or from the Greek *pinto*, to drink.

A pint of water weighs nearly one pound. This gave rise to the old adage, "A pint's a pound the world around," an idea often used for certain household measurements.

Quart is from the Latin *quartus*, a fourth, the quart being a fourth of a gallon.

Gallon. The derivation of this unit is in doubt.

Peck. The origin is uncertain.

Bushel is derived from a word meaning box.

Avoirdupois Weight	* Troy Weight

Avoirdupois Weight

16 ounces (oz.) = 1 pound (lb.)

100 pounds = 1 hundredweight (cwt.)

2000 pounds = 1 ton (T.)

1 long or gross ton = 2240 pounds

* Troy Weight

24 grains (gr.) = 1 pennyweight

20 pennyweights = 1 ounce

12 ounces = 1 pound

1 troy lb. = 5760 gr. = $\frac{5760}{7000}$ av. lb.

The long ton is used at United States customhouses and in weighing certain ·mineral products at the mines.

Origin of Units. *Pound* is from the Latin *pendo*, I bend or weigh.

Ounce comes from the Latin *uncia*, a twelfth part, the ounce (troy) being one twelfth of a pound. *Pennyweight* was the weight of an old English penny.

Grain has its origin in a grain of wheat, the primitive standard of weights in England.

Standard Weight of a Bushel

In New York and most other states:

Oats, 32 lb.   Buckwheat, 48 lb.   Barley, 48 lb.

Wheat, 60 lb.   Corn (shelled), 56 lb.   Potatoes, 60 lb.

A barrel of flour contains 196 lb.; of salt, 280 lb.; of pork, 200 lb.

* Apothecaries' Weight

This is used to some extent in filling prescriptions. The grain, ounce, and pound are the same as in troy weight, but the ounce is divided differently.

20 grains (gr.) = 1 scruple . . . sc. or ϶

3 scruples = 1 dram . . . dr. or ʒ

8 drams = 1 ounce . . . oz. or ℥

12 ounces = 1 pound . . . lb. or ℔

* Apothecaries' Liquid Measures

60 drops (gtt.) or minims (♏︎) = 1 fluid dram . . . ƒʒ

8 fluid drams = 1 fluid ounce . . . ƒʒ

16 fluid ounces = 1 pint . . . . . O.

8 pints = 1 gallon . . . . . Cong.

Time Measures

60 seconds = 1 minute (min.)

60 minutes = 1 hour (hr.)

24 hours = 1 day (da.)

7 days = 1 week (wk.)

365 days = 1 year (yr.)

366 days = 1 leap year

Thirty days have September,

April, June, and November.

All the rest have thirty-one,

Save February, which alone

Has twenty-eight, and one day more

We add to it one year in four.

The earth revolves around the sun in 365 days 5 hr. 48 min. 46 sec. This is the solar year, and is nearly 365¼ days.

To correct the errors in the calendar, made by disregarding the fraction of a day over 365 days, centennial years divisible by 400 and other years divisible by 4 are lengthened 1 day, Feb. 29. These years are leap years.

Circular Measures

60 seconds ($''$) $= 1$ minute ($'$)
60 minutes $= 1$ degree ($°$)
360 degrees $= 4$ right angles or 1 circumference
$90°$ of angle $= 1$ right angle; $90°$ of arc $= 1$ quadrant

United States Money

10 mills $= 1$ cent ($\not$)
10 cents $= 1$ dime
10 dimes $= 1$ dollar ($)
10 dollars $= 1$ eagle

English Money

4 farthings $= 1$ penny ($d.$)
12 pence $= 1$ shilling ($s.$)
20 shillings $= 1$ pound, or sovereign

Counting Table

20 $= 1$ score
12 $= 1$ dozen
12 dozen $= 1$ gross
12 gross $= 1$ great gross

French Money

100 centimes (c.) $= 1$ franc (fr.)

German Money

100 pfennigs (pf.) $= 1$ mark (M.)

Foreign Money Equivalents

1 pound (£) $= 4.8665
1 franc (fr.) $= $ .193$
1 mark (M.) $= $ .238$

Stationers' Measures

24 sheets $= 1$ quire
20 quires $= 1$ ream
2 reams $= 1$ bundle
5 bundles $= 1$ bale

* Metric tables are given on pages 486–493; equivalents follow:

* COMMON TO METRIC		* METRIC TO COMMON	
1 yd.	$= .9144$ m.	1 m.	$= 39.37$ in.
1 mi.	$= 1.60935$ Km.	1 Km.	$= .62137$ mi.
1 sq. yd.	$= .836$ sq. m.	1 sq. m.	$= 1.196$ sq. yd.
1 A.	$= .4047$ Ha.	1 Ha. .	$= 2.471$ A.
1 cu. yd.	$= .765$ cu. m.	1 cu. m.	$= 1.308$ cu. yd.
1 qt. (dry)	$= 1.1012$ l.	1 l.	$= .908$ qt. (dry)
1 qt. (liq.)	$= .94636$ l.	1 l.	$= 1.0567$ qt. (liq.)
1 bu.	$= .35239$ Hl.	1 Hl.	$= 2.8377$ bu.
1 lb. (av.)	$= .45359$ Kg.	1 Kg.	$= 2.2046$ lb. (av.)
1 T.	$= .90718$ M. T.	1 M. T.	$= 1.1023$ T.

MULTIPLICATION TABLES

$1 \times 1 = 1$	$1 \times 2 = 2$	$1 \times 3 = 3$	$1 \times 4 = 4$
$2 \times 1 = 2$	$2 \times 2 = 4$	$2 \times 3 = 6$	$2 \times 4 = 8$
$3 \times 1 = 3$	$3 \times 2 = 6$	$3 \times 3 = 9$	$3 \times 4 = 12$
$4 \times 1 = 4$	$4 \times 2 = 8$	$4 \times 3 = 12$	$4 \times 4 = 16$
$5 \times 1 = 5$	$5 \times 2 = 10$	$5 \times 3 = 15$	$5 \times 4 = 20$
$6 \times 1 = 6$	$6 \times 2 = 12$	$6 \times 3 = 18$	$6 \times 4 = 24$
$7 \times 1 = 7$	$7 \times 2 = 14$	$7 \times 3 = 21$	$7 \times 4 = 28$
$8 \times 1 = 8$	$8 \times 2 = 16$	$8 \times 3 = 24$	$8 \times 4 = 32$
$9 \times 1 = 9$	$9 \times 2 = 18$	$9 \times 3 = 27$	$9 \times 4 = 36$
$10 \times 1 = 10$	$10 \times 2 = 20$	$10 \times 3 = 30$	$10 \times 4 = 40$
$11 \times 1 = 11$	$11 \times 2 = 22$	$11 \times 3 = 33$	$11 \times 4 = 44$
$12 \times 1 = 12$	$12 \times 2 = 24$	$12 \times 3 = 36$	$12 \times 4 = 48$
$1 \times 5 = 5$	$1 \times 6 = 6$	$1 \times 7 = 7$	$1 \times 8 = 8$
$2 \times 5 = 10$	$2 \times 6 = 12$	$2 \times 7 = 14$	$2 \times 8 = 16$
$3 \times 5 = 15$	$3 \times 6 = 18$	$3 \times 7 = 21$	$3 \times 8 = 24$
$4 \times 5 = 20$	$4 \times 6 = 24$	$4 \times 7 = 28$	$4 \times 8 = 32$
$5 \times 5 = 25$	$5 \times 6 = 30$	$5 \times 7 = 35$	$5 \times 8 = 40$
$6 \times 5 = 30$	$6 \times 6 = 36$	$6 \times 7 = 42$	$6 \times 8 = 48$
$7 \times 5 = 35$	$7 \times 6 = 42$	$7 \times 7 = 49$	$7 \times 8 = 56$
$8 \times 5 = 40$	$8 \times 6 = 48$	$8 \times 7 = 56$	$8 \times 8 = 64$
$9 \times 5 = 45$	$9 \times 6 = 54$	$9 \times 7 = 63$	$9 \times 8 = 72$
$10 \times 5 = 50$	$10 \times 6 = 60$	$10 \times 7 = 70$	$10 \times 8 = 80$
$11 \times 5 = 55$	$11 \times 6 = 66$	$11 \times 7 = 77$	$11 \times 8 = 88$
$12 \times 5 = 60$	$12 \times 6 = 72$	$12 \times 7 = 84$	$12 \times 8 = 96$
$1 \times 9 = 9$	$1 \times 10 = 10$	$1 \times 11 = 11$	$1 \times 12 = 12$
$2 \times 9 = 18$	$2 \times 10 = 20$	$2 \times 11 = 22$	$2 \times 12 = 24$
$3 \times 9 = 27$	$3 \times 10 = 30$	$3 \times 11 = 33$	$3 \times 12 = 36$
$4 \times 9 = 36$	$4 \times 10 = 40$	$4 \times 11 = 44$	$4 \times 12 = 48$
$5 \times 9 = 45$	$5 \times 10 = 50$	$5 \times 11 = 55$	$5 \times 12 = 60$
$6 \times 9 = 54$	$6 \times 10 = 60$	$6 \times 11 = 66$	$6 \times 12 = 72$
$7 \times 9 = 63$	$7 \times 10 = 70$	$7 \times 11 = 77$	$7 \times 12 = 84$
$8 \times 9 = 72$	$8 \times 10 = 80$	$8 \times 11 = 88$	$8 \times 12 = 96$
$9 \times 9 = 81$	$9 \times 10 = 90$	$9 \times 11 = 99$	$9 \times 12 = 108$
$10 \times 9 = 90$	$10 \times 10 = 100$	$10 \times 11 = 110$	$10 \times 12 = 120$
$11 \times 9 = 99$	$11 \times 10 = 110$	$11 \times 11 = 121$	$11 \times 12 = 132$
$12 \times 9 = 108$	$12 \times 10 = 120$	$12 \times 11 = 132$	$12 \times 12 = 144$

GLOSSARY

Abstract number. A number that is used without reference to any particular kind of object or unit.

Acceptance. The written promise of the drawee of a draft to pay the draft when it is due, usually made by writing across the face of the draft the word "Accepted" with the date and the drawee's name underneath.

Account. A record of business transactions.

Account purchase. A statement in detail rendered by a commission merchant to his principal showing the cost of goods bought, the charges connected therewith, and the amount charged.

Account sales. A statement in detail rendered by a commission merchant to his principal showing the goods sold, the charges connected therewith, and the net proceeds of the sale.

Accurate interest. Interest computed by taking the exact number of days between dates and reckoning 365 days for a year.

Addends. Numbers added.

Addition. The process of uniting two or more numbers into one number.

Ad valorem duty. A duty that is a certain per cent of the cost of the goods in the country from which they are imported.

Agent. Same as *commission merchant.*

Aliquot part. A part of a number that exactly divides the number.

Altitude. The height of a figure measured on a line perpendicular to the base.

Amount. In percentage, the sum of the base and the percentage. In interest, the sum of the principal and the interest.

Analysis. Any course of reasoning by which a problem is solved.

Angle. The difference in the direction of two lines that meet.

Antecedent. The first number in a ratio.

Apex of a cone. Its vertex.

Arabic notation. Notation by use of the figures, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

Area. The number of square units that a surface contains.

Assessment. A sum levied upon the stockholders of a corporation to cover losses.

Assessors. The officers who estimate the value of the property subject to taxation.

Austrian division in decimals. The method of placing the decimal point in the quotient directly *over* the decimal point in the dividend in long division (*under* in short division) after changing the divisor to an integer (when it is not one) and making a corresponding change in the dividend.

Austrian subtraction. Subtraction by addition, that is, finding the number that added to the subtrahend gives the minuend.

Balance. The difference between the sum of the debits and the sum of the credits of an account.

Balancing an account. *Computing* the balance, *entering* it on the lesser side, and *ruling* up the account.

Bank. An establishment for the custody, loan, exchange, or issue of money, and for facilitating the transmission of funds from one place to another.

Bank discount. Interest, collected by a bank in advance, upon the value of a note at maturity.

Bank draft. A check drawn by one bank on another bank.

Base. The side on which a figure is assumed to stand. In percentage, the number of which some per cent is found.

Bill. A detailed written statement of indebtedness for goods sold or for services rendered.

Bill of exchange. Same as *bank draft.*

Board foot. A board or part of a board 1 inch (or less) in thickness and having a surface area on one side of 1 square foot.

Brokerage. The compensation of a broker.

Cancellation. The process of shortening work in division by rejecting equal factors from both dividend and divisor.

Capital stock. The amount of money pledged by the members of a corporation, or a stock company.

Cash account. A record of money paid out and money received.

Cash discount. A discount allowed for immediate payment.

Central angle. An angle whose vertex is at the center of a circle.

Check. Any method of verifying a result. An order upon a bank to pay to a depositor or his order a specified sum of money on demand.

Circle. A figure bounded by a curved line every point of which is equally distant from a point within called the *center.*

Circular cone. A solid whose base is a circle and whose surface tapers uniformly to a point, the *vertex.*

Circular cylinder. A solid bounded by a uniformly curved surface and having for its bases circles that are parallel to each other.

Circumference. The bounding line of a circle. Of a sphere, the circumference of a great circle of the sphere.

Closing an account. Balancing an account when it is settled.

Collector. The officer who collects taxes.

Commercial discount. A general name referring to either trade, cash, or time discount.

Commission. The compensation of a commission merchant.

Commission merchant. A person who buys or sells goods or transacts business for another.

Common denominator. A denominator common to two or more fractions.

Common divisor of two or more numbers. A number that is an exact divisor of those numbers.

Common fraction. Same as *fraction.*

Common multiple of several numbers. A number that is divisible by each of those numbers.

Composite number. A number that has other factors than itself and 1.

Compound denominate number. A denominate number that is composed of units of two or more related denominations.

Compound interest. Interest on the principal and its unpaid interest, combined at regular intervals.

Concrete number. A number that is used with reference to some particular kind of object or unit.

Consequent. The second number in a ratio.

Corporate bond. A corporation's formal written promise under seal to pay a certain sum of money to the purchaser, on or before a specified time, with interest at regular intervals at a fixed rate.

Corporation. A group of persons authorized by law to transact business as an individual.

Coupon bonds. Bonds to which interest coupons are attached.

Creditor. A person to whom a debt is owed.

Credits. Amounts paid.

Cube. A solid that has six equal square sides or faces.

Cube of a number. The third power of a number.

Cube root of a number. One of the three equal factors of the number.

Customs. Same as *duties*.

Debits. Amounts owed.

Debtor. A person who owes a debt.

Decimal. Same as *decimal fraction*.

Decimal fraction. A fraction that expresses tenths, or hundredths, or thousandths, etc.

Decimal point. The period written at the left of tenths.

Demand note. A note payable on the *demand* of the *holder*.

Denominate number. A concrete number in which the unit of measure is established by law or custom.

Denominator. The number in a common fraction that shows into how many equal parts the unit is divided. It is written below the line.

Diagonal. A straight line joining two opposite vertices of a polygon.

Diameter. Of a circle, a straight line passing through the center and terminating at both ends in the circumference. Of a sphere, a straight line passing through the center and terminating at both ends in the surface.

Difference. The result obtained by subtracting.

Digits. The figures that are used to represent a number.

Discount. Any deduction made from a price, debt, or amount.

Discount series. Two or more successive discounts.

Dividend. The number divided by another number.

Dividends. Profits of a corporation that are divided among the stockholders.

Division. The process of finding how many times one number contains another, or the process of separating a number into equal parts.

Divisor. The number by which another number is divided.

Domestic exchange. Exchange between two places in the same country.

Draft. A written order of one party on another to pay a specified sum of money at a certain time to a person named therein, to order, or to bearer.

Drawee. The person who is directed to pay the face of a draft.

Drawer. The *maker* of a note. The person who signs a draft.

Duties. Charges on goods imported from other countries.

Equation. An expression of the equality of two numbers or quantities.

Equator. The imaginary line around the earth running east and west halfway between the poles.

Even number. A number that is divisible by 2.

Exact divisor. A number that divides another number without a remainder.

Exact interest. Same as *accurate interest.*

Exchange. Paying debts or collecting credits in distant places without actually transferring money. The amount paid for the collection of a draft, bill of exchange, check, etc.

Exponent. A small figure written at the right of a number and a little above, to indicate how many times the number occurs as a factor.

Extremes. The first and last terms of a proportion.

Face of a note. The *sum*, written in the note, to be paid.

Face of a policy. The amount of indemnity or protection.

Factor. An exact divisor of a number.

Factoring. The process of separating a number into its factors.

Foreign exchange. Exchange between two places in different countries.

Formula. The expression of a principle, a rule, or a law in brief form by means of letters and signs.

Fraction. One or more of the equal parts of a unit.

Government bonds. Bonds issued by countries, states, cities, counties, and other political divisions of a country to raise money for various purposes of government.

Great circle of a sphere. A circle whose plane passes through the center.

Greatest common divisor of two or more numbers. The largest number that is an exact divisor of those numbers.

Gross amount of a bill. The amount before it is discounted.

Hemisphere. One of the two equal parts into which a great circle divides a sphere.

Holder of a note. The person who lawfully owns a note.

Hypotenuse of a right triangle. The side opposite the right angle.

Improper fraction. A fraction whose numerator equals or exceeds its denominator.

Index of a root. A small figure written in the opening of the root sign, indicating what root is to be taken.

Indorsement. That which is written across the back of a note, check, or draft, pertaining to its transfer or payment.

Indorsement in blank. An indorsement consisting of the name of the indorser only and guaranteeing the payment of the note, check, or draft, when due.

Indorsement in full. An indorsement in which the indorser directs payment to be made to the order of a specified person.

Insurance. Indemnity against loss or damage.

Integer. A number that stands for one or more whole units.

Interest. Money paid for the use of money.

Interest term. The time between the dates at which interest is paid.

Least common denominator. The *smallest* common denominator to which two or more fractions can be reduced.

Least common multiple of several numbers. The *least* number that is divisible by each of those numbers.

Legs of a right triangle. The sides that form the right angle.

Like numbers. Numbers that are made up of the same kind of units.

Long division. Division in which all the steps are written.

Longitude. Distance east or west of the prime meridian, measured in degrees along the equator.

Lowest terms. A fraction is expressed in *lowest terms* when its terms are prime to each other.

Maker of a note. The person who promises to pay.

Maturity of a note. The date upon which the note becomes payable.

Means. The second and third terms of a proportion.

Meridian. An imaginary line passing north and south from one pole of the earth to the other.

Metric system. A *decimal* system of weights and measures, having the meter for its principal unit.

Minuend. The number from which another number is subtracted.

Mixed decimal. A number expressed by an integer and a decimal.

Mixed number. A number expressed by an integer and a fraction.

Multiple of a number. A number that is divisible by that number.

Multiplicand. The number multiplied by another number.

Multiplication. The process of taking one number as many times as there are units in another.

Multiplier. The number by which another number is multiplied.

Negotiable note. A note that is or may be made payable to a person other than the payee.

Net amount of a bill. The gross amount less all discounts.

Net cost. Same as *net price.*

Net price. The price, or cost, after all discounts have been taken off.

Net proceeds. The sum left after the commission and other expenses have been paid.

Non-negotiable note. A note payable to the payee only.

Notation. Any method of representing numbers by characters.

Note. Same as *promissory note.*

Number. A unit or a collection of units.

Numeration. Any method of naming, or reading, numbers represented by characters.

Numerator. The number in a common fraction that shows how many parts form the fraction. It is written above the line.

Odd number. A number that is not divisible by 2.

Orders of units. The units, tens, and hundreds of each period in a number.

Parallel lines. Lines that cannot meet, however far they are extended.

Parallelogram. A four-sided plane figure whose opposite sides are parallel.

Partial payment. The payment of a part of the sum due on a note.

Partition. The process of separating a number into equal parts.

Partitive proportion. The process by which a number is separated into parts proportional to two or more given numbers.

Par value. The face value of a stock or bond.

Payee. The person to whom a note, draft, or check is to be paid.

Per cent means per *hundred*, or *hundredths*.

Percentage. The general name given to that part of arithmetic that treats of per cents. The result obtained by finding a per cent of the base.

Perimeter. The total length of the bounding lines of a polygon.

Periods of figures. Groups of three (or less) figures.

Perpendicular. A line that forms a right angle with another line.

Personal property. Movable property, such as money, bonds, cattle, etc.

Plane figure. Any portion of a plane surface bounded by lines, straight or curved.

Plane surface. A surface such that a straight line joining any two points of it lies wholly in the surface.

Policy. The contract or written agreement between an insurance company and the person protected.

Polygon. Any plane figure bounded by straight lines.

Power of a number. The product arising from using a number a certain number of times as a factor.

Premium. The price paid for insurance.

Prime factor. A factor that is a prime number.

Prime number. A number that has no factors except itself and 1.

Prime to each other. Two numbers are *prime to each other* when they have no common factor except 1.

Principal. In interest, the money for the use of which interest is paid. In commission and brokerage, the person for whom business is done.

Prism. A solid whose sides are parallelograms and whose ends, or bases, are equal polygons, parallel to each other.

Proceeds of a note. The value of a note at maturity less the bank discount.

Product. The result found by multiplying.

Promissory note. A written promise made by one person to pay to another a definite sum of money at a specified time.

Proper fraction. A fraction whose numerator is less than its denominator.

Proportion. An equality of ratios.

Pyramid. A solid whose base is a polygon and whose faces are triangles meeting at a point.

Qualified indorsement. An indorsement in which the indorser relieves himself of the responsibility for payment by writing the words, *without recourse*, over his name.

Quotient. The result obtained by dividing.

Radical sign. Same as *root sign.*

Radius. Of a circle, a straight line drawn from the center to the circumference. Of a sphere, the distance from the center to the surface.

Rate. In percentage, the numbers of hundredths found.

Rate of discount. The per cent of discount.

Rate of interest. The per cent paid for the use of the principal for *one year.*

Rate per cent. Same as *rate.*

Ratio of one number to another. The relation expressed by the quotient of the first number divided by the second.

Ratio sign is a colon (:).

Real estate. Fixed property, such as land and structures built upon it.

Receipt. A statement acknowledging the taking or receiving of anything.

Rectangle. A plane surface that is bounded by four straight lines and has four *equal* angles.

Rectangular solid. A solid that has six rectangular faces.

Reduction. The process of changing the form of any number without changing its value.

Reduction ascending. Reduction to a *higher* unit.

Reduction descending. Reduction to a *lower* unit.

Registered bonds. Bonds recorded by number, and in the name of the person owning them.

Remainder. In subtraction, same as *difference.* In division, the part of the dividend that is left when the division is inexact.

Right angle. When a straight line meets another straight line, forming two *equal* angles, each angle is a *right angle.*

Right triangle. A triangle that has a right angle.

Roman notation. Notation by the use of the seven letters I, V, X, L, C, D, M.

Root. One of the equal factors of a number.

Root sign is $\sqrt{}$. When placed over a number, it indicates that a root is to be taken.

Scale drawing. A drawing whose linear dimensions are in a certain ratio to the corresponding dimensions of the thing represented.

Shareholder. Same as *stockholder.*

Shares. The equal parts into which the stock of a corporation is divided.

Short division. Division in which the operations of multiplication and subtraction are done mentally.

Sight draft. A draft payable on demand, that is, "at sight."

Signs of aggregation. The parentheses ( ), the brackets [ ], the braces { }, and the vinculum ⁻, used to group numbers or expressions.

Similar figures. Figures of exactly the same shape differing in size.

Similar fractions. Fractions that have the same denominator.

Similar solids. Solids of exactly the same shape differing in volume.

Simple denominate number. A denominate number composed of units of one denomination only.

Specific duty. A fixed sum upon the weight or quantity of imported goods, without regard to their value or cost.

Sphere. A solid bounded by a curved surface every point of which is equally distant from a point within called the center.

Square. A plane surface that is bounded by *four equal* straight lines and has *four equal* angles. In roofing, an area of 100 square feet.

Square of a number. The second power of a number.

Square root of a number. One of the two equal factors of the number.

Statement. A summary of the debits and credits of a personal account.

Stock broker. A person whose business is to buy and sell stocks and bonds.

Stock company. Same as *corporation.*

Stockholder. A person who owns one or more shares in a corporation.

Subtraction. The process of finding how much greater one number is than another, or the process of finding what number added to one of two numbers will give the other.

Subtrahend. The number subtracted from another number.

Sum. The result obtained by adding.

Tariff. A schedule of duties on merchandise.

Taxes. Sums of money levied on persons, on property, or on incomes, for the purpose of defraying the expenses of government.

Term of discount. The number of days from the time a note is discounted to the time when it legally matures.

Terms of a bill. The conditions as to discounts and time of payment upon which goods are sold.

Terms of a fraction. The numerator and the denominator.

Terms of a ratio. The antecedent and the consequent.

Time discount. A discount made for payment within a definite time.

Time draft. A draft payable at a specified time after sight or after date.

Time note. A note payable at a specified time after date.

Trade discount. A discount from a list price.

Trapezoid. A four-sided plane figure that has only two sides parallel.

Triangle. A plane surface bounded by three straight lines.

Unit. Any one thing.

Unit of measure. The standard by which any quantity is ascertained.

Unlike numbers. Numbers that are made up of different kinds of units.

Value of a fraction. Quotient of the numerator by the denominator.

Vertex of a pyramid. The point where the triangular faces meet.

Volume. The number of cubic units that a solid contains.

INDEX

509